

CMA Final Strategic Cost Management

Volume 2
New Syllabus

"Live as if you were to die tomorrow. Learn as if you were to live forever."

Mahatma Gandhi



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Contents

Sl. No.	Chapter Name	Page No.
I.	Quantitative Techniques in Decision Making	
	1. Learning Curve	3 – 18
	2. Linear Programming	19 – 43
	3. Assignment	44 – 67
	4. Transportation	68 – 96
	5. Simulation	97 – 115
	6. Project Management	116 – 148
	7. Game Theory	149 – 167
	8. Business Application of Maxima and Minima	168 – 189
	9. Business Forecasting Models	190 – 209
	10. Introduction to Tools for Data Analytics	210 – 213
II.	Decision Making with Probability	
	Decision Making with Probability	215 – 222
III.	Theories	
	1. Kaizen Costing	225 – 227
	2. Value Analysis and Value Engineering	228 – 236
	3. Business Process Reengineering	237 – 239
	4. Lean Accounting	240 – 244
	5. Decision Making Techniques	245 – 259
	6. Uniform Costing in Profit Planning	260 – 265
	7. Quality Cost Management	
	(a) Total Quality Management	267 – 271
	(b) Praise Analysis	272 – 275
	(c) Six Sigma	276 – 279
	(d) Pareto Analysis	280 – 284
	8. Bench Marking	285 – 290
	9. Yield Management	291 – 302
IV.	Practice Questions	
	1. CVP Analysis	305 – 309
	2. Transfer Pricing	310 – 311
	3. Marginal Costing	312 – 314

4.	Decision Making	315 – 316
5.	Relevant Costing	317 – 319
6.	Target Costing	320 – 325
7.	Life Cycle Costing	326 – 327
8.	Activity Based Cost Management	328 – 332
9.	Standard Costing	333 – 340
10.	Linear Programming	341 – 364
11.	Transportation	365 – 381
12.	Assignment	382 – 400
13.	Simulation	401
14.	Network Analysis - PERT, CPM	402 – 407
15.	Learning Curve	408 – 416
V.	Case Studies	419 – 462

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Part I

Quantitative Techniques in Decision Making

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Learning Curve



Learning Objective

THEORY

1. PHASE IN LEARNING CURVE
2. AREAS OF CONSEQUENCE
3. GRAPHICAL PRESENTATION OF LEARNING CURVE
4. USES OF LEARNING CURVE
5. LIMITATIONS OF LEARNING CURVE
6. FACTORS AFFECTING LEARNING CURVE
7. THE EXPERIENCE CURVE
8. PHASES AND POSSIBLE AREAS OF APPLICATION

PRACTICAL

1. LEARNING CURVE TABLE
2. LEARNING CURVE EQUATION
3. DOUBLING THE PRODUCTION

1. APPLICABILITY AND NON APPLICABILITY

Applicability	Non - applicability
Human efforts	Automated efforts
New product / process	Existing product / process
Repetitive	Non repetitive
Regular workers	Non regular / Casual workers
Skilled / unskilled workers	Experienced workers / Experts
On Variable cost or Direct Costs	On Fixed Cost or Indirect Costs
On fixed costs - if absorption costing is followed	On fixed costs - if marginal costing is followed

2. LEARNING RATE

$$= \frac{\text{Average time or cost per unit of first } 2n \text{ units}}{\text{Average time or cost per unit of first } n \text{ units}}$$

[where n is the lot size]

- # 1. **Learning Rate + Improvement Rate = 1**
- # 2. **Higher learning rate implies lower improvement rate.**
- # 3. **Learning rate of highly experienced people or machines is 100%**

3. PREDICTION BY DOUBLE OF PRODUCTION VOLUME

Assuming a 90% learning rate

Cumulative output (units or batch)	Cumulative Average time per unit or batch	Total time (hours)	Incremental time (hours)
1	50	50	50
2	45	90	40
4	40.50	162	72
8	36.45	291.6	129.6

- #1. **We use average time per unit to estimate the cost of an order comprising of all the cumulative units or the initial order.**
- #2. **We use incremental time to estimate the cost for a repeat order.**
- #3. **Time of a specific batch E.g. 25th batch**
 - (a) Average time of first 25 batches
 - (b) Average time of first 24 batches
 - (c) Incremental Time = Time of 25th batch = (Average of first 25 × 25 batches) – (Average of first 24 × 24 batches)

4. PREDICTION AT ANY VOLUME USING LEARNING CURVE EQUATION

Preferred when the required average time or cost per unit cannot be determined by doubling the production volume.

$$Y = K \times X^S$$

Where,

K = Total time of first unit or batch or average time of first unit or batch

X = Volume (units or batches) whose average is required

S = Learning curve index = $\text{Log LCR} / \text{Log } 2$

Solve this equation by using **given values** or **log and antilog** or **by dirty power concept** in calculator

5. LEARNING CURVE THEORY [JUN'17; JUN'18; JUN'19; MTP JUN'22]

Learning Curve Theory is concerned with the idea that when a **new job, process or activity commences for the first time** it is likely that the workforce involved will not achieve maximum efficiency immediately. **Repetition** of the task is likely to make the people more confident and knowledgeable and will eventually result in a more efficient and rapid operation. Eventually the learning process will stop after continually repeating the job. As a consequence the time to complete a task will initially decline and then stabilise once efficient working is achieved. The cumulative average time per unit is assumed to decrease by a constant percentage every time that output doubles. **Cumulative average time** refers to the average time per unit for all units produced so far, from and including the first one made.

Learning is the process by which an individual acquires skill, knowledge and ability. When a new product or process is started, the performance of a worker is not at its best and learning phenomenon takes place. As the experience is gained, the performance of a worker improves, time taken per unit of activity reduces and his productivity goes up. This improvement in productivity of a worker is due to learning effect. Cost predictions especially those relating to direct labour cost must allow for the effect of learning process. This technique is a mathematical technique. It can be very much used to accurately and graphically predict cost. It is a geometrical progression, which reveals that there is steadily decreasing cost for the accomplishment of a given repetitive operation, as the identical operation is increasingly repeated. The amount of decrease is less and less with each successive unit produced. The slope of the decision curve can be expressed as a percentage. **Experience curve**, improvement curve and progress curve are other terms which can be synonymously used. Learning curve is essentially a measure of the experience gained in production of an article by an individual or organization. As more units are produced, people involved in production become more efficient than before. Each subsequent unit takes fewer man-hours to produce. The amount of improvement will differ with each type of article produced. This improvement or experience gain is reflected in a decrease in man-hours or cost.

6. GRAPHICAL PRESENTATION OF LEARNING CURVE

The learning curve (not to be confused with experience curve) is a graphical representation of the phenomenon explained by Theodore P. Wright in his "Factors Affecting the Cost of Airplanes", 1936. It refers to the effect that learning had on labour productivity in the aircraft industry, which translates into a **relation between the cumulative number of units produced (X) and the average time (or labour cost) per unit (Y)**, which resulted in a **convex downward slope**, as seen in the adjacent diagram.

There is a simple rationalisation behind all this: the more units produced by a given worker, the less time this same worker will need to produce the following units, because he will learn how to do it faster and better. Therefore, when a firm has higher cumulative volume of production, its time (or labour cost) per unit will be lower. **Wright's learning curve model** is defined by the following function:

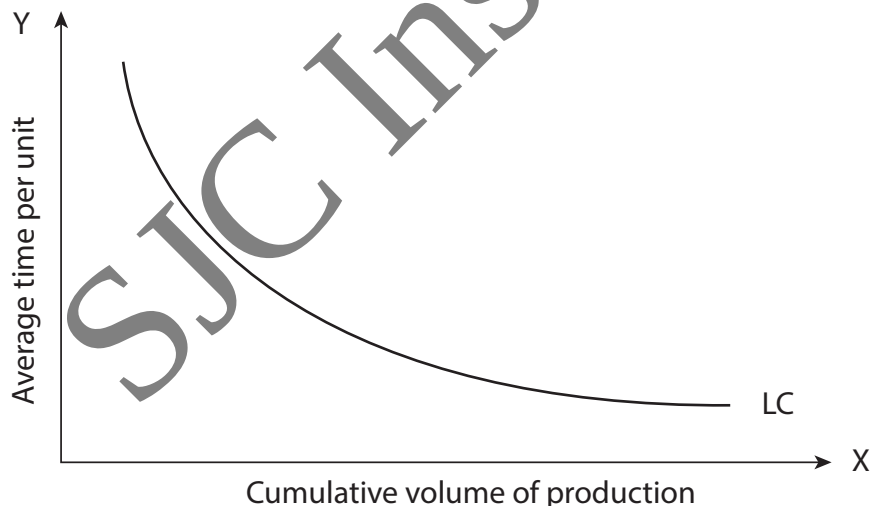
$$Y = a \times \frac{\log b}{\log 2}$$

Where: Y = average time (or labour cost) per unit

a = time (or labour cost) per unit

X = cumulative volume of production

b = learning rate (%)



Cumulative volume of production

Some important implications arise from this curve. If the time (or labour cost) per unit decreases as the cumulative output increases, this will mean that firms that have been producing more and for a longer period, will have lower average time per unit and thus dominate the market.

7. USES OF LEARNING CURVE.

Learning curve is now being widely issued in business. Some of the uses are as follows:

1. Where applicable the learning curve **suggest great opportunities** for cost reduction to be achieved by improving learning.
2. The learning curve concept **suggests a basis for correct staffing** in continuously expanding production. The curve shows that the work force need not be increased at the same rate as the prospective output. This also helps in proper production planning through proper scheduling of work; providing manpower at the right moment permitting more accurate forecast of delivery dates.
3. Learning curve concept provides a means of **evaluating the effectiveness of training programs**. **What** level of cumulative cost reduction do they accomplish? **How** does the learning curve for this group or shop compare with others? **Whether** any of the employees who lack the aptitude to meet normal learning curve should be eliminated.
4. Learning curve is frequently **used in conjunction with establishing bid price for contracts**. Usually, the bid price is based on the cumulative average unit cost for all the units to be produced for a given contract. If production is not interrupted. Additional units beyond this quantity should be costed at the increment costs incurred, and not at the previous cumulative average. If the contract agreement so provides, a **contract may be cancelled and production stopped before the expected efficiency is reached**. This would mean that the company having quoted on the basis of cumulative average unit cost is at a disadvantage because it can not reap the benefit of leaning. The contractor must provide for these contingencies so that it will be reimbursed for such loss.
5. The use of learning curve, where applicable, is **important in the working capital required**. If the requirement is based on average cumulative unit cost, the revenues from the first few units may not cover the actual expenditures. For instance, if the price was based on the average cumulative unit cost of 328 hours the first unit when produced and sold will cause a deficit of 4.72 hours (8.00 – 3.28). Provision should therefore, be made to cover the deficit of working capital in the initial stages of production.
6. As **employees become more efficient**, the rate of production increases and so more materials are needed, **the work-in-progress inventory turns over faster**, and finished goods inventory grows at an accelerated rate. A knowledge of the learning curve assists in planning the inventories of materials. **Work-in-progress, and finished goods**.
7. **Learning curve techniques** are useful in exercising control, Variable norms can be established for each situation, and a comparison between these norms and actual expenses can be made. **Specific or average** incremental unit cost should be used for this purpose.
8. The learning curve may be used for **make-or-buy** decisions especially if the outside manufacturer has reached the maximum on the learning curve. Help to calculate the sensitive rates in wage bargaining.

8. LIMITATIONS TO THE USEFULNESS OF THE LEARNING CURVE

The following points **limiting** the usefulness of learning curves should be noted:

1. The learning curve is **useful only for new operations** where machines do not constitute a major part of the production process. It is not applicable to all productions.
E.g. new and experienced workmen.
2. The learning curve **assumes** that the production will continue without any major interruptions. If for any reason the work is interrupted, the curve may be deflected or assume a new slopes
3. **Charges** other than learning may effect the learning curve. **For example**, improvement in facilities, arrangements, and equipment as well as personnel morale and performance may be factors influencing the curve. On the other hand, **negative developments** in employee attitudes may also affect the curve and reverse or retard the progress of improvement.
4. The characteristic **80 percent learning curve** as originally obtaining in the air force industry in U.S. A. has been usually accepted as the percentage applicable to all industries. Studies show that there cannot be a **unique percentage** which can be universally applied.

9. FACTORS AFFECTING LEARNING CURVE [JUN'18; JUN'19]

1. While **pricing for bids**, general tendency is to set up a **very high initial labour cost** so as to show a high learning curve. This shows that learning curve is useless and sometimes misleading.
2. The **method of production**, i.e. whether it is labour oriented or machine oriented influences the slop of the learning.
3. When **labour turnover rate** is high management has to train new workers frequently. In such situations the company may never reach its maximum efficiency potential. One of the important requisites of the learning curve concept is that there should be uninterrupted flow of work. The fewer the interruptions, the greater will be the improvement in efficiency.
4. **Changes in a product or in the methods** of production, designs, machinery, or the tools/ used affect the slope of the learning curve. All these have the effect of starting learning a fresh because of new conditions If the changes are frequent, there may be no learning at all.
5. Also **other factors influencing the learning curve** are labour strikes, lock outs and shut downs due to other cause also/affect the learning curve. In each such case there is interruption in the progress of learning.

As far as possible the effects of above factors should be carefully separated from the data used to establish the curve. The effects of these factors must also be separated from the actual costs used to measure the performance. Unless this is done analysis of the projected cost or the actual cost will not be meaningful.

10. THE EXPERIENCE CURVE

The more experience a firm has in producing a particular product, the lower its costs.

The **experience curve** is an idea developed by the **Boston Consulting Group (BCG)** in the mid-1960s. Working with a leading manufacturer of semiconductors, the consultants noticed that the company's unit cost of manufacturing fell by about 25% for each doubling of the volume that it produced. This relationship they called the **experience curve**: the more experience a firm has in producing a particular product, the lower are its costs. **Bruce Henderson**, the founder of BCG, put it as follows:

Costs characteristically decline by 20-30% in real terms each time accumulated experience doubles. This means that when **inflation** is factored out, costs should always decline. The decline is fast if growth is fast and slow if growth is slow.

There is **no fundamental economic law** that can predict the existence of the experience curve, even though it has been shown to apply to industries across the board. Its truth has been proven inductively, not deductively. And if it is true in service industries such as investment banking or legal advice, the lower costs are clearly not passed on to customers.

By itself, the curve is not particularly **earth shattering**. Even when BCG first expounded the relationship, it had been known since the second world war that it applied to direct labour costs. Less labour was needed for a given output depending on the experience of that labour. In aircraft production, for instance, labour input decreased by some 10–15% for every doubling of that labour's experience.

The strategic implications of the experience curve came closer to shattering earth. For if costs fell (fairly predictably) with experience, and if experience was closely related to market share (as it seemed it must be), then the competitor with the biggest market share was going to have a big cost advantage over its rivals.

11. PHASES AND POSSIBLE AREAS OF APPLICATION [JUN'19]

Learning Curve Theory is concerned with the idea that when a new job, process or activity commences for the first time it is likely that the workforce involved will not achieve maximum efficiency immediately. Repetition of the task is likely to make the people more confident and knowledgeable and will eventually result in a more efficient and rapid operation. Gradually the learning process will stop after continually repeating the job. As a consequence the time to complete a task will initially decline and then stabilise once efficient working is achieved.

Phases in Learning Curve

The learning curve will pass through three different phases:

In the first phase, there will be gradual increase in production rate until the maximum expected rate is reached and this phase is generally steep.

In the second phase, the learning rate will gradually deteriorate because of the limitations of equipment.

In the third phase, the production rate begins to decrease due to a reduction in customer requirements and increase in costs.

Learning curve is essentially a measure of the experience gained in production of an article by an organization. As more and more units are re-produced, workers involved in production become more efficient than before.

The learning curve ratio can be calculated with the help of the following formula:

Learning curve ratio = Average labour cost of first 2 units / Average labour cost of first units

Following are the possible areas of application of Learning Curve:

1. Learning curve suggests **great opportunities** for cost reduction to be achieved by improving learning.
2. The learning curve concept suggests a **basis for correct staffing** in continuously expanding production. The curve shows that the work force need not be increased at the same rate as the prospective output.
3. Learning curve concept provides a **means of evaluating the effectiveness** of training programs.
4. Learning curve is frequently used in conjunction with establishing bid price for contracts.
5. Learning curve is applicable in relation to the **working capital** required. If the requirement is based on **average cumulative unit cost**, the revenues from the first few units may not cover the actual expenditures.
6. It **affects the rate of production** by increasing.
7. Learning curve **techniques are useful in exercising control**. Specific or average incremental unit cost should be used for this purpose.
8. The learning curve **may be used for make-or-buy decisions** especially if the outside manufacturer has reached the maximum on the learning curve.

Q1

Ex. Book No.

Pg. No.

CMA Mat; RTP Jun'19

The learning curve as a management accounting has now become or going to become an accepted tool in industry, for its applications are almost unlimited. When it is used correctly, it can lead to increase business and higher profits; when used without proper knowledge, it can lead to lost business and bankruptcy.

State precisely:

- (i) Your understanding of the learning curve;
 - (ii) The theory of learning curve;
 - (iii) The areas where learning curves may assist in management accounting; and
 - (iv) Illustrate the use of learning curves for calculating the expected average units cost of making.
- (a) 4 machines (b) 8 machines

Using the data below:

Data:

Direct Labour need to make first machine	= 1,000 hrs.
Learning curve	= 90%
Direct Labour cost	= ₹15/- per hour.
Direct materials cost	= ₹1,50,000
Fixed cost for either size orders	= ₹60,000

Reference	What's New
Learning curve equation	

Answer

Statement showing computation of cost of making 4 machines & 8 machines:

No of machines	Average time Hours	Labour cost ₹	Material ₹	Fixed cost ₹	Total ₹
1	1,000	15,000	1,50,000	60,000	2,25,000
2	900	13500	1,50,000	30,000	1,93,500
4	810	12150	1,50,000	15,000	1,77,150
8	729	10935	1,50,000	7,500	1,68,435

Average cost of making 4 machines ₹ 1,77,150 Average cost of making 8 machines ₹ 1,68,435

Q2

Ex. Book No.

Pg. No.

CMA Mat; MTP Jun'17

A firm received an order to make and supply eight units of standard product which involves intricate labour operations. The first unit was made in 10 hours. It is understood that this type of operations is subject to 80% learning rate. The workers are getting a wages rate of ₹ 12 per hour.

- (i) **What** is the total time and labour cost required to execute the above order?
- (ii) If a repeat order of 24 units is also received from the same customer, **what** is the labour cost necessary for the second order?

Reference

Doubling the Production

What's New

Answer

80% Learning Curve results are given below:

Production (Units)	Cumulative Average Time (hours)	Total Time (hours)
1	10	10
2	8	16
4	6.4	25.6
8	5.12	40.96
16	4.096	65.54
32	3.2768	104.86

Labour time required for first eight units = 40.96 hours

Labour cost required for 8 units = 40.96 hours x ₹ 12/hr = ₹ 491.52

Labour time for 32 units = 104.86 hours

Labour time for first eight units = 40.96 hours

Labour time required for 2nd order for 24 units = 63.90 hours

Labour cost for 24 units = 63.90 hours x ₹ 12/hr = ₹ 766.80

Q 3

Ex. Book No.

Pg. No.

CMA Mat

The usual learning curve model is $Y = ax^b$ where

Y is the average time per unit for x units.

a is the time for first unit

x is the cumulative number of units

b is the learning coefficient and is

equal to $\frac{\log 0.8}{\log 2} = -0.322$ for a learning rate of 80%

Given that $a = 10$ hours and learning rate 80%.

You are **required** to **Calculate**:

- (i) The average time for 20 units.
- (ii) The total time for 30 units.
- (iii) The time for units 31 to 40.

Given that $\log 2 = 0.301$, Antilog of 0.5811 = 3.812
 $\log 3 = 0.4771$, Antilog of 0.5244 = 3.345.
 $\log 4 = 0.6021$, Antilog of 0.4841 = 3.049.

Reference

Learning curve equation

What's New

Answer

- (i) $Y = AX^b$

$$Y = 10(20)^{-0.322}$$

Taking log on both sides

$$\log y = \log 10 + \log 20^{(-0.322)}$$

$$\log y = \log 10 - (0.322) \log 20$$

$$= 1 - (0.322) \log 20$$

$$= 1 - (0.322) \times (1.3010)$$

$$= 1 - 0.41892 = 0.5811$$

$$\log y = 0.5811$$

$$Y = \text{Anti log } (0.5811) = 3.812 \text{ hrs (average time)}$$

$$\text{Total Time} = 3.812 \times 20 = 76.24 \text{ hours}$$

$$(ii) \text{ Log } y = \log 10 + \log 30^{(-0.322)}$$

$$\text{Log } y = 1 - (0.322) \times (1.4771)$$

$$= 1 - (0.4756) = 0.5244$$

$$Y = \text{anti log } (0.5244) = 3.345 \text{ hrs (average time)}$$

$$\text{Total time} = 3.345 \times 30 = 100.35 \text{ hrs}$$

$$(iii) \text{ Log } y = \log 10 + \log 40^{(-0.322)}$$

$$= 1 - (0.322) \times (1.6021) \text{ Log } y = 0.4841$$

$$Y = \text{anti log } (0.4841) = 3.049 \text{ hrs}$$

$$\text{Total time} = 40 \times 3.049 = 121.96 \text{ hrs}$$

$$\text{Time from 31 to 40 units} = 121.96 - (100.35) = 21.61 \text{ hrs}$$

Q4

Ex. Book No.	Pg. No.

CMA Mat

Z.P.L.C experience difficulty in its budgeting process because it finds it necessary to qualify the learning effect as new products are introduced.

Substantial product changes occur and result in the need for retraining.

An order for 30 units of a new product has been received by Z.P.L.C So far, 14 have been completed; the first unit required 40 direct labour hours and a total of 240 direct labour has been recorded for the 14 units. The production manager expects an 80% learning effect for this type of work.

The company use standard absorption costing. The direct costs attributed to the centre in which the unit is manufactured and its direct materials costs are as follows:

	₹
Direct material	30.00 per unit.
Direct Labour	6.00 per hour.
Variable overhead	0.50 per direct labour hour.
Fixed overhead	6,000 per four-week operating period.

There are ten direct employees working a five-day week, eight hours per day. Personal and other downtime allowances account for 25% of total available time.

The company usually quotes a four-week delivery period for orders.

You are **required** to:

- (i) **Determine** whether the assumption of an 80% learning effect is a reasonable one in this case, by using the standard formula $y = ax^b$

Where Y = the cumulative average direct labour time per unit (productivity)

a = the average labour time per unit for the first batch.

x = the cumulative number of batches produced.

b = the index of learning.

- (ii) **Calculate** the number of direct labour hours likely to be required for an expected second order of 20 units.
- (iii) Use the cost data given to **produce** an estimated product cost for the initial order, examine the problems which may be created for budgeting by the presence of the learning effect.



Reference

Learning curve equation

What's **New**

Answer

- (i) **Total time taken to produce 14 units**

$$Y = ab^x$$

$$Y = 40 (14)^{-0.322}$$

$$\log Y = \log 40 + \log 14^{-0.322}$$

$$\log Y = \log 40 - (0.322) \log 14$$

$$= 1.60221 - (0.322) \times 1.1461$$

$$= 1.60221 - 0.3690 = 1.233$$

$$Y = \text{Antilog } (1.233) = 17.14$$

$$\text{Total time} = 17.14 \times 14 = 239.96$$

$$= 240 \text{ hours}$$

It is true that learning ratio 80% is effective.

- (ii) **30 units**

$$Y = 40 (30)^{-0.322} = 13.380 \text{ hours (Average time)}$$

50 units

$$Y = 40 (50)^{-0.322} = 11.35 \text{ hours (Average time)}$$

$$\text{Total time for 30 units} = 13.38 \times 30 = 401.4 \text{ hours}$$

$$\text{Total time for 50 units} = 11.35 \times 50 = 567.5 \text{ hours}$$

$$\text{Time taken for 20 units from 31 to 50 units } (567.5 - 401.4) = 166.1 \text{ hours}$$

(iii)

Man hours = $10 \times 8 \times 5 \times 4$	=	1,600
(-) down time	=	400
		1,200

Fixed Cost per hour = $6,000/1,200 = ₹ 5$

Computation of total cost for the initial order

		₹
Material (30 x 30)	=	900.0
Labour (401.4 x 6)	=	2,408.4
Variable Overheads (0.5 x 401.4)	=	200.7
Fixed Overheads (5 x 401.4)	=	2,007.0
	=	5,516.1

Q5

Ex. Book No.

Pg. No.

RTP Jun'18

An electronics firm which has developed a new type of Fire-Alarm System has been asked to quote for a prospective contract.

The customer requires separate price quotations for each of the following possible orders -

Order	First	Second	Third
Number of Fire-Alarm Systems	100	60	40

The Firm estimates the following cost per Unit for the first order -

Direct Materials	₹ 500
Direct Labour: Department A (Highly automatic)	20 hours at ₹ 10 per hour
Department B (Skilled Labour)	40 hours at ₹ 15 per hour
Variable Overheads	20% of Direct Labour
Fixed Overheads absorbed: Department A	₹ 8 per hour
Department B	₹ 5 per hour

Determine a price per unit for each of the orders, assuming the Firm uses a mark-up of 25% on total costs and allows for as 80% Learning Curve. **Extract** from 80% Learning Curve table -

X	1.0	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Y%	100.0	91.7	89.5	87.6	86.1	84.4	83.0	81.0	80.0

X represents the cumulative total volume produced to date expressed as a multiple of the initial order.

Y is the Learning Curve Factor, for a given X value, expressed as a percentage of the cost of the initial order.

Reference**Learning curve table****What's New****Answer****1. Computation of Selling Price per unit of First Order (100 units)**

Particulars	Computation	₹
Direct Materials	100 units × 500 per unit	50,000
Direct Labour: Department A	100 units × 20 hours × 10 per hour	20,000
Department B	100 units × 40 hours × 15 per hour	60,000

Particulars	Computation	₹
Variable OH	20% of (20,000 + 60,000)	16,000
Fixed OH:		
Department A	100 units × 20 hours × 8 per hour	16,000
Department B	100 units × 40 hours × 5 per hour	20,000
Total Cost	1,82,000 × 25%	1,82,000
Add: Profit at 25% of cost		45,500
Sale Value		2,27,500
Sale Price per unit	2,27,500 ÷ 100 units	2,275

Note: Department A is highly automatic C and there is no Learning Effect in respect of machine/ mechanical operations. Hence, Learning Effect is applicable for Department B only, where Skilled Labour is involved.

2. Computation of Selling Price per unit of Second Order (60 units) (See Note below for Learning Effect)

Particulars	Computation	₹
Direct Materials	60 units × 500 per unit	30,000.00
Direct Labour: Department A	60 units × 20 hours × 10 per hour	12,000.00
Department B	1,510.40 hours × 15 per hour	22,656.00
Variable OH	20% of (12,000 + 22,656)	6,931.20
Fixed OH: Department A	60 units × 20 hours × 8 per hour	9,600.00
Department B	1,510.40 hours × 5 per hour	7,552.00
Total Cost	88,739.20 × 25%	88,739.20
Add: Profit at 25% of cost		22,184.80
Sale Value		1,10,924.00
Sales Price Per Unit	1,10,924.00 ÷ 60 units	1,848.73

Learning Effect: Since Second Order is for 60 units the relevant "X" factor is 1.6 (cumulative quantity = 160 units) and "Y" factor is 86.1%. Hence, the additional time taken for the Second Order is determined as under -

Total Time for First and Second Order	$(40 \text{ hrs. p.u} \times 86.1\%) \times 160 \text{ units}$	5,510.40 hours.
Less: Total Time for First Order only	$(40 \text{ hrs. p.u} \times 100 \text{ units})$	4,000.00 hours.
Additional Time for Second Order		1,510.40 hours.

3. **Computation of Selling Price per unit of Second Order (40 units)** (See Note below for Learning Effect)

Particulars	Computation	₹
Direct Materials	40 units $\times$ 500 per unit	20,000.00
Direct Labour : Department A	40 units $\times$ 20 hours $\times$ 10 per hour	8,000.00
Department B	889.60 hours $\times$ 15 per hour	13,344.00
Variable OH	20% of (8,000 + 13,344)	4,268.80
Fixed OH: Department A	40 units $\times$ 20 hours $\times$ 8 per hour	6,400.00
Department B	889.60 hours $\times$ 5 per hour	4,448.00
Total Cost		56,460.80
Add: Profit at 25% of cost	$56,460.80 \times 25\%$	14,115.20
Sale Value		70,576.00
Sales Price Per Unit	$70,576.00 \div 40 \text{ units}$	1,764.40

Learning Effect: Since Third Order is for 40 units, the relevant "X" factor is 2.0 (Cumulative quantity = 200 units) and "Y" factor is 80%. Hence, the additional time taken for the Third Order is determined as under -

Cumulative time upto Third Order =	$(40 \text{ hours p. u} \times 80\%) \times 200 \text{ units}$	= 6,400.00 hours.
Less: Cumulative time upto Second Order =	(as calculated earlier)	= 5,510.40 hours.
Additional Time for Third Order		= 889.60 hours

Linear Programming



Learning Objective

THEORY

1. APPLICATION AREAS OF LINEAR PROGRAMMING
2. FORMULATION OF LINEAR PROGRAMMING PROBLEMS
3. LIMITATION OF LINEAR PROGRAMMING

PRACTICAL

1. FORMULATION
2. FORMULATION USING SLACK
3. DUAL PROBLEM

1. CONDITIONS

Multiple Products and Multiple Constraints

Maximisation / Minimisation Objective

2. FLOW OF SOLVING

Original Problem ► Formulation ► Solution ► Interpretation

3. SOLUTION METHOD

Graphical method, Simplex Method

4. DEFINITION OF LINEAR PROGRAMMING

According to Kohlar "A method of planning and operation involved in the construction of a model of a real situation" containing the following elements: (a) variables representing the available choices, and (b) mathematical expressions

- (i) relating the variables to the controlling conditions, and
- (ii) reflecting the criteria to be used in measuring the benefits derivable from each of the several possible plans, and
- (iii) establishing the objective. The method may be so devised as to ensure the selection of the best of a large number of alternatives "

Samuelson, Dorfman and Solow defines LP as **"The analysis of problems in which a linear function of a number of variables is to be maximized (or minimized) when those variables are subject to a number of restraints in the form of linear inequalities"**.

In the words of Loomba, **"LP is only one aspect of what has been called a system approach to management wherein all programmes are designed and evaluated in terms of their ultimate affects in the realization of business objectives"**.

5. LIMITATIONS OF LINEAR PROGRAMMING [MTP DEC'21]

Although linear programming is a very **useful technique for solving optimization problems**, **there are certain important limitations** in the application of linear programming. Some of these are discussed below:

1. Firstly, the linear programming models can be applied only in those situations where the constraints and the objective function can be stated in terms of linear expressions.

2. In linear programming problems, coefficients in the objective function and the constraint equations must be completely known and they should not change during the period of study.
3. Yet another important limitation of linear programming is that it may give fractional valued answers.
4. Linear programming will fail to give a solution if management have conflicting multiple goals.
5. Linear programming problem requires that the total measure of effectiveness and total resource usage resulting from the joint performance of the activities must equal the respective sums of these quantities resulting from each activity being performed individually.
6. Many real-world problems are so complex, in terms of the number of variables and relationships constrained in them, that they tax the capacity of even the largest computer.
7. **Other limitations of LP includes:**
 - Does not take into consideration the effect of time and uncertainty.
 - Parameters appearing in the model are assumed to be constants but in real-life situations they are frequently neither known nor constants.

6. APPLICATION AREAS OF LINEAR PROGRAMMING

In practice linear programming has proved to be one of the most widely used technique of managerial decision making in business, industry and numerous other fields.

1. Industrial Applications:

Linear programming is extensively used to solve a variety of industrial problems. In each of these applications, the general objective is to determine a plan for production and procurement in the time period under consideration. It is necessary to satisfy all demand requirements without violating any of the constraints. **Few examples** of industrial applications are as follows:

- (a) Product Mix-Problem.
- (b) Production Scheduling.
- (c) Production Smoothing Problem.
- (d) Blending Problems.
- (e) Transportation Problems.
- (f) Production distribution problems.
- (g) Trim Loss.
- (h) Linear programming is also used by oil refineries to determine the optimal mix of products to be produced by the refinery during a given period.

- (i) **Communication Industry:** LP methods are used in solving problems involving facilities for transmission, switching, relaying etc.
- (j) **Rail Road Industry:** An LP model for optimal programming of railway freight, and train movements has been formulated to handle scheduling problems as found at large terminal switching rail points.

2. **Management Applications:**

- (a) Portfolio Selection.
- (b) Financial Mix Strategy.
- (c) Profit Planning.
- (d) Media Selection.
- (e) Travelling Salesmen Problem.
- (f) Determination of equitable salaries.
- (g) Staffing problem.

3. **Miscellaneous Applications:**

The additional application of Linear Programming are as follows:

- (a) Form planning.
 - The particular crops to be grown or cattle to keep during a period
 - The average to be devoted to each, and
 - The particular production methods to be used.
- (b) Airline routine.
- (c) Administration, Education and Politics have also employed linear programming to solve their problems.
- (d) **Diet Problems:** The diet problem, one of the earliest applications of linear programming was originally used by hospitals to determine the most economical diet for patients.

4. **Administrative applications of Linear Programming:**

Linear programming can be used for administrative applications. Administrative applications of Linear Programming are concerned with optimal usage of resources like men, machine and material.

5. **Non-Industrial applications of linear programming:**

Linear programming techniques/tools can be applied in the case of non-industrial applications as well. **Examples** of the use of L.P techniques for non-industrial applications are given below:

- Agriculture.
- Environmental Protection.

- Urban Department.
- Facilities Location.

6. Further applications of Linear Programming are:

- In structural design for maximum product.
- In balancing assembly lines.
- In scheduling of a military tanker fleet.
- In determining which parts to make and which to buy to obtain maximum profit margin.
- In selecting equipment and evaluating methods improvements that maximize profit margin.
- In planning most profitable match of sales requirements to plant capacity that obtains a fair share of the market.
- In design of optimal purchasing policies.

7. FORMULATION OF LINEAR PROGRAMMING PROBLEM

The formulation of linear programming problem as a mathematical model involves the following basic steps:

Step 1:	Find the key-decision to be made from the study of the solution. (In this connection, looking for variables helps considerably).
Step 2:	Identify the variables and assume symbols $x_1, x_2, \dots$ for variable quantities noticed in step 1.
Step 3:	Express the possible alternatives mathematically in terms of variables. The set of feasible alternatives generally in the given situation is: $[(x_1, x_2); x_1 \geq 0, x_2 \geq 0]$
Step 4:	Mention the objective quantitatively and express it as a linear function of variables.
Step 5:	Express the constraints also as linear equalities/inequalities in terms of variables.

Notes: Use slack variables for formulation as per simplex in constraints of type $x \leq a$ as $x + s_1 = a$. Neutralise Slack variables in objective function.

Q1

Ex. Book No.

Pg. No.

MTP Dec'17

Four products A, B, C and D have ₹5, ₹7, ₹3 and ₹9 profitability respectively. First type of material (limited supply of 800 kg) is required by A, B, C, and D at 4 kg, 3 kg, 8 kg, and 2 kg, respectively per unit. Second type of material has a limited supply of 300 kg, and is for A, B, C, and D at 1 kg, 2 kg, 0 kg, and 1 kg per unit. Supply of other types of material consumed is unlimited. Machine hours available are 500 hours and requirements are 8, 5, 0, and 4 hours for A, B, C, and D each per unit. Labour hours are limited to 900 hours and requirements are 3, 2, 1 and 5 hours for each unit?

Formulate this as Linear Programming Problem. You are not required to solve the LPP.



Reference

Formulation

What's **New**

Variables

Answer

	Material I	Material II	Machine hours	Labour Hours
A	4	1	8	3
B	3	2	5	2
C	8	0	0	1
D	2	1	4	5
Max. availability	800	300	500	900

Let X_1 , X_2 , X_3 and X_4 are the quantity produced for products A, B, C and d respectively.

The objective function is –

$$\text{Max. } Z = 5X_1 + 7X_2 + 3X_3 + 9X_4$$

Subject to constraints

$$4X_1 + 3X_2 + 8X_3 + 2X_4 \leq 800 \text{ (Material I constraint)}$$

$$X_1 + 2X_2 + 0X_3 + X_4 \leq 300 \text{ (Material II constraint)}$$

$$8X_1 + 5X_2 + 0X_3 + 4X_4 \leq 500 \text{ (Machine hours constraint)}$$

$$3X_1 + 2X_2 + X_3 + 5X_4 \leq 900 \text{ (Labour hours constraint)}$$

And non-negativity constraints

$$X_1, X_2, X_3, X_4 \geq 0$$


Q2

Ex. Book No.

Pg. No.

MTP Dec'18

SMC has three Departments – Assembly, Painting and Packing with the capability of making three types of Almirah. An Almirah of Type I requires one hour of Assembly, 40 minutes of Painting and 20 minutes of Packing time respectively. Similarly, Type II requires 80 minutes, 20 minutes and one hour respectively. The Type III requires 40 minutes each of Assembly, Painting and Packing time. The total time available at Assembly, Painting and Packing Departments are 600 hours, 400 hours and 800 hours respectively.

The unit profits for types I, II and III are ₹40, 80 and 60 respectively.

Formulate the problem as a LPP.

[10]

**Reference****Formulation**What's **New****Hours per Unit**

Answer

Let X_1 , X_2 and X_3 be the number of units of Type I, Type II and Type III Almirah respectively. The data given in the question can be converted into a matrix form as under –

Particulars	Type I	Type II	Type III	Time Available
Assembly Hours per unit	1	4/3	2/3	600
Painting Hours per unit	2/3	1/3	2/3	400
Packing Hours per unit	1/3	1	2/3	800
Profit per unit	₹40	₹80	₹60	

The LPP is given as under -

Maximise Revenue $Z = 40 X_1 + 80 X_2 + 60 X_3$

Subject to:

$X_1 + 4/3 X_2 + 2/3 X_3 \leq 600$ (Assembly Time Condition)

$2/3 X_1 + 1/3 X_2 + 2/3 X_3 \leq 400$ (Painting Time Condition)

$1/3 X_1 + X_2 + 2/3 X_3 \leq 800$ (Packing Time Condition)

$X_1, X_2, X_3 \geq 0$ (Non-Negativity Assumption)

Q3

Ex. Book No.

Pg. No.

CMA Mat

A company possesses two manufacturing plants each of which can produce three products X, Y and Z from a common raw material. However, the proportions in which the products are produced are different in each plant and so are the plant's operating costs per hour. Data on production per hour and costs are given below, together with current orders in hand for each product.

	Product			Operating cost/ hour in ₹
	X	Y	Z	
Plant A	2	4	3	9
Plant B	4	3	2	10
Orders on hand	50	24	60	

You are **required** to use the simplex method to find the number of production hours needed to fulfill the orders on hand at minimum cost.

(Only formulation is required)



Reference

Formulation

What's New

Minimisation

Answer

Let a be no. of hours of plant A in use

Let b be no. of hours of plant B in use

Objective function:

$$\text{Min } Z = 9a + 10b$$

Subject to constraints:

$$2a + 4b \geq 50$$

$$4a + 3b \geq 24$$

$$3a + 2b \geq 60$$

And $a, b \geq 0$

Q4

Ex. Book No.

Pg. No.

CMA Mat

A firm manufactures and sells two products Alpha and Beta. Each unit of Alpha requires 1 hour of machining and 2 hours of skilled labour, whereas each unit of Beta uses 2 hours of machining and 1 hour of labour. For the coming month the machine capacity is limited to 720 machine hours and the skilled labour is limited to 780 hours. Not more than 320 units of Alpha can be sold in the market during a month. Unit contribution from Alpha is ₹6 and from Beta is ₹4.

Develop a suitable model that will enable determination of the optimal product mix. **Reformulate** Using Slack Variables.

Reference

Formulation

What's New

Slack Variables

Answer

Products	Machining	Skilled Labour	Contribution
Alpha	1 hr	2 hr	6/-
Beta	2 hr	1 hr	4/-
Available hours	720 hr	780 hr	

Let x_1 be the no. of units of Alpha produced x_2 be the no. of units of Beta produced.

Objective function:

$$\text{Max. } Z = 6x_1 + 4x_2$$

Subject to constraints

$$x_1 + 2x_2 \leq 720$$

$$2x_1 + x_2 \leq 780$$

$$x_1 \leq 320 \text{ and}$$

$$x_1, x_2 \geq 0$$

Reformulation using slack variable

$$x_1 + 2x_2 + S_1 = 720$$

$$2x_1 + x_2 + S_2 = 780$$

$$x_1 + S_3 = 320$$

$$x_1, x_2, S_1, S_2, S_3 \geq 0$$

$$\text{Max. } Z = 6x_1 + 4x_2 + 0.S_1 + 0.S_2 + 0.S_3$$

Q5

Ex. Book No.

Pg. No.

CMA Mat; MTP Jun'17

A Factory manufactures 3 products which are processed through 3 different production stages. The time required to manufacture one unit of each of the three products and the daily capacity of the stages are given in the following table:

Stage	Time/unit in minutes			Stage capacity (minutes)
	Product 1	Product 2	Product 3	
1	1	2	1	430
2	3	-	2	460
3	1	4	-	420
Profit/unit	₹3	₹2	₹5	

Reformulate using slack variable



Reference

Formulation

What's New
Simplex

Answer

Let x_1 be the no. of units of product 1

Let x_2 be the no. of units of product 2

Let x_3 be the no. of units of product 3

Objective function:

$$\text{Max } Z = 3x_1 + 2x_2 + 5x_3$$

Subject to constraints:

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_3 \leq 460$$

$$x_1 + 4x_2 \leq 420$$

$$\text{And } x_1, x_2, x_3 \geq 0$$

Reformulation

$$x_1 + 2x_2 + x_3 + S_1 = 430$$

$$3x_1 + 2x_3 + S_2 = 460$$

$$x_1 + 4x_2 + S_3 = 420$$

$$\text{Max } Z = 3x_1 + 2x_2 + 5x_3 + 0.S_1 + 0.S_2 + 0.S_3$$

Q 6

Ex. Book No.

Pg. No.

Jun'18; MTP Dec'19

The management of SAB Ltd. has suggested that a linear programming model might be used for selecting the best mix of five possible products — A, B, C, D and E. The following information are available:

	Per Unit of Product				
	A	B	C	D	E
Selling Price (₹):	96	84	76	62	54
Costs (₹):					
Material	30	28	32	30	32
Direct Labour	36	32	12	8	8
Fixed Overhead	18	16	6	4	4
Total Costs	84	76	50	42	44

Expected maximum unit demand per week for each product at the prices indicated:

A	B	C	D	E
3,000	24,000	1,800	1,200	1,200

Cost of material includes a special component which is in short supply. It costs ₹ 6 per unit. Only 11,600 units are available to the company during the week. The number of units of the special component needed for a unit of each product is:

A	B	C	D	E
2	1	4	3	6

The management of SAB Ltd. has ruled that expenditure on materials must not exceed a sum of ₹ 60,000.

All other resources are freely available in sufficient quantities for planned need.

Formulate a linear programming model stating clearly the criterion you use.

[6]

Reference Formulation	What's New

Answer

	A	B	C	D	E
Selling Price	96	84	76	62	54
Variable Cost	66	60	44	38	40
Contribution	30	24	32	24	14

Let a, b, c, d, e be the number of units respectively of A,B,C,D and E to be produced. Objective function: **Maximise contribution:** $Z = 30a + 24b + 32c + 40d + 14e$

Subject to:

Demand Constraint

a	≤	3,000
b	≤	2,400
c	≤	1,800
d	≤	1,200
e	≤	1,200

Assumed that FOH is an irrelevant cost

Q7

Ex. Book No.

Pg. No.

MTP Jun'22

The manager of an oil refinery must decide on the optimum mix of 2 possible blending processes of which the input and output per production runs are as follows:

Process	Input		Output	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	6	4	6	9
2	5	6	5	5

The maximum amount available of crude A and B are 250 units and 200 units respectively. Market demand shows that at least 150 units of gasoline X and 130 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are ₹ 4 and ₹ 5 respectively.

- Applying Linear Programming, **formulate** the objective function.
- What** are the constraints in the LP Formulation?

Reference

Formulation

What's **New**

Constraints

Answer

N1 Analysis

	P I	P II	
Input per run			max
Crude A	6	5	200
Crude B	4	6	200
O/P per run			min
Gasoline X	6	5	150
Gasoline Y	9	5	130
Pft/Run	4	5	

N 2

Let the number of runs produced of P_1, P_2 be X_1, X_2 runs respectively

Formulation of LP

Objective function : Maximise $Z = 4x_1 + 5x_2$

Subject to constraints :

(1) **I/P** : Crude A : $6x_1 + 5x_2 \leq 250$

Crude B : $4x_1 + 6x_2 \leq 200$

(2) **O/P** : Gasoline X : $6x_1 + 5x_2 \geq 150$

Crude B : $9x_1 + 5x_2 \geq 130$

NNA : $x_1, x_2 \geq 0$

Q 8

Ex. Book No.

Pg. No.

Dec'18

Coffee provider is made by a shop by blending different flavours of coffee seeds. 520 gms of Plantation A seeds and 510 gms of Plantation B seeds are ground to yield 1,000 gms of Special Blend power. 520 gms of Peaberry seeds and 560 gms of Plantation A seeds are ground to yield 1,050 gms of Special Peaberry powder. 500 gms of Plantation B seeds and 510 gms of Robusta seeds are ground to get 980 gms of Normal Blend powder. The contribution per kg of Special Blend, Special Peaberry and Normal Blend are Rs. 100, 120 and 140 respectively. The following stock are available for the production period:

Plantation A : 200 kgs ; **Plantation B** : 300 kgs ; **Peaberry** : 250 kgs ; **Robusta** : 51 kgs. Grinding capacity on a total is limited to 500 kgs of output in a production period.

Required :

Formulate the above as a linear programme with the objective to maximize contribution. Identify the variables and give the constraints. (Consider 1,000 gms = 1 kg). [8]

Reference Formulation	What's New

Answer

Let x, y and z be the no. of kgs of Special Blend, Special Peaberry and Normal Blend respectively to be produced in the production period.

Then, Objective function : **Max Z = 100x + 120y + 140z** The following are resources per kg of output :

	x	Y	Z	Availability
Plantation A	520 / 1,000	560/1,050		200
Plantation B	510 / 1,000		500/980	300
Peaberry		520/1,050		250
Robusta			510/980	51

Resource constraint :

$$\frac{520}{1,000}x + \frac{560}{1,050}y \leq 200$$

$$\frac{510}{1,000}x + \frac{500}{980}z \leq 300$$

$$\frac{520}{1,050}y \leq 250$$

$$\frac{510}{980}z \leq 51$$

Production Capacity : x + y + z ≤ 500

x, y, z ≥ 0

Q9

Ex. Book No.	Pg. No.

Jun'17

In a processing industry two products A and B are made involving two operations. The production of B also results in a by-product C. The product A can be sold at a profit of ₹ 3 per unit and B at a profit of ₹ 8 per unit. The by-product C has a profit of ₹ 2 per unit. Forecast shows that upto 5 units of C can be sold. The company gets 3 units of C for each unit of B produced. The manufacturing times are 3 hours per unit for A on each of the operation one and two and 4 hours and 5 hours per unit for B on operation one and two, respectively. Because the product C results from producing B, no time is used in producing C. The available times are 18 hours and

21 hours of operation one and two respectively. The company desires to know how much of A and B should be produced keeping C in mind to make the highest profit.

Required:

Formulate LP model for this problem.

[8]

 Reference Formulation-Maximise	What's New By Product Effect

Answer

Let X_1, X_2, X_3 be the number of units produced of products A, B and C respectively.

Objective function:

Then the profit gained by the industry is given by

$$Z = 3x_1 + 8x_2 + 2x_3$$

Here it is assumed that all the units of products A and B are sold.

Condition-1:

In first operation, A takes 3 hrs of manufacturer's time and B takes 4 hrs of manufacturer's time. Therefore, total number of hours required in first operation becomes - $3x_1 + 4x_2$

In **second operation**, per unit of A takes 3 hrs of manufacturer's time and per unit B takes 5 hrs of manufacturer's time. Therefore, the total number of hours used in second operation becomes - $3x_1 + 5x_2$

Since, there are 18 hours available in first operation and 21 hours in second operation, the restrictions become

$$3x_1 + 4x_2 \leq 18$$

$$3x_1 + 5x_2 \leq 21$$

Condition-2:

Since, the maximum number of units of C that can be sold is 5, therefore, $X_3 \leq 5$

Condition-3:

Further, the company gets three units of by product C for every unit of product B produced, therefore,

$$X_3 = 3X_2$$

Now, the allocation problem of the industry can be finally put in the following linear programming problem:

Maximise

$$Z = 3x_1 + 8x_2 + 2x_3$$

Subject to the Constraints

$$3x_1 + 4x_2 \leq 18$$

$$3x_1 + 5x_2 \leq 21$$

$$x_3 \leq 5$$

$$x_3 = 3x_2$$

$$x_1, x_2, x_3 \geq 0$$

A better solution is by using the 4th variable

x_3 = units sold of By Prod c

x_4 = units unsold of By Prod c

units produced = $x_3 + x_4$

$$\text{Max } Z = 3x_1 + 8x_2 + 2x_3$$

subject to: $3x_1 + 4x_2 \leq 18$

$$3x_1 + 5x_2 \leq 21$$

$$x_3 \leq 5$$

$$x_3 + x_4 = 3x_2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Q10

Ex. Book No.

Pg. No.

Dec'19

Nava Bharat Industries Ltd. manufactures four products (1,2, 3,4) on two machines (X and Y). The time (in minutes) to process one unit of each product on each machine is shown below:

		Machine	
		X	Y
Product	1	12	26
	2	15	19
	3	18	30
	4	10	25

The profit per unit for each product (1,2,3,4) is ₹ 120, ₹ 150, ₹ 190 and ₹ 100 respectively. Product 1 must be produced on both machines X and Y but products 2, 3 and 4 can be produced on either machine.

Due to acute space constraints in the company's works, only one week's production is stored in 4,000 square feet for floor space where the floor space taken up by each product is 1.0, 1.5, 5.0 and 0.50 (square ft.) for products 1,2,3 and 4 respectively.

As per customer requirements the output of Product 2 is related with that of Product 3 and over a week approximately twice as many units of product 2 should be produced as product 3.

Machine X is out of action (for maintenance/because of breakdown) 8% of the time and machine Y 10% of the time.

Required:

Assuming a working week 42 hours, **formulate** the problem of how to manufacture these products as a linear programme. [8]

 Reference Formulation	What's New Variables

Answer

Variables:

Essentially the company is interested in the quantity produced on each machine.

Hence let:

x_i = amount of product i ($i = 2, 3, 4$) produced on machine X per week y_i = amount of product i ($i = 2, 3, 4$) produced on machine Y per week where $x_i \geq 0$ $i = 1, 2, 3, 4$ and $y_i \geq 0$ $i = 2, 3, 4$

It may be stated here that as Product 1 must be processed on both machines X and Y, y_1 has not been defined here.

Then objective function will be :

Maximize $120x_1 + 150(x_2 + y_2) + 190(x_3 + y_3) + 100(x_4 + y_4)$ subject to constraints

- Floor space**

$$1x_1 + 1.5(x_2 + y_2) + 5(x_3 + y_3) + 0.5(x_4 + y_4) \leq 4,000$$

- Customer requirements**

$$x_2 + y_2 = 2(x_3 + y_3)$$

- Available time**

$$12x_1 + 15x_2 + 18x_3 + 10x_4 \leq 0.92(42)(60) \text{ (machine X)}$$

$$26y_1 + 19y_2 + 30y_3 + 25y_4 \leq 0.90(42)(60) \text{ (machine Y)}$$

With non negative constraints to be inserted: X_1, X_2, X_3, X_4 and $Y_1, Y_2, Y_3, Y_4 \geq 0$.

Q11

Ex. Book No.

Pg. No.

RTP Jun'18; MTP Jun'22

The owner of Excel Sports wishes to determine how many advertisements to place in the selected three monthly magazines A B and C. His objective is to advertise in such a way that total exposure to principal buyers of expensive sports goods is maximized. Percentages of readers for each magazine are known. Exposure in any particular magazine is the number of

advertisements placed multiplied by the number of principal buyers. The following data may be used:

	Magazine		
	A	B	C
Readers	1 lakh	0.6 lakh	0.4 lakh
Principal Buyers	20%	15%	8%
Cost per Advertisement (₹)	8,000	6,000	5,000

The budgeted amount is at most ₹ 1 lakh for the advertisements. The owner has already decided that magazine A should have not more than 15 advertisements and that B and C each have at least 8 advertisements.

Formulate on LP model for the problem .

Reference

Formulation

What's **New**

Objective function

Answer

Step 1. The key decision to be made is to determine how many advertisements to place in the selected three monthly magazines A. B. and C so that total exposure to principal buyers of expensive sports goods is maximised

Step 2. Decision variables

Let x_1 = number of insertions in magazine A ,
 x_2 = number of insertions in magazine B , and
 x_3 = number of insertions in magazine C ,

Step 3. Feasible alternatives are sets of values of x_1, x_2, x_3 .

where x_1, x_2, x_3 each ≥ 0

Formulation of LP Model

The objective is to maximise (total exposure)

$$\begin{aligned} \text{Maximise } Z &= (20\% \text{ of } 1,00,000)x_1 + (15\% \text{ of } 60,000)x_2 + (8\% \text{ of } 4,000)x_3 \\ &= 20,000x_1 + 9,000x_2 + 3,200x_3 \end{aligned}$$

Subject to constraints

$$8,000x_1 + 6,000x_2 + 5,000x_3 \leq 1,00,000 \quad (\text{Budgeted amount})$$

$$x_1 \leq 15, x_2 \geq 8 \text{ and } x_3 \geq 8 \quad (\text{Advertisement})$$

$$x_1 \geq 0, x_2 \geq 0 \text{ and } x_3 \geq 0 \quad (\text{Insertions assumed to be non-negative})$$

Q12

Ex. Book No.

Pg. No.

RTP Dec'18

A farmer has a farm with 125 acres. He produces Carrot, Beetroot and Potato. Whatever he produces is fully sold in the market. He gets ₹ 5 per kg for Carrot, ₹ 4 per kg for Beetroot and ₹ 5 per kg for Potato. The average yield is 1,500 kg of Carrot per acre, 1,800 kg of Beetroot per acre and 1,200 kg of Potato per acre. To produce each 100 kg of Carrot and Beetroot and 80 kg of Potato, a sum of ₹ 12.50 has to be spent for manure. Labour required for each acre to raise the crop is 6 man-days for Carrot and Potato each and 5 man-days for Beetroot. A total of 500 man-days of labour at the rate of ₹ 40 per man-day are available.

Formulate a LPP to maximise the farmer's total profit.



Reference

Formulation

What's New

Objective function

Answer

Let C, B and P be the number of acres allotted for cultivating Carrot, Beetroot and Potato respectively. The profit from the produces is determined in the following manner –

Particulars per acre	Carrot	Beetroot	Potato
Selling Price	₹ 5 / Kg x 1,500 kgs = ₹ 7,500	₹ 4 / kg x 1,800 kgs = ₹ 7,200	₹ 5 / kg x 1,200 kgs = ₹ 6,000
Less: Manure Cost	1,500 kgs x ₹12.50/100 = ₹ 187.50	1,800 kgs x ₹12.50/100 = ₹ 225.00	1,200 kgs x ₹ 12.50/80 = ₹ 187.50
Less: Labour Cost	₹ 40 x 6 = ₹ 240	₹ 40 x 5 = ₹ 200	₹ 40 x 6 = ₹ 240
Profit per acre	₹ 7,072.50	₹6,775	₹ 5,572.50

Maximise Profit $Z = 7,072.50 C + 6,775 B + 5,572.5 P$

subject to,

$$C + B + P \leq 125$$

(Land Availability)

$$6C + 5B + 6P \leq 500$$

(Man Days Availability)

$$C, B, P \geq 0$$

(Non-Negativity Assumption)

Q13

Ex. Book No.

Pg. No.

The following matrix gives the unit cost of transporting a product from production plants P_1 , P_2 and P_3 to destinations D_1 , D_2 and D_3 . Plants P_1 , P_2 and P_3 have a maximum production of 65, 24 and 111 respectively and destinations D_1 , D_2 and D_3 must receive at least 60, 65 and 75 units respectively:

To From	D_1	D_2	D_3	Supply
P_1	400	600	800	65
P_2	1,000	1,200	1,400	24
P_3	500	900	700	111
Demand	60	65	75	200

You are **required** to **formulate** the above as a Linear Programming Problem. (Only formulation is needed. Please do not solve.)

Reference

Formulation

What's New

Variables

Answer

Let p_{ij} be the variable to denote the number of units of product from the i th Plant to the j th destination, so that p_{11} = Transport Cost from Plant P_1 to Destination D_1 , and p_{22} = transport from Plant P_2 to Destination D_2 , etc.

Objective Function:

$$\text{Minimize } Z = 400 p_{11} + 600 p_{12} + 800 p_{13} + 1,000 p_{21} + 1,200 p_{22} + 1,400 p_{23} + 500 p_{31} + 900 p_{32} + 700 p_{33}$$

Subject to:

$$\left. \begin{aligned} P_1 d_1 + p_{12} + p_{13} &\leq 65 \\ P_2 d_1 + p_{22} + p_{23} &\leq 24 \\ P_3 d_1 + p_{32} + p_{33} &\leq 111 \end{aligned} \right\} \text{ (Plant Constraints)}$$

and

$$\left. \begin{aligned} p_{11} + p_{21} + p_{31} &\geq 60 \\ p_{12} + p_{22} + p_{32} &\geq 65 \\ p_{13} + p_{23} + p_{33} &\geq 75 \end{aligned} \right\} \text{ (Destination Constraints)}$$

all $p_{ij} \geq 0$ (Non-Negativity Assumption)

Q14

Ex. Book No.

Pg. No.

RTP Jun'18

A leading Firm of Cost Accountants is attempting to determine a best investment portfolio and is considering six alternative investment proposals. The following table indicates point estimates for the price per share, the annual growth rates in the price per share, the amount of dividend per share and a measure of the risk associated with each investment.

Portfolio Data

Particulars	Shares under consideration					
	A	B	C	D	E	F
Current Price per Share (₹)	80	100	160	120	150	200
Projected Annual Growth	0.08	0.07	0.10	0.12	0.09	0.15
Projected Annual Dividend per Share (₹)	4.00	4.50	7.50	5.50	5.75	0.00
Projected Risk In Return	0.05	0.03	0.10	0.20	0.06	0.08

The total amount available for investment is ₹25 Lakhs and the following conditions are to be satisfied.

- The maximum rupee amount to be invested in alternative F is ₹ 2,50,000
- Not more than ₹5,00,000 should be invested in alternatives A and B combined.
- Total Weighted Risk should not be greater than 0.10 where -
- **Total Weighted Risk** =
$$\frac{(\text{Amount Invested in Alternative } i) \times (\text{Risk of Alternative } i)}{\text{Total Amount invested in all alternatives}}$$
- For the sake of diversity, at least 100 shares of each stock should be purchased.
- At least 10% of the total investment should be in alternatives A and B combined.
- Dividends for the year should be at least ₹ 10,000

Rupees Return per Share is defined as the Price per Share one year hence less Current Price per Share plus Dividend per Share.

If the objective is to maximize Total Rupee Return, **formulate** (but do not solve) the LPP for determining the optimum number of share & to be purchased In each of the Shares under consideration. Assume that the time horizon for the Investment is one year.

Reference

Formulation

What's New

**Return per share and
Constraints**

Answer

Let A, B, C, D, E and F be the number of shares to be purchased in each of the above investment proposals. Given, Return Per Share = Price Per Share one year hence Less Current Price Per Share Plus Dividend Per Share

Hence, Return per Share = (Current Price per Share $\times$ Projected Annual Growth Rate) + Dividends per Share.

Particulars Number of shares purchase	Shares under consideration					
	A	B	C	D	E	F
a. Current price per share (₹)	80	100	160	120	150	200
b. Projected Annual growth Rate (given)	0.08	0.07	0.10	0.12	0.09	0.15
c. Increase in price per share = $a \times b$ (in ₹)	6.40	7.00	16.00	14.40	13.50	30.00
d. Projected Annual Dividend per share (in ₹)	4.00	4.50	7.50	5.50	5.75	0.00
e. Rupee return per share = $c + d$ (in ₹)	10.40	11.50	23.50	19.90	19.25	30.00

Maximise $Z = 10.40A + 11.50B + 23.50C + 19.90D + 19.25E + 30.00F$ (objective Function)

Subject to the following constraints

(i) $80A + 100B + 160C + 120D + 150E + 200F \leq 25,00,000$ (Funds Availability)

(ii) $200F \leq 2,50,000$ (Investment F Condition)

(iii) $80A + 100B \leq 5,00,000$ (investment A & B Condition)

$$\frac{[(5\% \times 80A) + (3\% \times 100B) + (10\% \times 160C) + (20\% \times 120D) + (6\% \times 150E) + (8\% \times 200F)]}{80A + 100B + 160C + 120D + 150E + 200F} \leq 10$$

On simplification, we have $-4A - 7B + 12D - 6E - 4F \leq 0$ (Weighted risk Condition)

(iv) $A, B, C, D, E, F \geq 100$ (shares condition)

(v) $(80a + 100B) \geq (80A + 100B + 160C + 120D + 150E + 200F) \times 10\%$

Simplifying, we have, $72A + 90B - 16C - 12D - 115E - 20F \geq 0$ (Minimum Investment in A & B)

(vi) $4A + 4.50B + 7.50C + 5.50D + 5.75E \geq 10,000$ (Dividend Condition).

Note: Non-negativity Assumption is not necessary in the given case, since the Diversity condition is applicable.

Q15

Ex. Book No.

Pg. No.

MTP Dec'17

A company manufactures items X_1 and X_2 which are sold at a profit of ₹35 per unit of X_1 and ₹25 per unit of X_2 . X_1 requires 3 kgs of materials, 4 man-hours and 2 machine-hours per unit. X_2 requires 2 kgs of materials, 3 man-hours and 2 machine-hours per unit. During each production run, there are 350 kgs, of materials available, 600 man-hours and 550 machine-hours for use.

Formulate under Simplex method of linear programming:

- The objective function and the linear constraints, and
- The equations after introducing slack variables
- What** are the various methods of solving a linear programming problem?

Reference

Formulation

 What's New
Slack Variables

Answer

- The objective function to be maximized is - $Z = 35X_1 + 25X_2$ Subject to constraints**

$$3X_1 + 2X_2 \leq 350$$

$$4X_1 + 3X_2 \leq 600$$

$$2X_1 + 2X_2 \leq 550 \quad X_1, X_2 \geq 0$$

- By introducing slack variable the equation will be-**

$$3X_1 + 2X_2 + S_1 + 0S_2 + 0S_3 = 350$$

$$4X_1 + 3X_2 + 0S_1 + S_2 + 0S_3 = 600$$

$$2X_1 + 2X_2 + 0S_1 + 0S_2 + S_3 = 550 \text{ and}$$

$$Z = 35X_1 + 25X_2 + 0S_1 + 0S_2 + 0S_3$$

$$x, y, s_1, s_2, s_3 \geq 0$$

- Methods of solving a linear programming problem which are as follows:**

Simplex Method, Graphical Method, Transportation Method, Assignment Method

Q16

Ex. Book No.

Pg. No.

CMA Mat

A Company produces the products P, Q and R from three raw materials A, B and C. One unit of product P requires 2 units of A and 3 units of B. A unit of product Q requires 2 units of B and 5 units of C and one unit of product R requires 3 units of A, 2 unit of B and 4 units of C. The Company has 8 units of material A, 10 units of B and 15 units of C available to it. Profits/unit of products P, Q and R are ₹3, ₹5 and ₹4 respectively.

- (a) **Formulate** the problem mathematically,
(b) **Write** the Dual problem.

Reference

Formulation & Dual Problem

What's New

Primal and Dual

Answer

Raw Materials	P	Q	R	Available units
A	2	-	3	8
B	3	2	2	10
C	-	5	4	15

Profits **3** **5** **4**

Let x_1 be the no. of units of P Let x_2 be the no. of units of Q Let x_3 be the no. of units of R

Objective function: Max. $Z = 3x_1 + 5x_2 + 4x_3$

Subject to constraints:

$$2x_1 + 3x_3 \leq 8$$

$$3x_1 + 2x_2 + 2x_3 \leq 10$$

$$5x_2 + 4x_3 \leq 15$$

And $x_1, x_2, x_3 \geq 0$.

Primal

Max. $Z = 3x_1 + 5x_2 + 4x_3$

Subject to

$$2x_1 + 3x_3 \leq 8$$

$$3x_1 + 2x_2 + 2x_3 \leq 10$$

$$5x_2 + 4x_3 \leq 15$$

And $x_1, x_2, x_3 \geq 0$

Dual Min. $Z = 8y_1 + 10y_2 + 15y_3$

Subject to

$$2y_1 + 3y_2 \geq 3$$

$$2y_2 + 5y_3 \geq 5$$

$$3y_1 + 2y_2 + 4y_3 \geq 4$$

And $y_1, y_2, y_3 \geq 0$

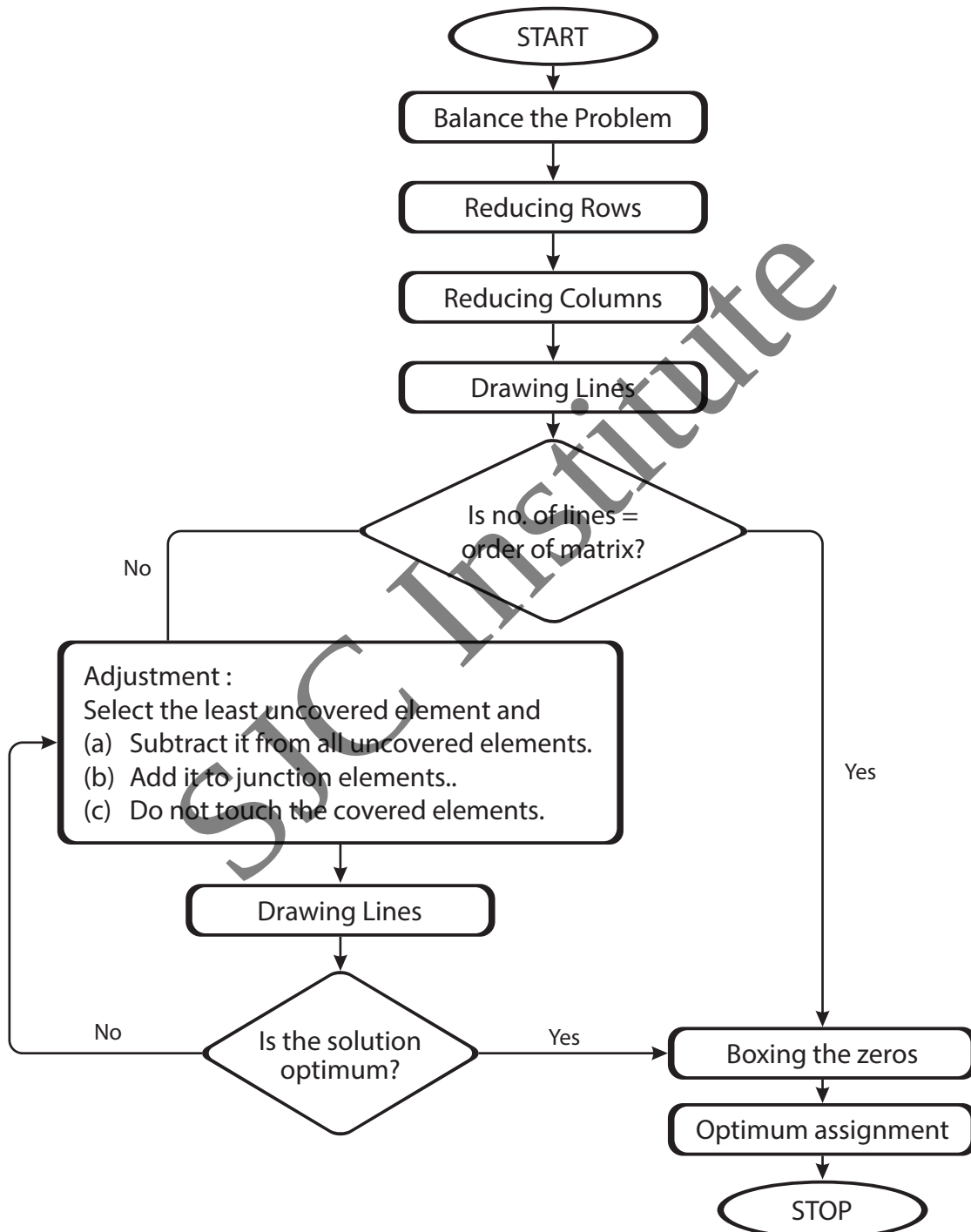
SJC Institute

Chapter **3**
Assignment**Learning
Objective**

1. OBJECTIVE
2. STEPS FOR HUNGARIAN RULE
3. SPECIAL CASES
 - (a) Maximisation Assignment Problem
 - (b) Unbalanced Assignment Problem
 - (c) Implication of dummy
 - (d) Prohibited Assignment
 - (e) Conditional Assignment
 - (f) Travelling salesmen cost minimisation

A. OBJECTIVE

Minimisation of Time / Cost

B. HUNGARIAN RULE STEPS

C. UNBALANCED PROBLEM

Use dummy in original matrix

D. MAXIMISATION OBJECTIVE

Opportunity Loss Matrix

E. MULTIPLE SOLUTION

Arbitrary boxing

F. PROHIBITED ASSIGNMENT

Use M ($M \rightarrow \infty$)

G. CONDITIONAL ASSIGNMENT

Eliminate the row and column

H. TRAVELLING SALESMAN CYCLE

- (1) Check cycle is completed or not in the optimum assignment
- (2) If not, box any row column at 1, instead of 0 and check. Try to break the loop in the last solution

1. ASSIGNMENT

Assignment is a special linear programming problem. There are many situations where the assignment of people or machines etc. may be called for. Assignment of workers to machines, clerks to various check-out counters, salesmen to different sales areas are typical examples of these. The Assignment is a problem because people possess varying abilities for performing different jobs and therefore the costs of performing jobs by different people are different. Thus, in an assignment problem, the question is how the assignments should be made in order that the total cost involved is minimized.

There are four methods of solving an assignment problem and they are

- (1) Complete Enumeration Method
- (2) Simplex Method
- (3) Transportation Method and
- (4) Hungarian Method

2. HUNGARIAN METHOD

The following are the steps involved in the minimization of an assignment problem under this method:

Step 1: Row Operation

Locate the smallest cost element in each row of the cost table. Now subtract this smallest element from each element in that row. As a result, there shall be at least one zero in each row of this new table, called the reduced cost table.

Step 2: Column Operation

In the reduced cost table obtained, consider each column and locate the smallest element in it. Subtract the smallest value from every other entry in the column. As a consequence of this action, there would be at least one zero in each of the rows and columns of the second reduced cost table.

Step 3: Optimality

Draw the minimum no. of horizontal and vertical lines (not the diagonal ones) that are required to cover all the zero elements. If the no. of lines drawn is equal to 'n' (the no. of rows/columns) the solution is optimal and proceeds to step 6. If the no. of lines drawn is smaller than 'n' go to step 4.

Step 4: Improved Matrix

Select the smallest uncovered (by the lines) cost element. Subtract this element from all uncovered elements including itself and add this element to each value located at the intersection of any two lines. The cost elements through which only one line passes remain unaltered.

Step 5: Repeat step 3 and 4 until an optimal solution is obtained.

Step 6: Given the optimal solution, make the job assignments as indicated by the 'zero' elements. This is done as follows:

- (a) Locate a row which contains only one zero element. Assign the job corresponding to this element to its corresponding person. Cross out the zero's if any in the column corresponding to the element, which is indicative of the fact that the particular job and person are no more available.
- (b) Repeat (a) for each of such rows which contain only one zero. Similarly, perform the same operation in respect of each column containing only one 'zero' element, crossing out the zero(s), if any, in the row in which the elements lies.
- (c) If there is no row or column with only a single 'zero' element left, then select a row/column arbitrarily and choose one of the jobs (or persons) and make the assignment. Thus in such a case, alternative solutions exists.

SJC Institute

Q1

Ex. Book No.

Pg. No.

MTP Jun'19

A Departmental head has four subordinates and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of the time each man would take to perform each task, is given in the matrix below:

Hour

Tasks	Men			
	E	F	G	H
A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

- (i) **How** should the tasks be allocated to a man, so as to minimize the total man-hours?
- (ii) **Calculate** total man hours required to complete all the tasks as per assignment. [8 + 2]

Reference

Minimisation

What's New

Balanced

Answer

Subtracting the smallest elements of each row from every element of the corresponding row, we get the reduced matrix:

Tasks	Men			
	E	F	G	H
A	7	15	6	0
B	0	15	1	13
C	23	4	3	0
D	9	16	14	0

Subtracting the smallest element of each column of the reduced matrix from every element of the corresponding column, we get the following reduce matrix

7	11	5	0
0	11	0	13
23	0	2	0
9	12	13	0

Three lines are required to cover all zeros, whereas the order of the matrix is 4. Therefore, optimum assignment cannot be made at this stage. The minimum uncovered element is 5. Therefore, subtracting this element from all the uncovered elements and adding the same to all the elements lying at the intersections of the lines, we obtain the following reduced matrix:

2	6	0	0
0	11	0	18
23	0	2	5
4	7	8	0

Four lines are required to cover all zeros and order of the matrix is also 4. Therefore, Optimum assignment can be made at this stage as follows.

2	6	0	0
0	11	0	18
23	0	2	5
4	7	8	0

Now, each row and each column has one and only one assignment. Therefore, optimal solution is reached.

Optimum assignment is as follows:

A → G, B → E, C → F and D → H

The minimum total time for the assignment schedule is as follows:

Tasks	Men	Man-hours
A	G	17
B	E	13
C	F	19
D	H	10

Total man-hours = 59

Q2

Ex. Book No.

Pg. No.

CMA Mat

Six men are available for different jobs. From past records the time in hours taken by different persons for different jobs are given below.

Men	Jobs						
		1	2	3	4	5	6
	1	2	9	2	7	9	1
	2	6	8	7	6	14	1
	3	4	6	5	3	8	1
	4	4	2	7	3	10	1
	5	5	3	9	5	12	1
	6	9	8	12	13	9	1

Find out an allocation of men to different jobs which will lead to minimum operation time.

Reference

Minimisation

What's New

Balanced

Answer

Row Operation

2	9	2	7	9	1	1	8	1	6	8	0
6	8	7	6	14	1	5	7	6	5	13	0
4	6	5	3	8	1	3	5	4	2	7	0
4	2	7	3	10	1	3	1	6	2	9	0
5	3	9	5	12	1	4	2	8	4	11	0
9	8	12	13	9	1	8	7	11	12	8	0

Column Operation

0	7	0	4	1	0
4	6	5	3	6	0
2	4	3	0	0	0
2	0	5	0	2	0
3	1	7	2	4	0
7	6	10	10	1	0

Improved

0	7	0	4	1	1
3	5	4	2	5	0
2	4	3	0	0	1
2	0	5	0	2	1
2	0	6	1	3	0
6	5	9	9	0	0

Improved Matrix

0	9	0	6	3	1
1	5	2	2	5	0
0	4	1	0	0	1
0	0	3	0	2	1
0	0	4	1	3	0
4	5	7	9	0	0

Assignment

0	9	0	6	3	3
1	5	2	2	5	0
0	4	1	0	0	1
0	0	3	0	2	1
0	0	4	1	3	0
4	5	7	9	0	0

Optimal Assignment, then is

1	-	3	-	2
2	-	6	-	1
3	-	1	-	4
4	-	4	-	3
5	-	2	-	3
6	-	5	-	9

22 Minimum Hours

Q3

Ex. Book No.

Pg. No.

CMA Mat

Average time taken by an operator on a specific machine is tabulated below. The management is considering replacing one of the old machines by a new one and the estimated time for operation by each operator on the new machine is also indicated.

Operation	Machines						New
	M ₁	M ₂	M ₃	M ₄	M ₅	M ₆	
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6

- (a) **Find out** an allocation of operators to the old machines to achieve a minimum operation time.
- (b) **Reset** the problem with the new machine and find out the allocation of the operators to each machine and comment on whether it is advantageous to replace an old machine to achieve a reduction in operating time only.
- (c) **How** will the operators be reallocated to the machines after replacement?



Reference

Minimisation

What's New

Balanced and Replacement

Answer

Operation	M ₁	M ₂	M ₃	M ₄	M ₅	M ₆	New
01	2	3	2	1	4	5	6
02	4	4	6	3	2	5	1
03	6	10	8	4	7	6	1
04	8	7	6	5	3	9	4
05	7	3	4	5	4	3	12
06	5	5	6	7	8	1	6

(a)

Row Operation

2	3	2	1	4	5
4	4	6	3	2	5
6	10	8	4	7	6
8	7	6	5	3	9
7	3	4	5	4	3
5	5	6	7	8	1

1	2	1	0	3	4
2	2	4	1	0	3
2	6	4	0	3	2
5	4	3	2	0	6
4	0	1	2	1	0
4	4	5	6	7	0

Column Operation

0	2	0	0	3	4
1	2	3	1	0	3
1	6	3	0	3	2
4	4	2	2	0	6
3	0	0	2	1	0
3	4	4	6	7	0

Improved matrix

0	2	0	1	4	5
0	1	2	1	0	3
0	5	2	0	3	2
3	3	1	2	0	6
3	0	0	3	2	1
2	3	3	6	7	0

01	M_3	-	2
02	M_1	-	4
03	M_4	-	4
04	M_5	-	3
05	M_2	-	3
06	M_6	-	1

17 Hours Minimum Operation Time

(b) & (c)

Row Operation

2	3	2	1	4	5	6
4	4	6	3	2	5	1
6	10	8	4	7	6	1
8	7	6	5	3	9	4
7	3	4	5	4	3	12
5	5	6	7	8	1	6
0	0	0	0	0	0	0

1	2	1	0	3	4	5
2	2	4	1	0	3	0
2	6	4	0	3	2	0
5	4	3	2	0	6	1
4	0	1	2	1	0	9
4	4	5	6	7	0	5
0	0	0	0	0	0	0

Improved Matrix 1

0	1	0	0	3	4	5
2	2	4	2	1	4	0
4	8	6	3	6	5	0
4	3	2	2	0	6	1
4	0	1	3	2	1	10
3	3	4	6	7	0	5
0	0	0	1	1	1	1

In Place of M_3 new machine is to be replaced.

Improved Matrix 2

0	2	0	0	4	5	6
1	2	3	1	1	4	0
3	8	5	2	6	5	0
3	3	1	1	0	6	1
3	0	0	2	2	1	10
2	3	3	5	7	0	5
0	1	0	1	2	2	2

Improved Matrix 3

0	2	0	0	5	6	7
0	1	2	0	1	4	0
2	7	4	1	6	5	0
2	2	0	0	0	6	1
3	0	0	2	3	2	11
1	2	2	4	7	0	5
0	1	0	1	3	3	3

Optimum Assignment

01	M_1	-	2
02	M_4	-	3
03	New	-	1
04	M_5	-	3
05	M_2	-	3
06	M_6	-	1
07	M_3	-	0
			13 Hours Minimum time

Q4

Ex. Book No.

Pg. No.

RTP Jun'18

To stimulate interest and provide an atmosphere for intellectual discussion, a finance faculty in a management school decides to hold special seminars to four contemporary topics -leasing, portfolio management, private mutual funds, swaps and options. Such seminars should be held once in a week in the afternoons. However, scheduling these seminars (one for each topic, and not more than one seminar per afternoon) has to be done carefully so that the number of students unable to attend is kept to a minimum. A careful study indicates that the number of students who cannot attend a particular seminar on a specific day is as follows:

	Leasing	Portfolio management	Private mutual funds	Swaps and Options
Monday	50	40	60	20
Tuesday	40	30	40	30
Wednesday	60	20	30	20
Thursday	30	30	20	30
Friday	10	20	10	30

Find an optimum schedule of the seminars. Also find out the total number of students who will be missing at least one seminar.



Reference

Minimisation

What's New

Unbalanced

Answer

This is an unbalanced minimisation assignment problem. We first of all balance it by adding a dummy topic.

	Leasing	Portfolio Management	Private Mutual funds	Swaps and options	Dummy
Monday	50	40	60	20	0
Tuesday	40	30	40	30	0
Wednesday	60	20	30	20	0
Thursday	30	30	20	30	0
Friday	10	20	10	30	0

Subtracting the minimum element of each column from all elements of that column, we get the following matrix.

	Leasing	Portfolio Management	Private Mutual funds	Swaps and options	Dummy
Monday	40	20	50	0	0
Tuesday	30	10	30	10	0
Wednesday	50	0	20	0	0
Thursday	20	10	10	10	10
Friday	0	0	0	0	0

The minimum number of lines to cover all zeros is 4, which is less than the order of the matrix (i.e., 5), the above matrix will not give the optimal solution. Subtract the minimum uncovered element (i.e., 10) from all uncovered elements and add it to the elements lying on the intersection, we get the following matrix:

	Leasing	Portfolio Management	Private Mutual funds	Swaps and options	Dummy
Monday	30	20	40	0	0
Tuesday	20	10	20	10	0
Wednesday	40	0	10	0	0
Thursday	10	10	0	10	10
Friday	0	10	0	10	10

Since the minimum number of lines to cover all zeros is 5, which is equal to order of the matrix, the above matrix will give the optimum solution which is given below:

	Leasing	Portfolio Management	Private Mutual funds	Swaps and options	Dummy
Monday	30	20	40	0	0
Tuesday	20	10	20	10	0

Wednesday	40	0	10	0	0
Thursday	10	10	0	10	10
Friday	0	10	0	10	10

The optimal schedule is:

Day	Contemporary topics	No. of students missing
Monday	Swaps and Options	20
Tuesday	No Seminar	0
Wednesday	Portfolio	20
Thursday	Private mutual funds	20
Friday	Leasing	10
	Total	70

Thus number of students, who will be missing at least one seminar is 70.

Q5 Ex. Book No. Pg. No.

CMA Mat; Jun'19

Four jobs can be processed on four different machines, one job on one machine. Resulting profits vary with assignments. They are given below:

	Machines			
	1	2	3	4
I	42	35	28	21
II	30	25	20	15
III	30	25	20	15
IV	24	20	16	12

Find the optimum assignment of jobs to machines and the corresponding profit.

Reference	What's New
Maximisation (Opportunity loss matrix)	Balanced

Answer

Loss Matrix (Reducing all elements from highest element of profit matrix)

0	7	14	21
12	17	22	27
12	17	22	27
18	22	26	30

Row Operation

0	7	14	21
0	5	10	15
0	5	10	15
0	4	8	12

Column Operation

0	3	6	9
0	1	2	3
0	1	2	3
0	0	0	0

Improved Matrix

0	2	5	8
0	0	1	2
0	0	1	2
1	0	0	0

Further Improved

0	2	4	7
0	0	0	1
0	0	0	1
2	1	0	0

Allocation

I	-	1	-	42
II	-	2	-	25
III	-	3	-	20
IV	-	4	-	12
				99

Maximum Profit ₹ 99

Q6

Ex. Book No.

Pg. No.

CMA Mat; MTP Jun'17; MTP Jun'18; MTP Dec'18; MTP Jun'18

A captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows:

Batsmen	Batting Position					
		III	IV	V	VI	VII
	A	40	40	35	25	50
	B	42	30	16	25	27
	C	50	48	40	60	50
	D	20	19	20	18	25
	E	58	60	59	55	53

Make the assignment so that the expected total average runs scored by these batsmen are maximum.

Reference

Maximisation (Opportunity loss matrix)

What's New

Balanced

Answer

	III	IV	V	VI	VII	Loss Matrix				
A	40	40	35	25	50	20	20	25	35	10
B	42	30	16	25	27	18	30	44	35	33
C	50	48	40	60	50	10	12	20	0	10
D	20	19	20	18	25	40	41	40	42	35
E	58	60	59	55	53	2	0	1	5	7

Row Operation M_3

10	10	15	25	0
0	12	26	17	15
10	12	20	0	10
5	6	5	7	0
2	0	1	5	7

Column Operation

10	10	15	25	0
0	12	26	17	15
10	12	20	0	10
5	6	5	7	0
2	0	1	5	7

Maximum Average Runs

A	-	VII	-	50
B	-	III	-	42
C	-	VI	-	60
D	-	V	-	20
E	-	IV	-	60

Q7

Ex. Book No.

Pg. No.

CMA Mat

Six salesmen are to be allocated to six sales regions so that the cost of allocation of the job will be minimum. Each salesman is capable of doing the job at different cost in each region. The cost matrix is given below:

		Region					
		I	II	III	IV	V	VI
Salesmen	A	15	35	0	25	10	45
	B	40	5	45	20	15	20
	C	25	60	10	65	25	10
	D	25	20	35	10	25	60
	E	30	70	40	5	40	50
	F	10	25	30	40	50	15

(Figures are in ₹)

- (a) **Find** the allocation to give minimum cost what is the cost?
- (b) Now suppose the above table gives earning of each salesman at each region. **How** can you find an allocation so that the earning will be maximum? **Determine** the solution with optimum earning.

- (c) There are restrictions for commercial reasons that A cannot be posted to region V and E cannot be posted to region II. **Write** down the cost matrix suitably after imposing the restrictions.

Reference	What's New
Minimum and Maximisation (Opportunity loss matrix)	Unbalanced and Prohibited

Answer

						Row Operation					
15	35	0	25	10	45	15	35	0	25	10	45
40	5	45	20	15	20	35	0	40	15	10	15
25	60	10	65	25	10	15	50	0	55	15	0
25	20	35	10	25	60	15	10	25	0	15	50
30	70	40	5	40	50	25	65	35	0	35	45
10	25	30	40	50	15	0	15	20	30	40	5

Column Operation

15	35	0	25	0	45
35	0	40	15	0	15
15	50	0	55	5	0
15	10	25	0	5	50
25	65	35	0	25	45
0	15	20	30	30	5

Improved Matrix

20	35	0	30	0	45
40	0	40	20	0	15
20	50	0	60	5	0
15	5	20	0	0	45
25	60	30	0	20	40
0	10	15	30	25	0

A	III	-	0
B	II	-	5
C	VI	-	10
D	V	-	25
E	IV	-	5
F	I	-	10

₹ 55 Minimum Cost

(b)

Loss Matrix

55	35	70	45	60	25
30	65	25	50	55	50
45	10	60	5	45	60
45	50	35	60	45	10
40	0	30	65	30	20
60	45	40	30	20	55

Row Operation

30	10	45	20	35	0
5	40	0	25	30	25
40	5	55	0	40	55
35	40	25	50	35	0
40	0	30	65	30	20
40	25	20	10	0	35

Column Operation

25	10	45	20	35	0
0	40	0	25	30	25
35	5	55	0	40	55
30	40	25	50	35	0
35	0	30	65	30	20
35	25	20	10	0	35

Improved Matrix

5	10	25	20	35	0
0	60	0	45	50	45
15	5	35	0	40	55
10	40	5	50	35	0
15	0	10	65	30	20
15	25	0	10	0	35

Improved Matrix - II

0	10	20	20	30	0
0	65	0	50	50	50
10	5	30	0	35	55
5	40	0	50	30	0
10	0	5	65	25	20
15	30	0	15	0	40

A	-	I	-	15
B	-	III	-	45
C	-	IV	-	65
D	-	VI	-	60
E	-	II	-	70
F	-	V	-	50

 ₹ 305 Maximum

(c) The cost matrix after imposing the given restriction is

		Region					
		I	II	III	IV	V	VI
Sales men	A	15	35	0	25	α	45
	B	40	5	45	20	15	10
	C	25	60	10	65	25	10
	D	25	20	35	10	25	60
	E	30	α	40	5	40	50
	F	10	25	30	40	50	15

Q8

Ex. Book No.

Pg. No.

**CMA Mat; Dec'21; MTP Dec'17;
MTP Dec'18; MTP Dec'19**

A company has four zones open and four salesmen available for assignment. The zones are not equally rich in their sales potentials. It is estimated that a typical salesman operating in each zone would bring in the following annual sales:

Zone: A: 1,26,000; Zone B: 1,05,000; Zone C: 84,000; Zone D: 63,000.

The four salesmen are also considered to differ in ability. It is estimated that working under the same condition their yearly sales would be proportionately as follows:

Salesman P: 7; Salesman Q: 5; Salesman R: 5; Salesman S: 4. If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest zone, the next best to the second richest zone and so on.

Verify this by the method of assignment.

Reference

Maximisation (Opportunity loss matrix)

What's New

Answer

Ratio of zone wise sales = 6 : 5 : 4 : 3

Ratio of Salesman ability = 7 : 5 : 5 : 4

Sales Matrix

Sales Men	A	B	C	D
P	42	35	28	21
Q	30	25	20	15
R	30	25	20	15
S	24	20	16	12

Loss Matrix

(Reducing all elements from highest element)

0	7	14	21
12	17	22	27
12	17	22	27
18	22	26	30

Row Operation

0	7	14	21
0	5	10	15
0	5	10	15
1	4	8	12

Column Operation

0	3	6	9
0	1	2	3
0	1	2	3
0	0	0	0

Improved Matrix 1

0	2	5	8
0	0	1	2
0	0	1	2
1	0	0	0

Improved Matrix 2

0	2	4	7
0	0	0	1
0	0	0	1
2	1	0	0

Allocation

P	-	A	-	42
Q	-	B	-	25
R	-	C	-	20
S	-	D	-	12

99 Maximum sales

Q9

Ex. Book No.

Pg. No.

CMA Mat

A salesman has to visit five cities A,B,C,D and E. The inter-city distances are tabulated below. Note the distance between two cities need not be same both ways.

From / To	A	B	C	D	E
A	-	12	24	25	15
B	6	--	16	18	7
C	10	11	--	18	12
D	14	17	22	--	16
E	12	13	23	25	--

Note further that the distances are in km.

Required:

If the salesman starts from city A and has to come back to city A, **which** route would you advise him to take that total distance traveled by him is minimised?

Reference

What's New

Travelling Salesman cycle optimization

Answer

- 12 24 25 15
6 - 16 18 7
10 11 - 18 12
14 17 22 - 16
12 13 23 25 -

Row Operation

- 0 12 13 3
0 1 10 12 1
0 1 - 8 2
0 3 8 - 2
0 1 11 13 -

Column Operation

- 0 4 5 2
0 2 4 0
0 1 - 0 1
0 3 0 - 1
0 1 3 5 -

A → B → E

C → D - i.e. cycle is not completed

Reboxing to complete the circle [first boxing with C to A]

-	0	4	5	2
~~0~~	-	2	4	0
0	1	-	~~0~~	1
~~0~~	3	0	-	1
~~0~~	1	3	5	-

A → B → E → D → C → A

$$12 + 7 + 25 + 22 + 10 = 76 \text{ Kms}$$

Optimum Distance 76 Kms.

SJC Institute



Learning Objective

1. OBJECTIVE
2. METHODS OF INITIAL SOLUTION
3. STEPS OF NORTH WEST CORNER RULE
4. STEPS OF LEAST COST METHOD
5. STEPS OF VOGEL'S APPROXIMATION METHOD (VAM)
6. OPTIMALITY TEST (MODIFIED DISTRIBUTION METHOD)
7. INITIAL SOLUTION IS NOT THE OPTIMAL SOLUTION – LOOP AND REALLOCATION
8. SPECIAL CASES
 - (a) Unbalanced Transportation Problem
 - (b) Degeneracy
 - (c) Maximisation Transportation Problem
 - (d) Multiple Solution

1. INTRODUCTION

The basic transportation problem was originally developed by F.L. Hitchcock (1941) in his study entitled "the distribution of a product from several sources to numerous locations". In 1947, T.C. Koopmans independently published a study on "optimum utilization of the transportation system".

Transportation models deals with the transportation of a product manufactured at different plants or factories (supply origins) to a number of different warehouses (demand destinations). The objective is to satisfy the destination requirements within the plants capacity constraints at the minimum transportation cost. Transportation models thus typically arise in situations involving physical movement of goods from plants to warehouses, warehouses to wholesalers, wholesalers to retailers and retailers to customers. Solution of the transportation models requires the determination of how many units should be transported from each supply origin to each demands destination in order to satisfy all the destination demands while minimizing the total associated cost of transportation.

Feasible Solution:

A set of non-negative values x_{ij} , $i=1, 2, \dots, m$; $j = 1, 2, \dots, n$ that satisfies no. of rows to no. of columns is called a feasible solution to the transportation problem.

Basic Feasible Solution:

An initial feasible solution with an allocation of $(m + n - 1)$ number of variables, x_{ij} , $i=1, 2, \dots, m$; $j = 1, 2, \dots, n$, is called a basic feasible solution.

Optimum Solution:

A feasible solution (not necessarily basic) is said to be optimum if it minimizes the total transportation cost.

Balanced or Unbalanced Transportation Problems:

A transportation problem can be balanced or unbalanced. It is said to be balanced if the total demand of all the warehouses equals the amount produced in all the factories. If in reality, capacity is greater than requirement, then a dummy warehouse may be used to create desired equality. If capacity is less than requirement, then a dummy factory may be introduced. The transportation cost in both the dummy cases is assumed to be zero.

Where the number of rows and columns are not equal, it is called unbalanced transportation problem.

Loops in Transportation Table:

In a transportation table, an ordered set of four or more cells is said to form a loop if any two adjacent cells in the ordered set lie either in the same row or in the same column. Moreover every loop has an even number of cells. It may be noted that a feasible solution to a T.P is basic if and only if the corresponding cells in the transportation table do not contain a loop.

Degeneracy of a Transportation Problem:

When the quantities are allocated to cost cells within the matrix and if such allocations are less than $m + n - 1$ allocations (where 'm' stands for no. of rows and 'n' stands for no. of columns), such a situation is said to be Degeneracy of a Transportation Problem.

2. METHODS OF SOLVING TRANSPORTATION PROBLEM [DEC'18; RTP JUN'18; MTP DEC'21]

The following are the methods of solving transportation problem:

1. **North West Corner Method (NWCM):**

The simplest of the procedures used to generate an initial feasible solution is NWCM. It is so called because we begin with the north west or upper left corner cell of our transportation table. Various steps of this method can be summarized as under:

Step 1:

Select the north west (upper left-hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand requirement, i.e., $\min(s_1, d_1)$.

Step 2:

Adjust the supply and demand numbers in the respective rows and columns allocation.

Step 3:

- (a) If the supply for the first row is exhausted, then move down to the first cell in the second row and first column and go to step 2.
- (b) If the demand for the first column is satisfied, then move horizontally to the next cell in the second column and first row and go to step 2.

Step 4:

If for any cell, supply equals demand, then the next allocation can be made in cell either in the next row or column.

Step 5:

Continue the procedure until the total available quantity is fully allocated to the cells required.

2. **Least Cost Method (LCM):**

The allocation according to this method is very useful as it takes into consideration the lowest cost and therefore, reduces the computation as well as the amount of time necessary to arrive at the optimum solution. Various steps of this method can be summarized as under:

Step 1:

- (a) Select the cell with the lowest transportation cost among all the rows or columns of the transportation table.
- (b) If the minimum cost is not unique, then select arbitrarily any cell with this minimum cost.

Step 2:

Allocate as many units as possible to the cell determined in step 1 and eliminate that row (column) in which either supply is exhausted or demand is satisfied.

Step 3:

Repeat steps 1 and 2 for the reduced table until the entire supply at different factories is exhausted to satisfy the demand at different warehouses.

3. Vogel's Approximation Method (VAM):

This method is preferred over the other two methods because the initial basic feasible solution obtained is either optimum or very close to the optimum solution. Therefore, the amount of time required to arrive at the optimum solution is greatly reduced. Various steps of this method are summarized as under:

Step 1:

Compute a penalty for each row and column in the transportation table. The penalty for a given row and column is merely the difference between the smallest cost and the next smallest cost in that particular row or column.

Step 2:

Identify the row or column with the largest penalty. In this identified row or column, choose the cell which has the smallest cost and allocate the maximum possible quantity to the lowest cost cell in that row or column so as to exhaust either the supply at a particular source or satisfy demand at a warehouse.

If a tie occurs in the penalties, select that row/column which has minimum cost. If there is a tie in the minimum cost also, select that row/column which will have maximum possible assignments. It will considerably reduce computational work.

Step 3:

Reduce the row supply or the column demand by the amount assigned to the cell.

Step 4:

If the row supply is now zero, eliminate the row, if the column demand is now zero, eliminate the column, if both the row supply and the column demand are zero, eliminate both the row and column.

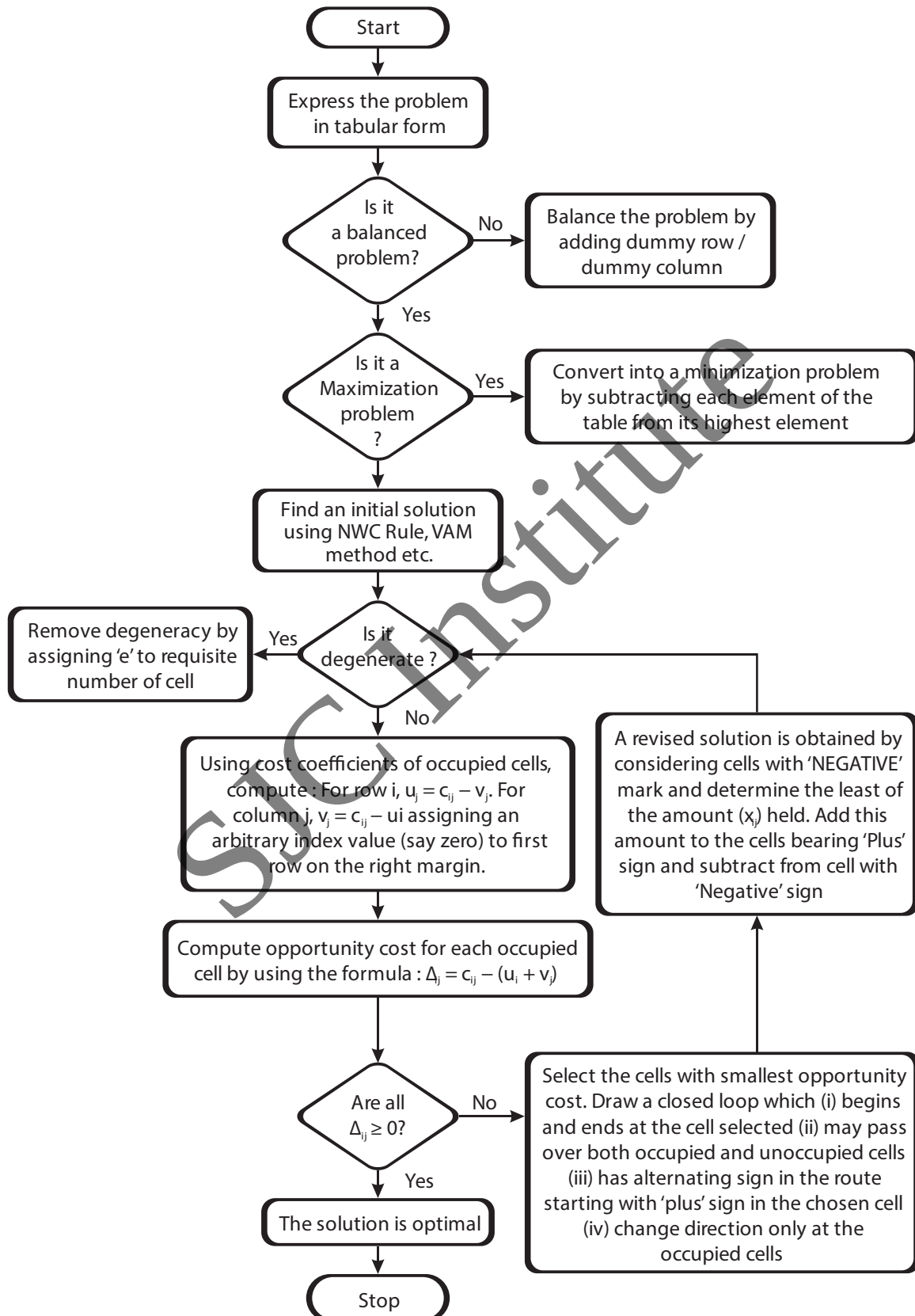
Step 5:

Recompute the row and column difference for the reduced transportation table, omitting rows or columns crossed out in the preceding step.

Step 6:

Repeat the above procedure until the entire supply at factories are exhausted to satisfy demand at different warehouses.

3. MODI METHOD - SCHEMATIC DIAGRAM



Can a transportation problem be solved using Assignment technique?

Ans : Yes if the problem is of type – equal number of rows and columns (ex. 3×3) with same requirements and availabilities in each column i.e.

	W1	W2	W3	Availability
F1				100
F2				100
F3				100
Requirements	100	100	100	

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Ex. Book No.

Pg. No.

CMA Mat

Anand Batteries have plants at X, Y and Z. Its products 'Torch Batteries' is sent in trucks to the warehouses situated at A, B, C and D for final delivery. The shipment, production runs and storage capacities are given on the basis of truckloads/week. The costing for transportation is also on the basis of truckloads.

The plants have their working capacities and the warehouses are of different sizes depending on market demand. The following tables show:

(i) The capacity details of the plants and the warehouses, and (ii) the transportation cost/truckload

Table 1:

Warehouse	Capacity Truckloads	Factory	Production Truckloads
A	16	X	48
B	20	Y	32
C	40	Z	40
D	44	-	-
Total	120		120

Table 2: Transportation cost per truckload in units of ₹100

	A	B	C	D
X	6	11	3.5	6
Y	2	6	5	4
Z	1.5	11	4.5	3

You are **required** to workout as to how the supplies from the plant be allocated to the warehouses to minimise total transportation cost. **Determine** the minimum total transportation cost.

Reference

Minimisation

What's New

Answer

Transport cost matrix					
Warehouses	A	B	C	D	Capacity/Production
Factory X	6	11	3.5	6	48
Y	2	6	5	4	32
Z	1.5	11	4.5	3	40
Demand	16	20	40	44	120

X	6	11	3.5	6	48/8/0	2.5/2.5/0/0
			40	8		
Y	2	6	5	4	32/12/0	2/2/2*
	12	20				
Z	1.5	11	4.5	3	40/36/0	1.5/1.5/1.5/1.5
	4			36		

$\frac{16}{4}$	$\frac{20}{0}$	$\frac{40}{0}$	$\frac{44}{8}$
0			0
$\frac{0.5}{0.5}$	5*	$\frac{1}{1}$	$\frac{1}{1}$
$\frac{0.5}{4.5*}$			$\frac{1}{3}$

	6	11	3.5	6	U _i
	1.5	2.5	40	8	0
	2	6	5	4	-2.5
12		20	4	0.5	
	1.5	11	4.5	3	-3
4		5.5	4	36	
V _j	4.5	8.5	3.5	6	

As $\Delta_{ij} \geq 0$, the solution is optimum.

		Quantity	Minimum Cost '000
X	C	40 x 3.5	140
	D	8 x 6	48
Y	A	12 x 2	24
	B	20 x 6	120
Z	A	4 x 1.5	6
	D	36 x 3	108
Minimum Cost			₹ 446

Minimum Cost ₹ 4,46,000

Q2

Ex. Book No.

Pg. No.

CMA Mat; MTP Dec'19

A manufacturer has distribution centres X, Y, and Z. These centres have 40, 20 and 40 units of his product. His retail outlets at A, B, C, D and E require 25, 10, 20, 30 and 15 units respectively. The transport cost in (Rupees/Unit) between each centre and each outlet is given in the following table:

Distribution Centre	Retail outlets				
	A	B	C	D	E
X	55	30	40	50	40
Y	35	30	100	45	60
Z	40	60	95	35	30

We have to **find out** the optimum distribution cost.

Reference

Minimisation

What's **New**
Loop Matrix

Answer

	A	B	C	D	E
X	55	30	40	50	40
Y	35	30	100	45	60
Z	40	60	95	35	30

40/20/10/5/0 10/10*/20/5/5

20/0 5/5/5/10/10

40/25/0 5/5/5/5

25	10	10	20	30	15
5	0	0	5	0	
0			0		
5	0	55*	10		10
5	0	—	10		10
5	—	—	10		
5	—	—	10		
20	—	—	—	5*	

$$V_j$$
$$V_j$$
$$V_j$$

X	→	B	$10 \times 30 =$	300
	→	C	$20 \times 40 =$	800
	→	E	$10 \times 40 =$	400
Y	→	A	$20 \times 35 =$	700

Z	A	$5 \times 40 =$	200
	D	$30 \times 35 =$	1,050
	E	$5 \times 30 =$	150
		100	₹ 3,600

Q3

Ex. Book No.

Pg. No.

CMA Mat

The cost conscious company requires for the next month 300, 260 and 180 tonnes of stone chips for its three constructions C1, C2 and C3 respectively. Stone chips are produced by the company at three mineral fields taken on short lease by the company. All the available boulders must be crushed into chips. Any excess chips over the demands at sites C1, C2 and C3 will be sold ex-fields.

The fields are M1, M2 and M3 which will yield 250, 320 and 280 tones of stone chips respectively.

Transportation costs from mineral fields to construction sites vary according to distances, which are given below in monetary unit (MU).

	To	C1	C2	C3
From	M1	8	7	6
	M2	5	4	9
	M3	7	5	5

- Determine** the optimal economic transportation plan for the company and the overall transportation cost in MU.
- What** are the quantities to be sold from M1, M2 and M3 respectively?



Reference

Minimisation

What's **New**

Unbalanced

Answer

As the demand is not equal to supply, problem is not balanced. Therefore, we will insert a dummy column to balance the matrix

	C ₁	C ₂	C ₃	Dummy		Difference
M ₁	7	8	6	0	250/140/0	6*/1/1/1
			140	110		
M ₂	5	4	9	0	320/20/0	4/1/5* –
	300	20				
M ₃	7	5	5	0	280/40/0	5/0/0/0
		240	40			
	300	260	180	110		
	0	240	0	0		
		0				
Difference	2	1	1	0	10	
	*2	1	1	–	10	
	–	1	1	–	10*	
	–	*2	1	–		

15*

Hence There are $m + n - 1$ allocations. Hence Optimality test is to be performed.

	C ₁	C ₂	C ₃	Dummy	U _i
M ₁	8	7	6	0	0
	1	1	140	110	
M ₂	5	4	9	0	–2
	300	20	5	2	
M ₃	7	5	5	0	–1
	1	240	40	1	
V _j	7	6	6	0	

Since $\Delta_{ij} \geq 0$ Solution is optimum.

			Qty	Minimum Cost
M ₁	→	C ₃	140 x 6 =	840
	→	C ₄	110 x 0 =	0
M ₂	→	C ₁	300 x 5 =	1,500
	→	C ₂	20 x 4 =	80
M ₃	→	C ₂	240 x 5 =	1,200
	→	C ₃	40 x 5 =	200
			850	₹ 3,820

Q4

Ex. Book No.

Pg. No.

CMA Mat

The products of two plants A and B are to be transported to 3 warehouses W_1 , W_2 and W_3 . The cost of transportation of each unit from plants to the warehouses are indicated below:

Plants	Warehouses			Capacities
	(W_1)	(W_2)	(W_3)	
A	25	17	25	300
B	15	10	18	500
Demand	300	300	500	800/1,100

Find the optimum distribution and the optimum cost.

Reference

Minimisation

What's New
Unbalanced

Answer

	W_1	W_2	W_3				
A	25	17	25	300/0	8	8	8
		100	200				
B	15	10	18	500/200/0	5	5	8
	300	200					
C	0	0	0	300/0	0	–	–
			300				
	300	300	500				
	0	100	200				
		0	0				
	15	10	18*				
	10*	7	7				
	–	7	7				

There are $m + n + 1$ allocation optimality is to be performed.

	25		17		25	U_i
	3	100		200		0
	15		10		18	-7
300		200			0	
	0		0		0	-25
	3		8	300		
V_j	22	17		25		

As $\Delta_{ij} \geq 0$, the solution is optimum. Therefore the minimum cost is as follows:

		Quantity	Minimum Cost
A	W_2	100 x 17	1,700
	W_3	200 x 25	5,000
B	W_1	300 x 15	4,500
	W_2	200 x 10	2,000
	Dummy W_3	300 x 0	0
		1,100	₹ 13,200

Optimum cost: ₹ 13,200

Q5

Ex. Book No.

Pg. No.

CMA Mat

The products of three plants F_1 , F_2 and F_3 are to be transported to 5 warehouses W_1 , W_2 , W_3 , W_4 and W_5 . The capacities of plants, demand of warehouses and the cost of transportation from one plant to various warehouses are indicated in the following table:

	W_1	W_2	W_3	W_4	W_5	Plant Capacity
F_1	74	56	54	62	68	400
F_2	58	64	62	58	54	500
F_3	66	70	52	60	60	600
Warehouse Demand	200	280	240	360	320	1,500/1,400

- Find** out a distribution plan of products from plants to the warehouses at a minimum cost. What is the minimum cost?
- Is** there any surplus capacity of the plants? If so, in which plant should we associate that surplus capacity?
- Is** there any alternate solution for the optimum solution achieved in above?

Reference

Minimisation

What's **New**

Unbalanced

Answer

								Difference	
74	56	54	62	68	0	400/300/20/0	54/2/6/6/6/6		
280		20		100					
58	64	62	58	54	0	500/300/0	*54/4/4/4/4 -		
200			300						
66	70	52	60	60	0	600/360/340/0	52/8/0/0/0/0/-		
		240	340	20					

$\frac{200}{0}$	$\frac{280}{0}$	$\frac{240}{0}$	$\frac{360}{340}$	$\frac{320}{20}$	$\frac{100}{0}$
			$\frac{0}{0}$	$\frac{0}{0}$	
$\frac{8}{8}$	$\frac{8}{8}$	$\frac{2}{2}$	$\frac{2}{2}$	$\frac{6}{6}$	$\frac{0}{-}$
$\frac{8}{8}$	$\frac{8}{8^*}$	$\frac{2}{-}$	$\frac{2}{2}$	$\frac{6}{6}$	$\frac{-}{-}$
$\frac{8}{8^*}$	$\frac{-}{-}$	$\frac{-}{-}$	$\frac{2}{2}$	$\frac{6}{6}$	$\frac{-}{-}$
$\frac{-}{-}$	$\frac{-}{-}$	$\frac{-}{-}$	$\frac{2}{2}$	$\frac{6}{6^*}$	$\frac{-}{-}$
$\frac{-}{-}$	$\frac{-}{-}$	$\frac{-}{-}$	$\frac{2}{2}$	$\frac{8}{8^*}$	$\frac{-}{-}$
$\frac{-}{-}$	$\frac{-}{-}$	$\frac{-}{-}$	$\frac{2}{2}$	$\frac{-}{-}$	$\frac{-}{-}$

	W_1	W_2	W_3	W_4	W_5	W_6	Dummy	U_i
F_1	74 8	56 280	54 0	62 20	68 6	0 100	0	
F_2	58 200	64 16	62 16	58 4	54 300	0 8	-8	
F_3	66 2	70 16	52 240	60 340	60 20	0 2	-2	
V_i	66	56	54	62	62		0	

Since $m + n - 1$ allocation, optimality test can be performed. Since Δ_{ij} is > 0 , the solution is optimal.

		Qty		Minimum Cost
F_1	W_2	280×56	=	15,680
	W_4	20×62	=	1,240
	Dummy	100×0	=	0
F_2	W_1	200×58	=	11,600
	W_5	300×54	=	16,200
F_3	W_3	240×52	=	12,480
	W_4	340×60	=	20,400
	W_5	20×60	=	1,200
		1,500		₹ 78,800

Alternative Solution:

	74		56		54		62		68		0
		280			20					100	
	58		64		62		58		54		0
200								300			
	66		70		52		60		60		0
				220		360		20			

F_1	W_2	$280 \times 56 =$	15,680
	W_4	$20 \times 54 =$	1,080
	Dummy	$100 \times 0 =$	0
F_2	W_1	$200 \times 58 =$	11,600
	W_5	$300 \times 54 =$	16,200
F_3	W_3	$220 \times 52 =$	11,440
	W_4	$360 \times 60 =$	21,600
	W_5	$20 \times 60 =$	1,200
		1,500	₹ 78,800

Q 6

Ex. Book No.

Pg. No.

CMA Mat

A manufacturing company has three plants at locations X, Y and Z which supply to the distributors located at A, B, C, D and E. Monthly capacities are 80, 50 and 90 units respectively. Monthly requirements of distributors are 40, 40, 50, 40 and 80 units respectively. Unit transportation costs are given below in Rupees.

	To	A	B	C	D	E
From	X	5	8	6	6	3
	Y	4	7	7	6	6
	Z	8	4	6	6	3

Determine an optimum distribution for the company in order to minimise the total transportation cost.

Reference

Minimisation

What's New

Unbalanced and Degenerated

Answer

X	5	8	6	6	3	80/0	2/2
Y	4	7	7	6	6	50/10/0	2/2/1/1
Z	8	4	6	6	3	90/50/0	1/1/2/2/0
Dummy	0	0	0	0	0	30/0	0
	40	40	50	40	80		
	0	0	0	10	0		
	4	4	6	6	3		
	1	3	1	0	3		
	4*	3*	1	0			
			1	0			



As $\Delta_{ij} \geq 0$, the solution is optimum. Hence the schedule and cost is as follows:

		Quantity	Minimum Cost
X	E	80 x 3	240
Y	A	40 x 4	160
	D	10 x 6	60
Z	B	40 x 4	160
	C	50 x 6	300
Dummy	D	30 x 0	0
		250	₹ 920

Minimum Cost ₹ 920 / -

Q7

Ex. Book No.

Pg. No.

CMA Mat; MTP Jun'17

A Company has 4 factories F_1, F_2, F_3 and F_4 , manufacturing the same product. Production and raw material costs differ from factory to factory and are given in the table below in the first two rows. The transportation costs from the factories to the sales depots S_1, S_2 and S_3 are also given. The last two columns in the table below give the sales price and total requirements at each depot and the production capacity of each factory is given in the last row.

	F_1	F_2	F_3	F_4	Sales Price/Unit (₹)	Requirement
Production Cost/Unit (₹)	15	18	14	13		
Raw Materials Cost/Unit (₹)	10	9	12	9		
Transportation Cost/Unit (₹)						
S_1	3	9	5	4	34	80
S_2	1	7	4	5	32	120
S_3	5	8	3	6	31	150
Production capacity	10	150	50	100		

Determine the optimal solution and the associated profit by using the Vogel's Approximation Method (VAM).

Reference

Maximisation (Opportunity loss matrix)

What's New

Answer

Setp 1: To convert the above matrix into cost matrix by adding production cost, raw material cost and transportation cost.

	F ₁	F ₂	F ₃	F ₄	
S ₁	28	36	31	26	80
S ₂	26	34	30	27	120
S ₃	30	35	29	28	150
	10	150	50	100	350
					310

Setp 2: Convert the cost matrix to profit matrix by reducing cost from sale price and inserting dummy column (F₅) to balance the matrix.

	F ₁	F ₂	F ₃	F ₄	F ₅	
S ₁	6	-2	3	8	0	80
S ₂	6	-2	2	5	0	120
S ₃	1	-4	2	3	0	150
	10	150	50	100	40	350
						350

Step 3: Convert the profit matrix to loss matrix.

	F ₁	F ₂	F ₃	F ₄	F ₅	
S ₁	2	10	5	0	8	80/0
			80			2
S ₂	2	10	6	3	8	120/110/90/0
	10	90	20			1/1/3*/2/2
S ₃	7	12	6	5	8	150/100/6/0
		60	50		40	1/1/1/2*/4
	10	150	50	100	40	
	0	0	0	20	0	
				0		
	0	0	1	3*	0	
5*	2	0	2	2	0	
	2	0	2	2	0	
	2	0	0			
	2					

$m + n - 1 = \text{No. of allocations}$

$$\therefore 3 + 5 - 1 = 7$$

$$\therefore 7 = 7$$

$\therefore$ There is no degeneracy.

Hence optimality test can be performed.

	2	10	5	0	8
	3	3	4	80	5
	2	10	6	3	8
10		90	2	20	2
	7	12	6	5	8
	3	60	50	0	40

V_j -1 7 1 0 3 0

U_i

0

3

5

As $\Delta_{ij} \geq 0$, the solution is optimum

		Qty		Maximum Profit
S_1	F_4	80×8	=	640
S_2	F_1	10×6	=	60
	F_2	$90 \times (-2)$	=	(-) 180
	F_4	20×5	=	100
S_3	F_2	60×-4	=	(-) 240
	F_3	50×2	=	100
	F_5 (Dummy)	40×0	=	0
		350		₹ 480

Q8

Ex. Book No.

Pg. No.

CMA Mat

Departmental store wishes to purchase the following quantities of Sprees:

Types of spreess	A	B	C	D	E
Quantity	150	100	75	250	200

Tenders are submitted by 4 different manufacturers who undertake to supply not more than the quantities mentioned below (all types of spreess combined):

Manufacturer	W	X	Y	Z
Total quantity	300	250	150	200

The store estimates that its profit/spree will vary with the manufacturer as shown in the following matrix.

Manufacturers	Spree				
	A	B	C	D	E
W	275	350	425	225	150
X	300	325	450	175	100
Y	250	350	475	200	125
Z	325	275	400	250	175

How should the orders be placed?

Reference

Maximisation (Opportunity loss matrix)

What's **New**

Unbalanced

Answer

As the demand is not equal to supply matrix is not balanced. Therefore we will insert a dummy column to balance the solution:

Profit Matrix

	A	B	C	D	E	F	
W	275	350	425	225	150	0	300
X	300	325	450	175	100	0	250
Y	250	350	475	200	125	0	150
Z	325	275	400	250	175	0	200
	100	75	250	200	125		

Given matrix is profit matrix, therefore, we will convert it into loss matrix by reducing all the elements from the highest element i.e., 475.

Loss Matrix

	200		125		50		250		325		475	
		25				50		200		25		
	175		150		25		300		375		475	
150										100		
	225		125		0		275		350		475	
		75		75								
	150		200		75		225		300		475	
						200						

300/275/225/25/0

250/100/0

150/75/0

200/0

$\frac{150}{0}$	$\frac{100}{25}$	$\frac{75}{0}$	$\frac{250}{50}$	$\frac{200}{0}$	$\frac{125}{0}$
	$\frac{25}{0}$		$\frac{50}{0}$		
$\frac{25}{25}$	$\frac{0}{0}$	25	$\frac{25}{25}$	$\frac{25}{25}$	$\frac{0}{0}$
$\frac{25}{25}$	$\frac{0}{25}$	–	$\frac{25}{25}$	$\frac{25}{25}$	$\frac{0}{0}$
$\frac{25}{25}$	$\frac{25}{-}$	–	$\frac{25}{25}$	$\frac{25}{25}$	$\frac{0}{0}$
$\frac{25}{-}$	$\frac{-}{-}$	–	$\frac{25}{25}$	$\frac{25}{25}$	$\frac{0}{0}$
$\frac{-}{-}$	$\frac{-}{-}$	–	$\frac{50}{50}$	$\frac{50}{50}$	$\frac{0}{0}$

Difference

75	75*	75	50	75	75	150
125	25	25	125	75	75	100
125*	100*	–	–	–	–	–
75	50	75	75*	–	–	–

$m + n - 1$ allocations are there, optimality test can be performed.

	200		125		50		250		325		475		0
	25	25			50	50		200		25			0
150	175		150		25		300		375		475		0
	25		25		50		50		100				0
	225		125		0		275		350		475		0
	50	75		75		25		25		0			0
	150		200		75		225		300		475		-25
	0		100		100	200		0		25			
	175		125		0		250		325		475		

As $\Delta_{ij} \geq 0$, maximum profit is as follows.

		Qty	Maximum Profit
W	B	25 x 350 =	8,750
	D	50 x 225 =	11,250
	E	200 x 150 =	30,000
	F	25 x 0 =	0
X	A	150 x 300 =	45,000
	F	100 x 0 =	0
Y	B	75 x 350 =	26,250
	C	75 x 475 =	35,625
Z	D	200 x 250 =	50,000
Max. Profit.			₹ 2,06,875

Q9

Ex. Book No.

Pg. No.

CMA Mat

A company manufacturing television sets has four plants with a capacity of 125, 250, 175 and 100 units respectively. The company supplies T.V. sets to its four show rooms which have demand of 100, 400, 90 and 60 units respectively. Due to the differences in the raw material cost and the transportation cost, the profit per unit (in ₹) differ which are given in the following table:

		Showroom			
		I	II	III	IV
Plants	I	90	100	120	110
	II	100	105	130	117
	III	111	109	110	120
	IV	130	125	108	113

By using Vogel's approximation method, **plan** the production programme so as to maximise the profit. Also **determine** the maximum total profit.



Reference

Maximisation (Opportunity loss matrix)

What's New

Degenerated

Answer

Profit matrix

		Show Room				
		I	II	III	IV	
Plants	I	90	100	120	110	125
	II	100	105	130	117	250
	III	111	109	110	120	175
	IV	130	125	108	113	100
		100	400	90	60	650

Loss Matrix:

	40		30		10		20
40		125					
	30		25		0		13
		100		90		60	
	19		21		20		10
		175					
	0		5		22		17
100							

125/0 10/10/10/10

250/160/100/0 13/13/13*/12

175/0 99/-/-/-

100/0 5/-/-/-

<u>100</u>	<u>400</u>	<u>90</u>	<u>60</u>
0	225	0	0
	125		
	0		
19*	16	10	3
-	16	10	3
-	5	10	7
-	5	-	7

As there are $m+n-1=7$ but there are 6 allocations only. Therefore a notional allocation called ϵ (epsilon) is to be placed in the least cost unallocated cell.

	40		30		10		20	U
	15	125			5		2	0
	30		25		0		13	-5
	10	100		90		60		
	19		21		20		10	-9
	3	175		130	24		1	
	0		5		22		17	-25
100			€		42		24	
25	30	5	18					

As $\Delta_{ij} = C_{ij} - (U_{ij} + V_{ij}) \geq 0$, the solution is optimum and therefore maximum profit is as follows:

		Quantity	Maximum Profit
I	II	125 x 100	12,500
II	II	100 x 105	10,500
	III	90 x 130	11,700
	IV	60 x 117	7,020
III	II	175 x 109	19,075
IV	I	130 x 100	13,000
Maximum profit			₹ 73,795

Q10

Ex. Book No. Pg. No.

CMA Mat

A company has 3 plants located at different places but producing an identical product. The cost of production, distribution cost of each plant to the 3 different warehouses, the sale price at each warehouse and the individual capacities for both the plant and warehouse are given below:

Plants	F ₁	F ₂	F ₃		
Raw material	15	18	14		
Other expenses	10	9	12		

Distribution cost to warehouse

	Sales Price in (₹)				Warehouse Capacity (No)
W_1	3	9	5	34	80
W_2	1	7	4	32	110
W_3	5	8	3	31	150
Capacity of Plant (No.)	150	100	130		

Establish a suitable table giving net profit/loss for a unit produced at different plants and distributed at different locations.

- Introduce** a suitable dummy warehouse / plant so as to match the capacities of plants and warehouses.
- Find** distribution pattern so as to maximise profit / minimise loss.
- Interpret** zero value of square evaluation of an empty cell and **find** alternative solutions.

Reference

Maximisation

What's New

Unbalanced and Alternate Solution

Answer

Profit matrix

6	-2	3	80
6	-2	2	110
1	-4	2	150
0	0	0	40
150	100	130	380

Loss Matrix:

	0		8		3
40		40			
	0		8		4
110					
	5		10		4
		20		130	
	6		6		6
		40			

80/40/0

3/3/5

110/0

4* --

150/20/0

1/1/6*

40/0

0/0/0

$$\begin{array}{r} 150 \\ 40 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 100 \\ 0 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 130 \\ 0 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 5 \\ 5^* \\ \hline - \\ - \end{array}$$

$$\begin{array}{r} 2 \\ 2 \\ \hline 2 \\ 2 \end{array}$$

$$\begin{array}{r} 1 \\ 1 \\ \hline 1 \\ - \end{array}$$

	8		4		3
40		40			1
	0		8		4
110			0		2
	5		10		4
	3	20		130	
	6		6		6
	8	40			6

U

0

0

2

-2

0

8

2

As there are $(m + n) - 1$ allocations, optimality test can be performed since $\Delta_{ij} \geq 0$,

		Quantity	Maximum Profit
F_1	W_1	40 x 6	240
	W_2	40 x -2	-80
F_2	W_1	110 x 6	660
F_3	W_2	20 x -4	-80
	W_3	130 x 2	260
F_4 Dummy	W_2	40 x 0	0
		380	₹ 1,000

Profit ₹ 1,000 / -

SJC Institute



Learning Objective

Theory

1. APPLICATION OF SIMULATION
2. ADVANTAGES OF SIMULATION
3. LIMITATION OF SIMULATION
4. STEPS INVOLVED IN CONSTRUCTING AND USING A SIMULATION MODEL
5. DESIGN OF SIMULATION MODEL
6. TYPES OF SIMULATION

Practical

1. DEMAND SIMULATION
2. PRODUCTION SIMULATION
3. CASH FLOW SIMULATION
4. PROFIT SIMULATION
5. WAITING LINE SIMULATION

1. SIMULATION TECHNIQUE [JUN'17; MTP DEC'19]

With a computer simulation model, a manager or system analyst is able to observe the behaviour of a process without the necessity of experimenting with the actual system.

In order to evaluate the system's performance given various disturbances, or to identify the bottlenecks, they may try out different manufacturing runs, new operational conditions, new equipment layouts or different cycle times.

A simple example of a simulation involves the tossing of a ball into the air. The ball can be said to "simulate" a missile, for instance. That is, by experimenting with throwing balls starting at different initial heights and initial velocity vectors, it can be said that we are simulating the trajectory of a missile. This kind of simulation is known as analog simulation since it involves a physical model of a ball. A flight simulator on a PC is a computer model of some aspects of the flight: it shows on the screen the controls and what the "pilot" (the youngster who operates it) is supposed to see from the "cockpit" (his armchair).

2. APPLICATION OF SIMULATION [RTP JUN'18; MTP DEC'17; MTP DEC'18]

- (i) Scheduling aircraft,
- (ii) Job-ship scheduling and personnel scheduling,
- (iii) Manpower-hiring decisions,
- (iv) Traffic light-timing,
- (v) Transport-scheduling,
- (vi) Evaluating alternative investment opportunities, and
- (vii) Design of parking lots, harbor, and communication systems etc.

3. ADVANTAGES OF SIMULATION [RTP JUN'18]

- (a) Enables to **experiment** and **study complex interactions** of a system (e.g. company operations, economic policies).
- (b) Possible to study the effects of organizational environment informational changes in the operations of a system (e.g. number of stocking points, industrial policies).
- (c) Better **insight** and understanding of a complex system to indication for improvement.
- (d) Assists in **teaching and training** (management games).
- (e) **New situations** policies can be protested.
- (f) **Probabilistic features** can be easily incorporated.
- (g) A process can be studies in **extended or compressed time**.
- (h) **Risks** involved in experimenting with real problems can be **eliminated**.

4. LIMITATIONS OF SIMULATION [DEC'21]

- (a) Simulated results are **not precise**. Unlike mathematical models, it does not give optimum solutions. At times one may not be able to assess the extent of error in a simulated result.
- (b) Some situations are **not amenable** to simulation.
- (c) Simulation may be **expensive** needing advanced computer supports
- (d) Simulation by itself does not generate solutions, but **only indicates a way of evaluating** solutions.
- (e) It is often a long, **complicated** process to develop a model.

5. STEPS INVOLVED IN CONSTRUCTING AND USING A SIMULATION MODEL

All simulation projects follow the following steps :

- (i) Define the problem
- (ii) Data collection
- (iii) Problem analysis
- (iv) Simulation model specification
- (v) Model programming
- (vi) Model validation
- (vii) Simulation experimentation
- (viii) Evaluation and interpretation of simulation results
- (ix) Report generation and plans for implementation of the results

6. DESIGN OF SIMULATION MODEL

One should consider the characteristic of the system in question and the nature of the problems to be tackled while choosing a modeling approach. In general the simulation model used in the area of Manufacturing Systems Engineering (MSE) can be classified as :

- (i) Static and Dynamic Simulation Models
- (ii) Continuous and Discrete Simulation Model
- (iii) Stochastic and Deterministic Simulation Model
- (iv) Social Simulation
- (v) Web based Simulation
- (vi) Parallel and Distributed Simulation

7. TYPES OF SIMULATION

Simulators are usually divided into the following categories or simulation modes :

- (i) Behavioral simulation
- (ii) Functional simulation
- (iii) Static timing analysis
- (iv) Gate-level simulation
- (v) Switch-level simulation
- (vi) Transistor-level or circuit-level simulation

This list is ordered from **high-level to low-level simulation** (high-level being more abstract, and low-level being more detailed). Proceeding from high-level to low-level simulation, the simulations become more **accurate**, but they also become **progressively more complex** and take longer to run. While it is just possible to perform a behavioral- level simulation of a complete system, it is **impossible to perform a circuit-level simulation of more than a few hundred transistors**.

(vii) Monte Carlo Simulation

Monte Carlo simulation, or probability simulation, is a technique used to understand the impact of risk and uncertainty in financial, project management, cost, and other forecasting models.

Uncertainty in Forecasting Models

When you develop a forecasting model – any model that plans ahead for the future – you make certain assumptions. These might be assumptions about the investment return on a portfolio, the cost of a construction project, or how long it will take to complete a certain task. Because these are projections into the future, the best you can do is estimate the expected value.

You can't know with certainty what the actual value will be, but based on historical data, or expertise in the field, or past experience, you can draw an estimate. While this estimate is useful for developing a model, it contains some inherent uncertainty and risk, because it's an estimate of an unknown value.

Estimating Ranges of Values

In some cases, it's possible to estimate a range of values. In a construction project, you might estimate the time it will take to complete a particular job; based on some expert knowledge, you can also estimate the absolute maximum time it might take, in the worst possible case, and the absolute minimum time, in the best possible case. The same could be done for project costs. In a financial market, you might know the distribution of possible values through the mean and standard deviation of returns.

By using a range of possible values, instead of a single guess, you can create a more realistic picture of what might happen in the future. When a model is based on ranges of estimates, the output of the model will also be a range.

This is different from a normal forecasting model, in which you start with some fixed estimates – say the time it will take to complete each of three parts of a project – and end up with another

value – the total time for the project. If the same model were based on ranges of estimates for each of the three parts of the project, the result would be a range of times it might take to complete the project. When each part has a minimum and maximum estimate, we can use those values to estimate the total minimum and maximum time for the project.

SJC Institute

Q1

Ex. Book No.

Pg. No.

MTP Jun'19; MTP Dec'19

The following was the pattern for demand of cars rented out by a tourist operator observed for 100 days :

No. of cars	5	7	10	15
No. of days	20	30	40	10

The random numbers are 88, 76, 10, 05, 23 Required :

- Simulate** the demand for cars over five days.
- How many** cars should the operator have in order to have at least 75% probability of fulfilling the demand based on your simulated results ? [5 + 3 = 8]

Reference

Demand Simulation

What's New

No. of cars to meet the designed probability

Answer

No. of cars	No. of Days	Cumulative Prob	Probability	Random No. Interval	Day	Random No.	Demand
5	20	0.20	0.20	00-19	1	88	10
7	30	0.30	0.50	20-49	2	76	10
10	40	0.40	0.90	50-89	3	10	5
15	10	0.10	1.00	90-99	4	05	5
					5	23	7

If 5 cars = 2 days demand will be fulfilled = $2/5 = 40\%$

If 7 cars = 3 days demand will be fulfilled = $3/5 = 60\%$

If 10 cars = 5 days demand will be fulfilled = $5/5 = 100\%$

As minimum requirement of demand fulfillment is 75%, it should have 10 cars.

Q2

Ex. Book No.

Pg. No.

CMA Mat; MTP Jun'22

A confectioner sells confectionery items. Past data of demand per week in hundred kilograms with frequency is given below:

Demand/Week	0	5	10	15	20	25
Frequency	2	11	8	21	5	3

Using the following sequence of random numbers, generate the demand for the next 10 weeks. Also find out the average demand per week

Random numbers	35	52	13	90	23	73	34	57
	35	83	94	56	67	66	60	

Reference	What's New
Demand simulation	Average Demand

Answer

Random No. Range Table for demand

Demand per week	Frequency	Probability	Cumulative Probability	Range
0	2	.04	.04	0-3
5	11	.22	.26	4-25
10	8	.16	.42	26-41
15	21	.42	.84	42-83
20	5	.10	.94	84-93
25	3	.06	1.00	94-99
	$\Sigma f = 50$	1.00		

Simulated Values for next 10 weeks

Weeks	R. Nos.	Demand
1	35	10
2	52	15
3	13	5
4	90	20
5	23	5
6	73	15
7	34	10
8	57	15
9	35	10
10	83	15
		120

$$\text{Average weekly demand} = \frac{120}{10} = 12$$

Q3

Ex. Book No.

Pg. No.

CMA Mat

The manager of a book store has to decide the number of copies of a particular tax law book to order. A book costs ₹ 60 and is sold for ₹ 80. Since some of the tax laws change year after year, any copies unsold while the edition is current must be sold for ₹ 30. From past records, the distribution of demand for this book has been obtained as follows:

Demand (No of copies)	15	16	17	18	19	20	21	22
Proportion	0.05	0.08	0.20	0.45	0.10	0.07	0.03	0.02

Using the following sequence of random numbers, **generate** the demand for 20 time periods (years). **Calculate** the average profit obtainable under each of the courses of action open to the manager. **What** is the optimal policy?

14	02	93	99	18	71	37	30	12	10
88	13	00	57	69	32	18	08	92	73

Reference

Demand Simulation

What's New

Assuming Stock levels 15,16,17,18

Average Profit and Optimal Solution

Answer

Random No. Range Table

Demand	Probability	Cumulative Probability	Random Range
15	.05	.05	0-4
16	.08	.13	5-12
17	.20	.33	13-32
18	.45	.78	33-77
19	.10	.88	78-87
20	.07	.95	88-94
21	.03	.98	95-97
22	.02	1.00	98-99
	1.00		

Calculation of demand and profit for next 20 years

Year	Random Numbers	Expected demand	No. of books unsold if stock is		
			16	17	18
1	14	17	-	-	1
2	02	15	1	2	3
3	93	20	-	-	-

4	99	22	-	-	-
5	18	17	-	-	1
6	71	18	-	-	-
7	37	18	-	-	-
8	30	17	-	-	1
9	12	16	-	1	2
10	10	16	-	1	2
11	88	20	-	-	-
12	13	17	-	-	1
13	00	15	1	2	3
14	57	18	-	-	-
15	69	18	-	-	-
16	32	17	-	-	1
17	18	17	-	-	1
18	08	16	-	1	2
19	92	20	-	-	-
20	73	18	-	-	-
Total			2	7	18

Statement Showing Computation of Profit

No. of Books order	No. of Books sold	Profit	Average Profit
15	$15 \times 20 = 300$	₹ 6000	₹ 300
16	$16 \times 20 - 2 = 318$	₹ 6300	₹ 315
		$(318 \times 20) - 2 \times 30$	
17	$(17 \times 20) - 7 = 333$	₹ 6450	₹ 322.5
		$(333 \times 20) - 7 \times 30$	
18	$(18 \times 20) - 18$	₹ 6300	₹ 315
		$(342 \times 20) - 18 \times 30$	

Since profit is more at 17 books order, it is the best quantity and ordering is more optimum.

Q4

Ex. Book No.

Pg. No.

CMA Mat; MTP Jun'18; MTP Dec'18

An automobile production line turns out about 100 cars a day, but deviations occur owing to many causes. The production is more accurately described by the probability distribution given below

Production/Day	Prob.	Production/Day	Prob.
95	0.03	101	0.15
96	0.05	102	0.10

97	0.07	103	0.07
98	0.10	104	0.05
99	0.15	105	0.03
100	0.20		
		Total	1.00

Finished cars are transported across the bay, at the end of each day, by ferry. If the ferry has space for only 101 cars, **what** will be the average number of cars waiting to be shipped, and **what** will be the average number of empty space on the boat? Random numbers are given below:

20	63	46	16	45	41	44	66	87	26	78	40	29	92	21

Reference

Production Simulation

What's New

Average no. of cars waiting and average no. of empty space

Answer

Simulation of data of an Automobile Production line

Production/ day	Probability	Cumulative Probability	Random No. Range
95	0.03	0.03	0-2
96	0.05	0.08	3-7
97	0.07	0.15	8-14
98	0.10	0.25	15-24
99	0.15	0.40	25-39
100	0.20	0.60	40-59
101	0.15	0.75	60-74
102	0.10	0.85	75-84
103	0.07	0.92	85-91
104	0.05	0.97	92-96
105	0.03	1.00	97-99
	1.00		

Simulated data

Day	Random No.	Production	No.of cars waiting to be shipped	No. of empty space on the boat
1	20	98	-	3
2	63	101	-	-

3	46	100	-	1
4	16	98	-	3
5	45	100	-	1
6	41	100	-	1
7	44	100	-	1
8	66	101	-	-
9	87	103	2	-
10	26	99	-	-
11	78	102	1	-
12	40	100	-	-
13	29	99	-	2
14	92	104	3	-
15	21	98	-	3
Total			6	12

Average no. of cars waiting to be shipped = $6 / 15 = 0.40$

Average no. of empty space on the boat = $12 / 15 = 0.8$

Q5

Ex. Book No.

Pg. No.

CMA Mat; Dec'18; MTP Jun'17;
MTP Dec'17

A Small retailer has studied the weekly receipts and payments over the past 200 weeks and has developed the following set of information:

Weekly Receipts (₹)	Probability	Weekly Payments (₹)	Probability
3000	0.20	4000	0.30
5000	0.30	6000	0.40
7000	0.40	8000	0.20
12000	0.10	10000	0.10

Using the following set of random numbers, **simulate** the weekly pattern of receipts and payments for the 12 weeks of the next quarter, assuming further that the beginning bank balance is ₹ 8000. **What** is the estimated balance at the end of the 12 weekly period? **What** is the highest weekly balance during the quarter? **What** is the average weekly balance for the quarter?

Random Numbers

For Receipts	03	91	38	55	17	46	32	43	69	72	24	22
For payments	61	96	30	32	03	88	48	28	88	18	71	99

According to the given information, the random number interval is assigned to both the receipts and the payments.

Reference Cash Flow Simulation	What's New Average Cash Balance

Answer

Range of random numbers

Receipt (₹)	Probability	Cumulative probability	Range	Payments (₹)	Probability	Cumulative probability	Range
3000	0.20	0.20	0-19	4000	0.30	0.30	0-29
5000	0.30	0.50	20-49	6000	0.40	0.70	30-69
7000	0.40	0.90	50-89	8000	0.20	0.90	70-89
12000	0.10	1.00	90-99	10000	0.10	1.00	90-99

Simulation of Data for a period of 12 weeks

Week	Random No. for receipt	Expected Receipt (₹)	Random No. for payment	Expected Payment (₹)	Week end Balance (₹)
Opening Balance					8000
1	03	3000	61	6000	5000 (8000 + 3000 – 6000)
2	91	12000	96	10000	7000
3	38	5000	30	6000	6000
4	55	7000	32	6000	7000
5	17	3000	03	4000	6000
6	46	5000	88	8000	3000
7	32	5000	48	6000	2000
8	43	5000	28	4000	3000
9	69	7000	88	8000	2000
10	72	7000	18	4000	5000
11	24	5000	71	8000	2000
12	22	5000	99	10000	(3000)

Estimated balance at the end of 12th week	=	₹ (3,000)
Highest balance	=	₹ 7,000
Average balance during the quarter	=	45,000/12 = ₹ 3,750

Q 6

Ex. Book No.

Pg. No.

RTP Jun'18

The management of ABC company is considering the question of marketing a new product. The fixed cost required in the project is ₹ 4,000. Three factors are uncertain, viz., the selling price, variable cost and the annual sales volume. The product has a life of only one year. The management has the data on these three factors as under:

Selling price (₹)	Probability	Variable cost (₹)	Probability	Sales volume (units)	Probability
3	0.2	1	0.3	2,000	0.3
4	0.5	2	0.6	3,000	0.3
5	0.3	3	0.1	5,000	0.4

Consider the following sequence of thirty random numbers:

81	32	60	04	46	31	67	25	24	10	40	02	39	68	08
59	66	90	12	64	79	31	86	68	82	89	25	11	98	16

Using the sequence (first 3 random numbers for the first trial, etc.), **simulate** the average profit for the above project on the basis of 10 trials.

Reference	What's New
Profit Simulation	Average Profit

Answer

First, we assign random numbers to the three uncertain factors.

	Probability	Cum. Probability	Random Numbers
Selling Price (₹)			
3	0.2	0.2	00 - 19
4	0.5	0.7	20 - 69
5	0.3	1.0	70 - 99
Variable Cost (₹)			
1	0.3	0.3	00 - 29
2	0.6	0.9	30 - 89
3	0.1	1.0	90 - 99
Sales Volume (units)			
2,000	0.3	0.3	00 - 29
3,000	0.3	0.6	30 - 59
5,000	0.4	1.0	60 - 99

Simulation sheet for finding average profit is developed as under:

S. No.	RN	Selling Price (₹)	RN	Variable Cost (₹)	RN	Sales Volume ('000 units)
1	81	5	32	2	60	5
2	04	3	46	2	31	3
3	67	4	25	1	24	2
4	10	3	40	2	02	2
5	39	4	68	2	08	2
6	59	4	66	2	90	5
7	12	3	64	2	79	5
8	31	4	86	2	68	5
9	82	5	89	2	25	2
10	11	3	98	3	16	2

Simulated profit in 10 trials would be as follows:

S. No	Simulated profit	Total Simulated profit
1	$(5 - 2) \times 5,000 - 4,000$	11,000
2	$(3 - 2) \times 3,000 - 4,000$	-1,000
3	$(4 - 1) \times 2,000 - 4,000$	2,000
4	$(3 - 2) \times 2,000 - 4,000$	-2,000
5	$(4 - 2) \times 2,000 - 4,000$	0
6	$(4 - 2) \times 5,000 - 4,000$	6,000
7	$(3 - 2) \times 5,000 - 4,000$	1,000
8	$(4 - 2) \times 5,000 - 4,000$	6,000
9	$(5 - 2) \times 2,000 - 4,000$	2,000
10	$(3 - 3) \times 2,000 - 4,000$	-4,000
		21,000

Average Profit = ₹21,000 ÷ 10 = ₹2,100.

Q7

Ex. Book No.

Pg. No.

RTP Jun'18

An Investment Company wants to study the investment projects based on market demand, profit and the investment required. Which are independent of each other. Following probability distribution are estimated for each of the three factors.

Annual Demand (units)	25,000	30,000	35,000	40,000	45,000	50,000	55,000
Probability	0.05	0.10	0.20	0.30	0.20	0.10	0.05

Profit pu (₹)	3	5	7	9	10
Probability	0.10	0.20	0.40	0.20	0.10

Investment (₹)	27,50,000	30,00,000	35,00,000
Probability	0.25	0.50	0.25

Using Simulation Process, **repeat** the trial 10 times and compute the investment on each trial taking these factors into trial. **What** is the most likely return? Use the following Random Numbers. (30, 12, 16), (59, 09, 69), (63, 94, 26), (27, 08, 74), (64, 60, 61), (28, 28, 72), (31, 23, 57), (54, 85, 20), (64, 68, 18), (32, 31, 87). In the bracket above, the first Random Number is for Annual Demand, the second one is for Profit and the last one is for the Investment required.

Reference	What's New
Profit Simulation	Most likely return = Average return

Answer

1. Random Number Allocation Tables

Table 1: Random Numbers for Demand

Demand	Prob.	Cum Prob .	Random Nos.
25,000 units	0.05	0.05	00 – 04
30,000 units	0.10	0.15	05 – 14
35,000 units	0.20	0.35	15 – 34
40,000 units	0.30	0.65	35 – 64
45,000 units	0.20	0.85	65 – 84
50,000 units	0.10	0.95	85 – 94
55,000 units	0.05	1.00	95 – 99

Table 2: Random Numbers for Profit

Profit	Prob.	Cum Prob	Random Nos.
₹3	0.10	0.10	00 – 09
₹5	0.20	0.30	10 – 29
₹7	0.40	0.70	30 – 69
₹9	0.20	0.90	70 – 89
₹10	0.10	1.00	90 – 99

Table 3: Random Numbers for Investment

Investment	Prob.	Cum Prob	Random Nos.
₹27,50,000	0.25	0.25	00 – 24

₹30,00,000	0.50	0.75	25 – 74
₹35,00,000	0.25	1.00	75 - 99

2. Simulation table

Trial	R. No. (Demand)	Demand Qty (units)	R. No. (Profit)	Profit p.u. (₹)	R. No. (Invst)	Investment (₹)	Return = Qty x Pft (₹)	ROI = (R Invst) × 100
1	30	35,000	12	5	16	27,50,000	1,75,000	6.36%
2	59	40,000	09	3	69	30,00,000	1,20,000	4.00%
3	63	40,000	94	10	26	30,00,000	4,00,000	13.33%
4	27	35,000	08	3	74	30,00,000	1,05,000	3.50%
5	64	40,000	60	7	61	30,00,000	2,80,000	9.33%
6	28	35,000	28	5	72	30,00,000	1,75,000	5.83%
7	31	35,000	23	5	57	30,00,000	1,75,000	5.83%
8	54	40,000	85	9	20	27,50,000	3,60,000	13.09%
9	64	40,000	68	7	18	27,50,000	2,80,000	10.18%
10	32	35,000	31	7	87	35,00,000	2,45,000	7.00%
Total								78.45%

Result: Most Likely Average Return = $78.45\% \div 10 \text{ trials} = 7.85\%$ (approximately).

Also, Highest ROI = 13.33% relating to Trial 3, Investment = ₹30 Lakhs and Demand = 40,000 units.

Q8

Ex. Book No.

Pg. No.

CMA Mat; MTP Dec'19; MTP Dec'21

Patients arriving at a village dispensary are treated by a doctor on a first-come-first-served basis. The inter-arrival time of the patients is known to be uniformly distributed between 0 and 80 minutes, while their service time is known to be uniformly distributed between 15 and 40 minutes. It is desired to simulate the system and determine the average time a patient has to be in the queue for getting service and the proportion of time the doctor would be idle.

Carry out the simulation using the following sequences of random numbers. The numbers have been selected between 00 and 80 to estimate inter-arrival times and between 15 and 40 to estimate the service times required by the patients.

Series 1	07	21	12	80	08	03	32	65	43	74
Series 2	23	37	16	28	30	18	25	34	19	21

Reference

Waiting Line Simulation

What's New

- No Random Interval
- Average waiting time of Patient and Doctor

Answer

Simulation of data at a village dispensary

No. of patients	Inter arrival time Random No. (minutes)	Entry time in to queue (hrs)	Service Time Random No. (minutes)	Service Start time (hrs)	End time (hrs)	Waiting time of patient (minutes)	Idle time of doctor (minutes)
1	07	8.07	23	8.07	8.30	-	07
2	21	8.28	37	8.30	9.07	2	-
3	12	8.40	16	9.07	9.23	27	-
4	80	10.00	28	10.00	10.28	-	37
5	08	10.08	30	10.28	10.58	20	-
6	03	10.11	18	10.58	11.16	47	-
7	32	10.43	25	11.16	11.41	33	-
8	65	11.48	34	11.48	12.22	-	07
9	43	12.31	19	12.31	12.50	-	09
10	74	01.45	21	01.45	02.06	-	55
Total (in minutes)						129	115

Average waiting time of patient = $129 / 10 = 12.9$ minutes

Average waiting time of doctor = $115 / 10 = 11.5$ minutes

It has been assumed that starting time be 8.00 A.M.


 Ex. Book No. Pg. No.

Jun'19

The following information are given:

Arrival of patients		Services	
Inter-arrival time (minutes)	Probability	Inter-Service time (minutes)	Probability
2	0-20	4	0-25
4	0-24	6	0-34
6	0-28	8	0-26
8	0-18	10	Balance
10	Balance		

The following random number are to be used for the simulation of arrival and service patterns:

Arrival	740	225	906	048	421
Service	402	183	706	923	638

Required:

Find out the average time spent by the patient in the queue by simulation. Assume that the time starts at 6:00 a.m. and that there is only one counter and there is no time gap between finishing with one patient and starting the next patient if the next patient is already in the queue.

A second counter is to be set up if the probability of waiting beyond 3 minutes exceeds 40% or if the average waiting time of a patient exceeds 5 minutes if there is a wait. **Should** the second counter be set up? **Substantiate** based on the simulation results. [6 + 2] = 8

Reference Waiting Line Simulation	What's New • 3 digits Random Nos. • Average waiting time of Patient, Evaluation of second counter • If there is wait meaning

Answer

Arrivals				Service			
Minutes	Probability	Cumulative Probability	Random No. Interval	Minutes	Probability	Cumulative Probability	Random No. Interval
2	0.20	0.20	000-199	4	0.25	0.25	000-249
4	0.24	0.44	200-439	6	0.34	0.59	250-589
6	0.28	0.72	440-719	8	0.26	0.85	590-849
8	0.18	0.90	720-899	10	0.15	1.00	850-999
10	0.10	1.00	900-999				

Sl. No	Random No. Arrival	Time	Entry time Inqueue	Service Start time	Random No For Service	Service Time	Service End Time	Waiting Time Of Patient	Idle Time Of Centre
1	740	8	6:08	6:08	402	6	6:14	-	08
2	225	4	6:12	6:14	183	4	6:18	2	-
3	906	10	6:22	6:22	706	8	6:30	-	4
4	048	2	6:24	6:30	923	10	6:40	06	-
5	421	4	6:28	6:40	638	8	6:48	12	

Average waiting time of patients = $20/5 = 4$ mins

Probability of waiting time being more than 3 minutes = $2/5 = 40\%$

Hence is not exceeding 40%.

Average waiting time of a patient, if there is a wait = Total of waiting time column / 3 instances of waiting
 $= 20/3 = 6.67$ mins

Hence the second counter should be set up based on simulated results.

Note: Ideally, for best results and interpretation, simulation should be carried out a large number of times. The decision based on small numbers, tend to be unrealistic and erroneous.

SJC Institute

Project Management



Learning Objective

1. NETWORK DIAGRAM
 - (a) Activity
 - (b) Sequencing of Activities
 - (c) Event
 - (d) Numbering of Events (FULKERSON's RULE)
 - (e) Duplicating
 - (f) Dangling
 - (g) Dummy Activities:
2. PROJECT DURATION CALCULATION AND CRITICAL PATH METHOD (CPM):
 - (a) Duration
 - (b) Path or Route
 - (c) Critical Path
 - (d) Project Duration or Duration of Critical Path
 - (e) Forward Pass Calculation (Left to Right)
 - (f) Backward Pass Calculation (Right to Left)
 - (g) Earliest Finish time of an activity
 - (h) Latest Start time of an activity
 - (i) Slack
 - (j) TOTAL FLOAT, FREE FLOAT, INDEPENDENT FLOAT
3. UPDATING A NETWORK
4. PROJECT CRASHING
 - Process of Crashing

1. MEANING OF PROJECT MANAGEMENT

A. PROJECT MEANS A WORK WITH THE FOLLOWING CHARACTERISTICS

- (1) Customized
- (2) Uniqueness
- (3) Requires time and resource
- (4) Divided into several activities

The project management is the use of tools and techniques of management to satisfy the need and expectations of stakeholders.

B. THE OBJECTIVE OF PROJECT MANAGEMENT IS TWOFOLD

- (1) Deliver the project within time
- (2) Satisfy the stakeholder from the quality

2. NETWORK ANALYSIS [JUN'19; MTP JUN'19]

Network analysis is the general name given to certain specific techniques which can be used for planning, management and control of project. It often acts as a network management tool for breaking down projects into components or individual activities and recording the result on a flow chart or network diagram. These results generally reveal information that is used to determine duration, resource limitations and cost estimates associated with the project.

It offers insight into what is occurring at each critical point of the network. **Project management and efficient resource allocation are two critical aspects of the production and operations managers' responsibilities.** Since a project is non-repetitive and temporal in nature, the mode of management differs from the usual job shop or other related types of scheduling.

Network analysis enables us to take a systematic quantitative structural approach to the problem of managing a project through to successful completion. Also, since it has a graphical representation, it can be easily understood and used by those with a less technical background.

Network is a graphical representation of all the Activities and Events arranged in a logical and sequential order. Network analysis plays an important role in project management. A project is a combination of interrelated activities all of which must be executed in a certain order for its completion. **Activity is the actual performance of the job.** This consumes resources (Time, human resources, money, and material). **An event refers to start or completion of a job.** This does not consume any resources.

3. APPLICATIONS [MTP JUN'19; DEC'21]

- (i) Construction of a Residential complex,
- (ii) Commercial complex,
- (iii) Petro-chemical complex
- (iv) Ship building
- (v) Satellite mission development
- (vi) Installation of a pipe line project etc...

4. NETWORK DIAGRAM/ARROW DIAGRAM

It is a diagrammatic presentation of work to be done in a project.

It is to be understood in the following aspects :-

A. ACTIVITIES :

- (1) Activity is represented by an arrow in left to right direction.
- (2) An activity must consume some time and resources.
- (3) The name of an activity is written above or below its arrow.
- (4) The duration of an activity is written in brackets besides its name.
- (5) The length of arrow of the activity has no relation to its duration.
- (6) Activities can be parallel or sequential.
- (7) Every activity must start from and end to an event.
- (8) All the preceding activities must be completed before the succeeding activities is to be started.

B. EVENTS/NODES :

These are the indicators of start and end of an activity. It is shown by a circle in network diagram.

Event can be of following types :

- (1) Start Event and End Event : W.r.t an activity
- (2) First Event and Last Event : W.r.t a project
 - Start of project = First event
 - End of project = Last event
 - There can be only one first and last event
- (3) Merge Event and Burst Event

Merge Event

Burst Event

C. ERROR IN NETWORK DIAGRAM :

- (1) **General Rule 1 :** All event should be numbered in such a manner that a higher number event would be towards the right side of lower number events or in the same vertical direction of lower number event.

If this rule is not follow, then there would be an error – “**Looping**”

Rectification of Looping - By correcting the event numbers or resizing the arrows

- (2) **General Rule 2 :** Every event must have at least one activity entering into it and one activity leaving from it, except the first and last event.

If this rule is not followed, then the error is called as “**DANGLING**” Error.

Rectification of Dangling –

Option 1 – By joining the dangling activity to the last event

Option 2 - By using a dummy activity, from the dangling event and joining to the last event.

#Dummy activity is used to rectify errors or to link two or more activities in order to make a network logical. It is shown by dotted arrows. Dummy does not consume any time or resource.

- (3) **General Rule 3 :** No two activities can have same head event and tail event.

If this rule is not followed, the error is called as “**Duplicating**” Error.

Rectification of Duplicating –

By using a dummy activity and a dummy event

D. RECTIFICATION OF ERRORS :

- (1) Looping Error:- Rectified by correcting the event numbers or re sizing the arrows.
- (2) Dangling Error:- Rectified by using a dummy activity. Dummy activity is used to rectify errors or to link two or more activities in order to make a network logical.
It is shown by dotted arrows.
Dummy does not consume any time or resource.
- (3) Duplicating Error :- Else dummy activities.
- (4) Redundancy Error :- Do not use dummy unnecessarily

5. DIFFERENCES BETWEEN PERT & CPM [MTP JUN'20]

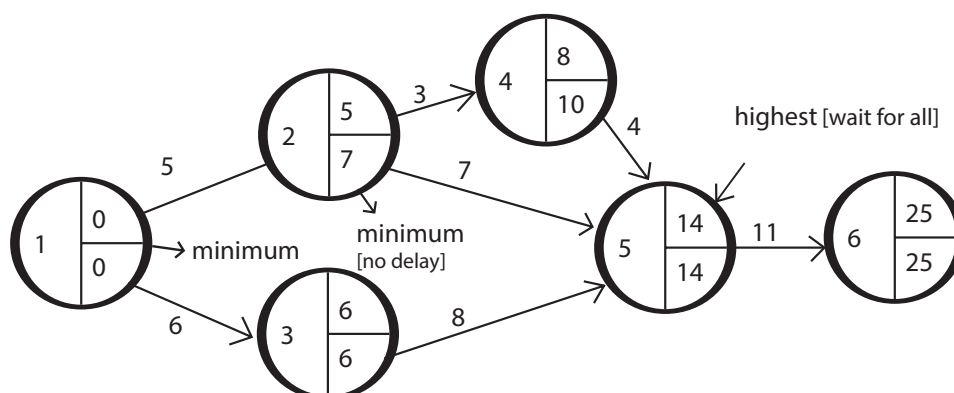
PERT	CPM
1. It is a technique for planning scheduling & controlling of projects whose activities are subject to uncertainty in the performance time. Hence it is a probabilistic model.	1. It is a technique for planning scheduling & controlling of projects whose activities not subjected to any uncertainty and the performance times are fixed. Hence it is a deterministic model.
2. It is an Event oriented system	2. It is an Activity oriented system
3. Basically does not differentiate critical and non- critical activities.	3. Differentiates clearly the critical activities from the other activities.
4. Used in projects where resources (men, materials, money) are always available when required.	4. Used in projects where overall costs is of primarily important. Therefore better utilized resources.
5. Suitable for Research and Development projects where times cannot be predicted.	5. Suitable for civil constructions.

6. CRITICAL PATH AND FLOAT ANALYSIS

Example :

Activities	Duration Days
1 – 2	5
1 – 3	6
2 – 4	3
2 – 5	7
3 – 5	8
4 – 5	4
5 – 6	11

A. NETWORK DIAGRAM :



B. LIST OF ALL THE PATHS AND THEIR DURATION

Paths means the link of several activities that would join its event to the last event.

Paths	Duration
1 – 2 – 4 – 5 – 6	23 [5+3+4+11]
1 – 2 – 5 – 6	23 [5+7+11]
1 – 3 – 5 – 6	25 [6+8+11]

C. WHICH IS THE CRITICAL PATH :

- Critical path is the path with longest duration.
- It is shown in network by bold or shaded arrows.
- The activities on the critical path are called as critical activities.

The critical path is 1 – 3 – 5 – 6 as its duration is longest at 25 days.

D. WHAT IS THE MINIMUM PROJECT DURATION :

The minimum project duration is the duration of critical path = 25 days.

E. EARLIEST START TIME OR FORWARD PASS FOR EACH ACTIVITY.

- Earliest start time is denoted by "E"
- For 1st event $E = 0$ and for next event duration of activity leading to it is added. If there are multiple activities leading to the event, take the highest duration.
- To check the E of an activity – check E of its Tail Event.

Earlier Finish Time = Earlier start time + Duration.

F. LATEST FINISH TIME OR BACKWARD PASS :

- Latest Finish Time is denoted by "L"
- For last event $L = E$, and for preceding events duration of activity emanating from it is subtracted.
- If there are multiple activity, emanating then take the last duration i.e. min. value
- To check the L of an activity – Check L of its Head Event
- Latest Start Time = Latest Finish Time – Duration.

G. FLOAT OF THE PATH :

It is applicable only for the non-critical path. It is the combinical float of all the activities of the path.

Float of a path = Duration of critical path – Duration of non- critical path.

Path	Float of Slack
1 – 2 – 4 – 5 – 6	= 25 – 23 = 2 days
1 – 2 – 5 – 6	= 25 – 23 = 2 days

H. FLOAT OF AN ACTIVITY OR TOTAL FLOAT :

- It is applicable only for non-critical activities.
- For critical activity it must be zero.

Total float = [Latest Finish Time – Earliest Start Time] – Duration

OR

= Latest Start Time – Earliest Start Time

OR

= Latest Finish Time – Earliest Finish Time

Time Schedule

Activity (TE) (HE)	Duration (D)	Earliest Start Time (E)	Earliest Finish Time (E + D)	Latest Finish Time (L)	Latest Start Time (L – D)	Total Float [LFT – EST] or [LST – EST]
1 – 2	5	0	5	7	2	2
1 – 3	6	0	6	6	0	0
2 – 4	3	5	8	10	7	2
2 – 5	7	5	12	14	7	2
3 – 5	8	6	14	14	6	0
4 – 5	4	8	12	14	10	2
5 – 6	11	14	25	25	14	0

Free Float –

It is the time period by which if an activity is delayed, it will not delay the start of its succeeding activity.

Free Float = (EST of Head Event – EST of Tail Event) – Duration

= Total Float – Slack of Head Event

Interfering Float –

It is the maximum interference that an activity can cause to its succeeding activity.

Interfering Float = Total Float – Free Float = Slack of Head Event

Independent Float –

It is the time period by which if an activity is delayed, it will neither delay the start of its succeeding activity nor it will interfere with the completion of its preceding activity.

Independent Float = (LFT of Tail Event - EST of Head Event) – Duration
 = Free Float – Slack of Tail Event

If independent float is negative, then take it as zero.

Observations

- (1) For critical activities, Total Float = Free Float = Interfering Float = Independent Float = 0 (use this as Chor Buddhi)
- (2) Total Float $\geq$ Free Float $\geq$ Independent Float
- (3) For events on the critical path, $E=L$. (use this as chor buddhi to identify the critical path)

7. PROJECT CRASHING [DEC'21]

A. MEANING

Crashing means completing a project earlier than schedule. This is done by increasing the man power or other resources.

Project crashing is a network technique with a focus on reducing the project duration to the optimum level. Reduction in project duration may involve extra costs. Hence, project crashing seeks to determine the optimum duration of the project, i.e. time that corresponds to the minimum costs. The activity cost slope indicates the additional cost incurred per unit of time saved in reducing the duration of an activity.

B. BENEFITS OF CRASHING

- (1) Savings in overhead cost allocated to the project
- (2) Reward for early completion

C. COST OF CRASHING

Additional cost of overtime, extra machine, hiring, etc.

D. CRASH COST SLOPE

It means additional cost to reduce one day of the project

$$= \frac{\text{Crash Cost} - \text{Normal Cost}}{\text{Normal Time} - \text{Crash Time}} = \frac{\text{Change in Cost}}{\text{Change in Time}}$$

E. TOTAL COST OF THE PROJECT

Direct Costs = normal costs of the project

Indirect Costs = overhead costs allocated to the project

Additional Crash Cost = additional cost to reduce the project duration

Due to crashing, the additional crash cost incurred saved the indirect costs allocated to the project.

F. TWO OBJECTIVES

(1) Optimum completion time –

The duration where the total project cost would be minimum.

Crash cost slope should be less than or equal to overhead cost saved due to crashing.

(2) Minimum Completion Time –

The duration where the project duration is minimum, but the cost may be higher than the optimum duration.

Crash cost slope may exceed the indirect cost saved.

G. FOCUS OF CRASHING

Ultimately the objective of crashing is to reduce the project duration



Project duration is determined from critical path duration



And critical path duration is determined from critical activity duration. Therefore, the focus of crashing should be to reduce the critical activity duration.

H. PROCESS OF CRASHING

Four basic ingredients

- (1) Network Diagram – with both normal duration and crash duration noted for each activity
- (2) All the Paths and their normal duration
- (3) Crash cost slope of each activity
- (4) Critical Path, Critical activities and their cost slopes.

Steps of Crashing

(1) Single Critical Path –

Crash or reduce the duration of a critical activity by 1 day (or week) in order of least cost slope, keeping in mind the indirect cost saved.

Crash the critical activities one by one and follow four cross checks –

B = Bracket the outflow

N = Reduce the normal duration of the critical activities in the **Network Diagram**

D = Reduce the normal **duration of the paths**

K = Use **khatiya** for dead (fully crashed) activities in the list of paths.

Continue crashing in this step, unless there are multiple critical paths

(2) Multiple Critical Paths –

- Select a combination of critical activities from all the critical paths, in such a way that if a set is crashed by a day, all the critical paths get reduced by a day.
- While preparing sets, keep following checks –
 - (a) Do not include a dead activity
 - (b) There can be a common critical activity
 - (c) Double check that there is no dual effect of the set on any critical path
- Find the crash cost slope of each set.
- Select a set in order of least cost slope and crash or reduce the duration of a set by 1 day (or week) keeping in mind the indirect cost saved.

A set can have multiple activities, but it reduces the project duration by 1 day only.

Follow four cross checks as above – B, N, D, K

Continue crashing until –

- (i) For Optimum Duration Objective – Crash cost slope $\geq$ Indirect cost saved
- (ii) For Minimum Duration Objective – One of the critical path has been fully crashed.

(3) Conclusion

Latest Project Duration will be the latest longest duration

Latest Total Cost will be the sum total of Direct Cost, Indirect Cost and Additional Crash Cost.

In some questions, where sales revenue or bonus or penalty is given, the optimum duration is computed by ignoring the indirect cost saved. We compute the minimum duration of the project and then prepare profitability statement at each different date of completion to determine the duration where the project profit is maximum.

8. PROJECT EVALUATION AND REVIEW TECHNIQUE (PERT)

PERT is more relevant for projects where there is a good measure of uncertainty in the estimation of activity duration. Here, we obtain three estimates of activity duration:

- (a) **The Optimistic Time Estimate:** This is the shortest possible time in which an activity can be completed. It is estimated under the best case situation and is denoted by t_o
- (b) **The Pessimistic Time Estimate:** This is the longest possible time in which an activity can be completed. It is estimated under the worst case situation and is denoted by t_p
- (c) **The Most Likely Time Estimate:** This is the normal time in which an activity can be completed. It is estimated under the most probable situation and is denoted by t_m

From the above three estimates an average or expected time of activity is calculated for any subsequent analysis.

A. EXPECTED TIME OF ACTIVITY

This is the duration of the activity found by calculating the weighted average of the three time estimates and is given by:

$$\text{Duration of activity} = T_E = \frac{t_o + 4t_m + t_p}{6}$$

B. EXPECTED PROJECT TIME

This is the longest duration or the critical path duration and is found by adding the expected times of the critical activities.

C. VARIANCES AND STANDARD DEVIATIONS:

- (1) **Standard Deviation of an activity:**

$$\text{S.D.} = (t_p - t_o) / 6$$

- (2) **Variance of an activity** = $(\text{S.D.})^2$

- (3) **Variance of the critical path:** This is obtained by adding variances of all the activities on the critical path.

- (4) **Standard Deviation of the Critical Path:** This is obtained by the formula given below and not by adding the SD's of the critical path activities.

$$\text{S.D.} = \sqrt{\text{variance of critical path}}$$

- (5) **Variances of events for TE and TL**

Sum of the variances along the path of TE and TL . If there are more than one longest path then select one having a larger sum of variances.

The Earliest time TE = the duration of the longest path leading to the event from the start.

The Latest time TL = the duration of the longest path emanating from the event to the end.

D. TO FIND THE PROBABILITIES OF COMPLETION OF PROJECT IN THE GIVEN TIME

- (1) The time of completion of project for which the probability is required is expressed as P(x)
- (2) To find the probability for a given value of x, first find z using the following formula:

$$Z = \frac{X - T_{cp}}{S_{cp}}$$

T_{cp} = Expected Project Duration i.e. Duration of the critical path

S_{cp} = Standard Deviation of the Project or Critical path.

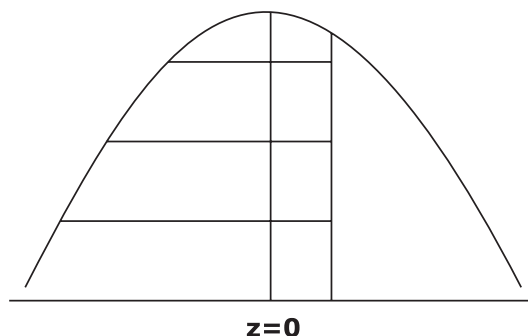
- (3) The required probability of x would be the area under the curve for respective value of z.

$$P(X) = A(Z).$$

E. TO FIND THE TIME OF THE PROJECT COMPLETION WHEN THE PROBABILITY IS GIVEN

- (1) Find the value of z against the probability or area from the tables. The area corresponding to P(X) is always the area to the left of the value Z.

**A (Z) : This area is the probability of
Completing the project in given time X.**



- (2) Calculate the respective x using the following:

$$Z = \frac{x - T_{cp}}{S_{cp}}$$

F. CHARACTERISTICS OF STANDARD NORMAL CURVE

- (1) It is a symmetrical bell shaped curve.
- (2) The total area under the curve is 1.
- (3) Because of symmetry, the area to the left and the right of central axis are both equal and is 0.5
- (4) The maximum value of Z for which area will be 0.5 on either side of axis can be 3.49
- (5) The area $A(z)$ would mean area between $z=0$ and the given value of z .
- (6) Because of symmetry, $A(-z) = A(z)$

SJC Institute

Q1

Ex. Book No.

Pg. No.

CMA Mat; Jun'17; MTP Dec'18; MTP Dec'19

XYZ Auto-manufacturing company has to prepare a design of its latest model of motorcycle. The various activities to be performed to prepare design are as follows:

Activity	Description of activity	Preceding activity
A	Prepare drawing	–
B	Carry out cost analysis	A
C	Carry out financial analysis	A
D	Manufacture tools	C
E	Prepare bill of material	B, C
F	Receive material	D, E
G	Order sub-accessories	E
H	Receive sub-accessories	G
I	Manufacture components	F
J	Final assembly	I, H
K	Testing and shipment	J

Prepare an appropriate network diagram.



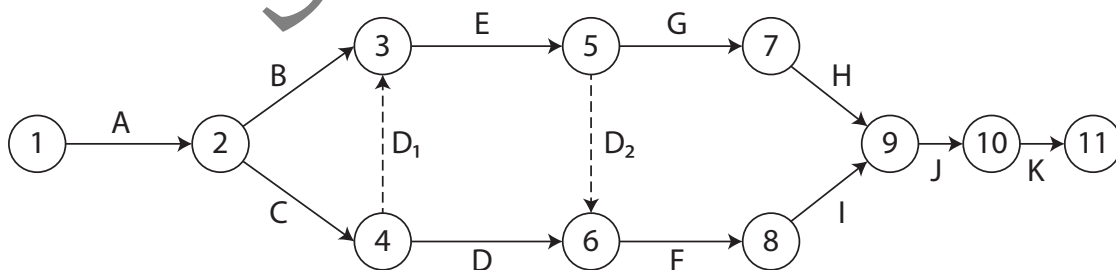
Reference

Network diagram

What's New

Answer

The network diagram will be as follows:



Where D_1 and D_2 are dummy activities.

Draw a network from the following activities. Find the critical path and total duration of the project.

Activity	Immediate predecessor activity	Duration (days)
A	—	10
B	A	5
C	A	4
D	A	7
E	B,C	6
F	C,D	4
G	E,F	7

Reference

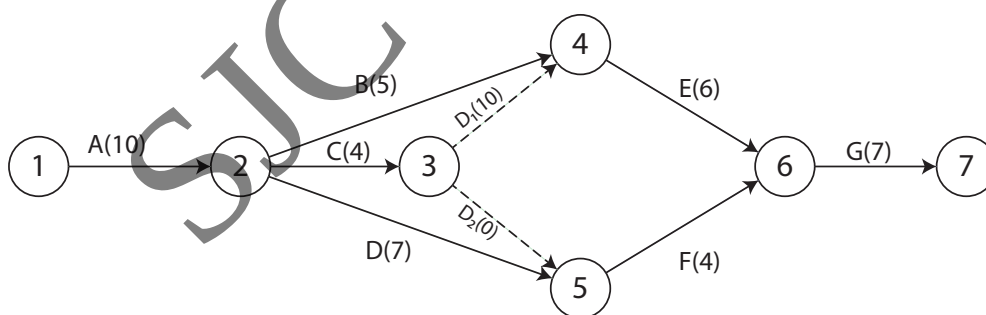
Critical Path Method

What's New

Network Diagram; Critical Path; Duration

Answer

The network is drawn as follows:



Network Diagram

Various paths Duration of paths (days)

(i)	1-2-4-6-7	$10 + 5 + 6 + 7 = 28$
(ii)	1-2-3-4-6-7	$10 + 4 + 0 + 6 + 7 = 27$
(iii)	1-2-3-5-6-7	$10 + 4 + 0 + 4 + 7 = 25$
(iv)	1-2-5-6-7	$10 + 7 + 4 + 7 = 28$

Critical paths are 1 - 2 - 4 - 6 - 7 and 1 - 2 - 5 - 6 - 7 with duration of 28 days and are marked with double lines.


Q3

Ex. Book No.

Pg. No.

CMA Mat

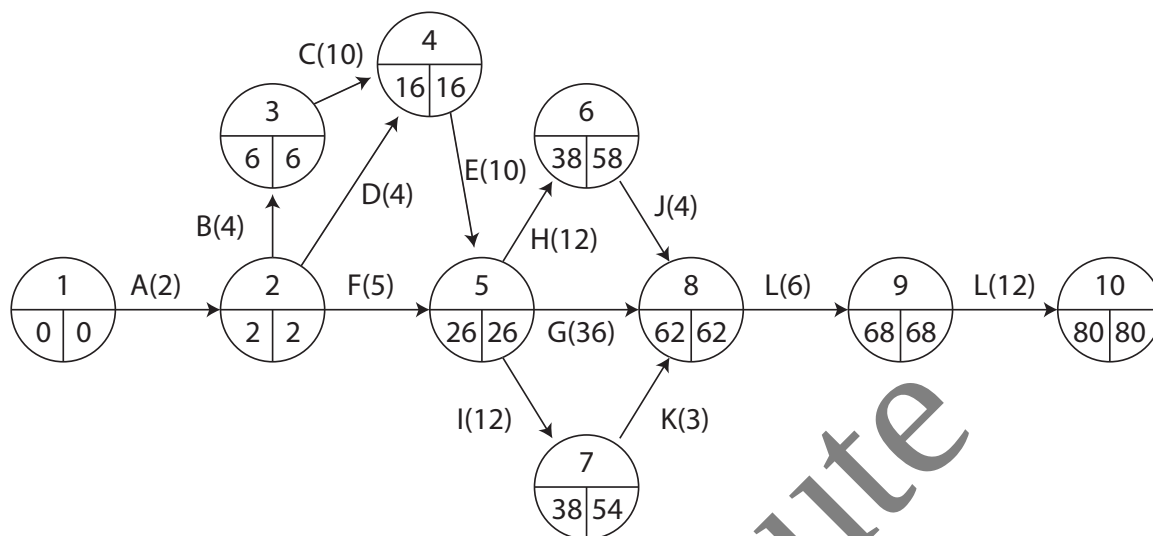
Draw a network of the following activities and tabulate earliest and latest starting and finishing times of each activity and the total and free floats of them:

Event numbers	Activity symbol	Activity description	No. of days
1 – 2	A	Study of plan layout	2
2 – 3	B	Clearance of site	4
3 – 4	C	Earth work	10
2 – 4	D	Procurement of line, sand, cement and concrete	4
4 – 5	E	Laying of foundations	10
2 – 5	F	Procurement of bricks	5
5 – 8	G	Construction of building	36
5 – 6	H	Laying of conduit pipe lines for electric wires	12
6 – 8	I	Laying of electric wires	4
5 – 7	J	Laying of drainage and sewage system	12
7 – 8	K	Laying of water pipe lines and taps	8
8 – 9	L	Connecting building to water and electricity	6
9 – 10	M	Finishing work in building	12


Reference
Critical Path Method
What's New
Float Analysis

Answer

The graphical representation of network and time schedule is shown in Fig. below:



The critical path is : A – B – C – E – G – L – M.

Table: Computation of scheduling time estimates and floats

Activity	Sequence code	Duration	Earliest time		Latest time		Float	
			Start	Finish	Start	Finish	TF	FF
A	1-2	2	0	2	0	2	0	0
B	2-3	4	2	6	2	6	0	0
C	3-4	10	6	16	6	16	0	0
D	2-4	4	2	6	12	16	10	10
E	4-5	10	16	26	16	26	0	0
F	2-5	5	2	7	21	26	19	19
G	5-8	36	26	62	26	62	0	0
H	5-6	12	26	38	46	58	20	0
I	6-8	4	38	42	58	62	20	20
J	5-7	12	26	38	42	54	16	0
K	7-8	8	38	46	54	62	16	16
L	8-9	6	62	68	62	68	0	0
M	9-10	12	68	80	68	80	0	0

Q4

Ex. Book No.

Pg. No.

Dec'18; MTP Dec'19; MTP Jun'22

Given the following information regarding a project and the time duration of each activity :

Activity	Preceding activity	Normal Time (days)
A	-	16
B	-	20
C	A	8
D	A	10
E	B,C	6
F	D,E	12

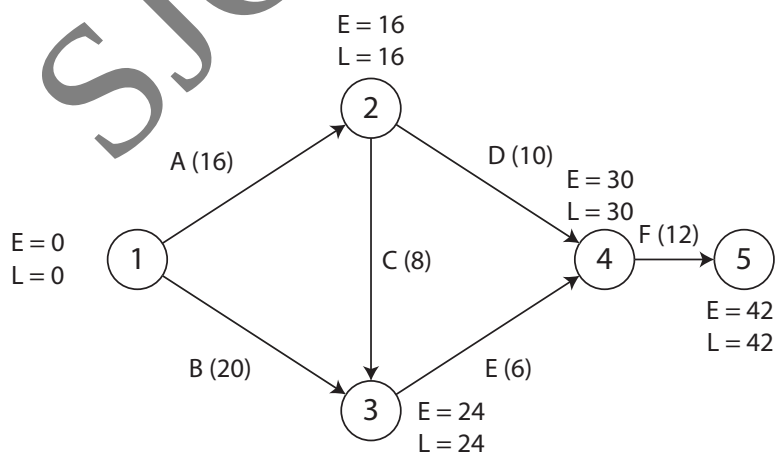
Required :

- Draw** the activity network of the project.
- Find** critical path and duration of the project.
- Find** the total float and free-float for each activity.

[2 + 2 + 4 = 8]

Reference
Critical Path Method
What's New
Float analysis
Answer

The network for the given problem



$$A \rightarrow D \rightarrow F = 16 + 10 + 12 = 38$$

$$B \rightarrow E \rightarrow F = 20 + 6 + 12 = 38$$

- $A - C - E - F = 16 + 8 + 6 + 12 = 42$ days (Critical path)

(ii) Total float and free float for each activity.

Activity	Normal time (Days)	EST	EFT	LST	LFT	Total Float	Free Float
A	16	0	16	0	16	0	0
B	20	0	20	4	24	4	4
C	12	30	42	30	42	0	2
D	6	24	30	24	30	0	0
E	10	16	26	20	30	4	4
F	8	16	24	16	24	0	0

Q5

Ex. Book No.

Pg. No.

MTP Jun'19

A small maintenance project consists of the following twelve jobs whose precedence relations are identified with their node number:

Job (i,j)	:	(1,2)	(1,3)	(1,4)	(2,3)	(2,5)	(2,6)
Duration (in days)	:	10	4	6	5	12	9
Job (i,j)	:	(3,7)	(4,5)	(5,6)	(6,7)	(6,8)	(7,8)
Duration (in days)	:	12	15	6	5	4	7

- Draw** an arrow diagram representing the project.
- Calculate** earliest start, earliest finish, latest finish time for all the jobs.
- Find** the critical path and project duration.
- Tabulate** total float, free float and independent float. [3 + 3 + 3 + 3 = 12]

Reference

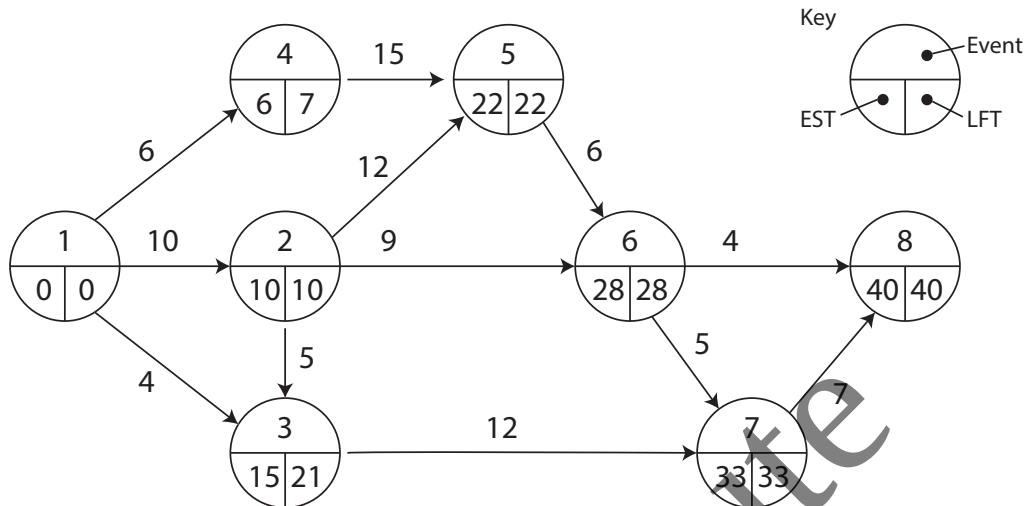
Critical Path

What's **New**

TF, FF, IF

Answer

- (i) The network diagram of the project corresponding to normal duration is given below:



- (ii) Statement showing Earliest Start Time (EST), Earliest Finish Time (EFT), Latest Start Time (LST) and Latest Finish Time (LFT) for all jobs.

Jobs	Duration in days	Earliest Time		Latest Time		Slack of event		Total Float (TF)	Free Float (FF)	Independent Float
		Start (EST)	Finish (EFT)	Start (LST)	Finish (LFT)	at start of activity	at end of activity			
						7	8	9	10	11
1-2	10	0	10	0	10	0	0	10-10 = 0	0-0 = 0	0-0 = 0
1-3	4	0	4	17	21	0	6	21-4 = 17	17-6 = 11	11-0 = 11
1-4	6	0	6	1	7	0	1	7-6 = 1	1-1 = 0	0-0 = 0
2-3	5	10	15	16	21	0	6	21-15 = 6	6-6 = 0	0-0 = 0
2-5	12	10	22	10	22	0	0	22-22 = 0	0-0 = 0	0-0 = 0
2-6	9	10	19	19	28	0	0	28-19 = 9	9-0 = 9	9-0 = 9
3-7	12	15	27	21	33	6	0	33-27 = 6	6-0 = 6	6-6 = 0
4-5	15	6	21	7	22	1	0	22-21 = 1	1-0 = 1	1-1 = 0
5-6	6	22	28	22	28	0	0	28-28 = 0	0-0 = 0	0-0 = 0
6-7	5	28	33	28	33	0	0	33-33 = 0	0-0 = 0	0-0 = 0
6-8	4	28	32	36	40	0	0	40-32 = 8	8-0 = 8	8-0 = 8
7-8	7	33	40	33	40	0	0	33-33 = 0	0-0 = 0	0-0 = 0

- (iii) Critical Path is 1 → 2 → 5 → 6 → 7 → 8

And project duration is 10 + 12 + 6 + 5 + 7 = 40 days

- (iv) Total Float, Free Float and Independent Float has been shown in the table at (ii).

Note:

- EST of the activity of the EST of node at the start of activity.
- EFT of activity is the EST of the activity plus time duration of the activity.
- LFT of the activity is the LFT of the node at the end of the activity.
- LST of the activity is difference between LFT of the activity minus time duration of the activity.
- Total Float (TF) is the difference between:
 - (LFT – EFT) of the activity or
 - (LST– EST) of the activity
- **Free Float** = Total Float – Slack of event at end of the activity
- **Independent Float** = Free Float – Slack of the event at start of the activity

Q6

Ex. Book No.

Pg. No.

CMA Mat; Dec'19; MTP Jun'17

The following table gives data on normal time & cost and crash time & cost for a project.

Activity	Normal		Crash	
	Time (days)	Cost (₹)	Time (days)	Cost (₹)
1 – 2	6	600	4	1,000
1 – 3	4	600	2	2,000
2 – 4	5	500	3	1,500
2 – 5	3	450	1	650
3 – 4	6	900	4	2,000
4 – 6	8	800	4	3,000
5 – 6	4	400	2	1,000
6 – 7	3	450	2	800

The indirect cost per day is ₹ 100.

- Draw** the network and identify the critical path.
- What** are the normal project duration and associated cost?
- Crash** the relevant activities systematically and determine the optimum project completion time and cost.



Reference

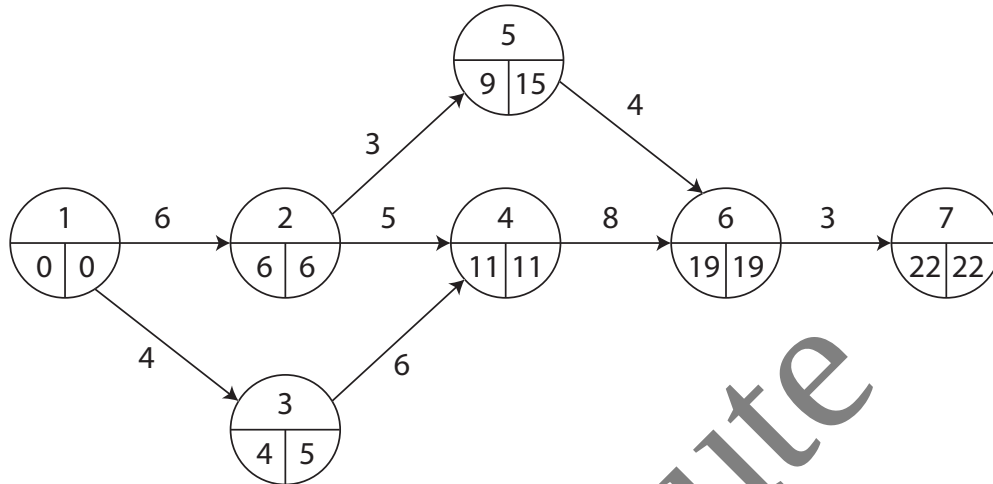
Crashing the Project

What's New

Optimum Duration

Answer

- (i) The network for normal activity times indicates a project time of 22 weeks with the critical path 1-2-4-6-7.



- (ii) Normal project duration is 22 weeks and the associated cost is as follows:

Total cost = Direct normal cost + Indirect cost for 22 weeks.

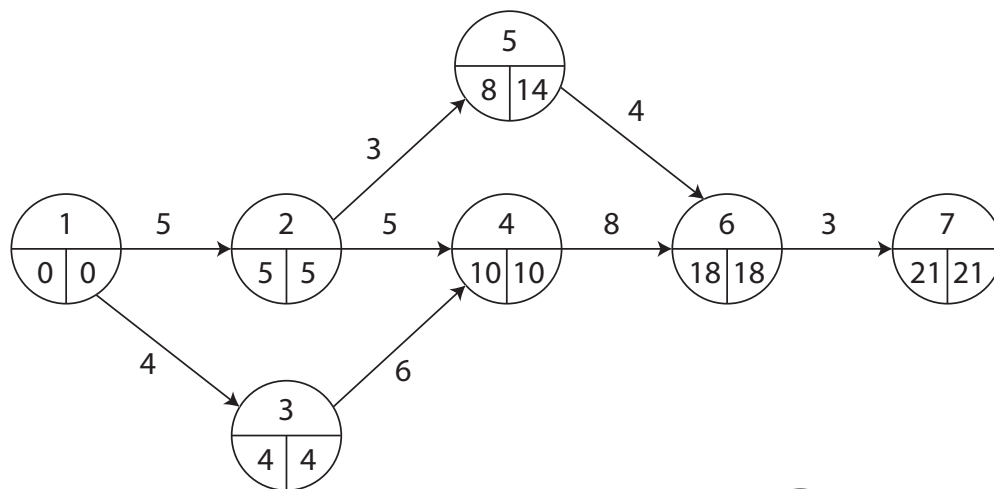
$$= 4,700 + 100 \times 22 = ₹ 6,900.$$

- (iii) For critical activities, crash cost - slope is given below:

Critical activity	Crash cost-slop
1 – 2	$\frac{1,000 - 600}{6 - 4} = 200$
2 – 4	$\frac{1,500 - 500}{5 - 3} = 500$
4 – 6	$\frac{3,000 - 800}{8 - 4} = 550$
6 – 7	$\frac{800 - 450}{3 - 2} = 350$

Of the activities lying on the critical path, activity 1 – 2 has lowest cost slope. Therefore, we shall first crash this activity by just one day.

$$\text{Duration} = 21 \text{ days, and cost} = 4700 + 1 \times 200 + 100 \times 21 = ₹ 7000.$$



Other activities too have become critical. Now we have 2 critical paths:

1 → 2 → 3 → 6 → 7 and 1 → 3 → 4 → 6 → 7.

To reduce duration of the activity further, we shall have to reduce duration of both the paths. We have following alternatives:

Crash activity 6 – 7 by 1 day at a cost of ₹ 350.

Crash activity 4 – 6 by 4 days at the cost of ₹ 550 per day.

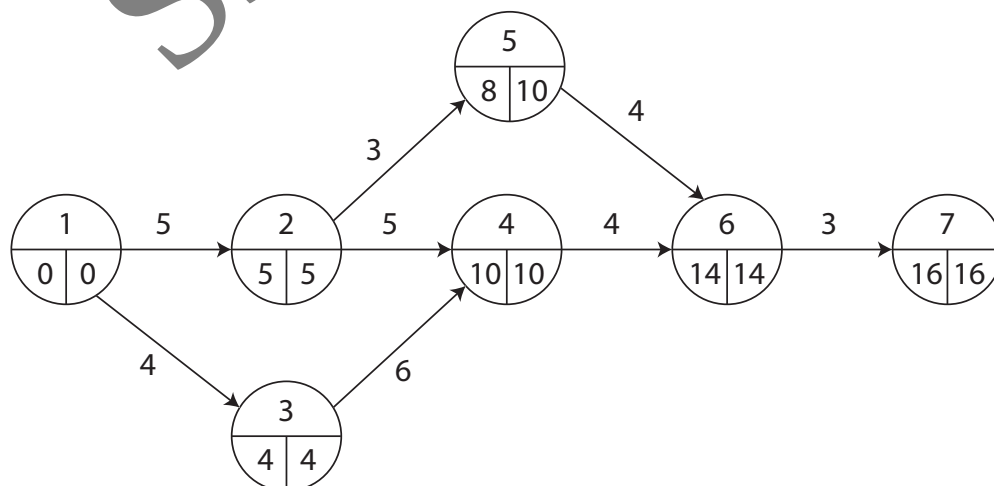
Crash activities 1 – 2 and 1 – 3 by 1 day each at a cost of ₹ (200 + 700) = ₹ 900.

Crash activities 2 – 4 and 3 – 4 by 2 days each at a cost of ₹ (500 + 550) = ₹ 1050/day.

Thus, we shall first crash activities 6 – 7 by 1 day and then activity 4 – 6 by 4 days.

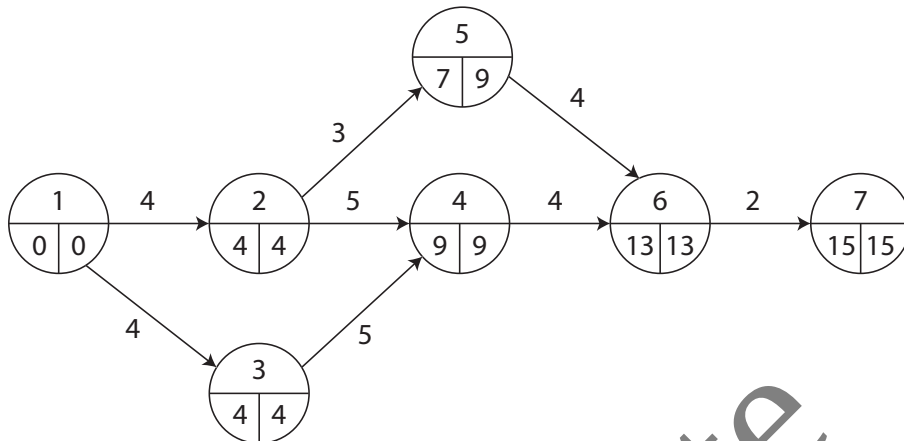
On crashing activity 6 – 7 by 1 day, cost = 4900 + 350 × 1 + 100 × 20 = ₹ 7250, and Duration = 20 days. Next we crash 4 – 6 by 4 days.

Cost = 5250 + 550 × 4 + 100 × 16 = ₹ 9050. Duration = 16 days.



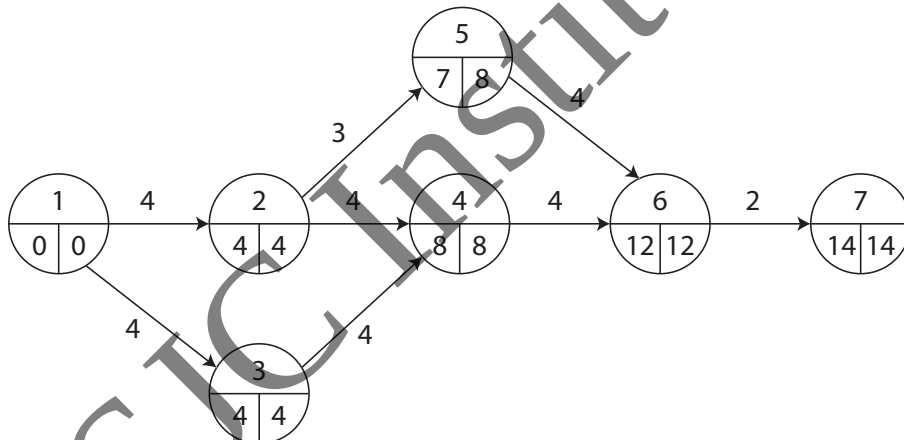
Next we crash activities 1 – 2 and 3 – 4 by 1 day each.

$$\text{Cost} = 7450 + 200 \times 1 + 550 \times 1 + 100 \times 15 = ₹ 9700.$$



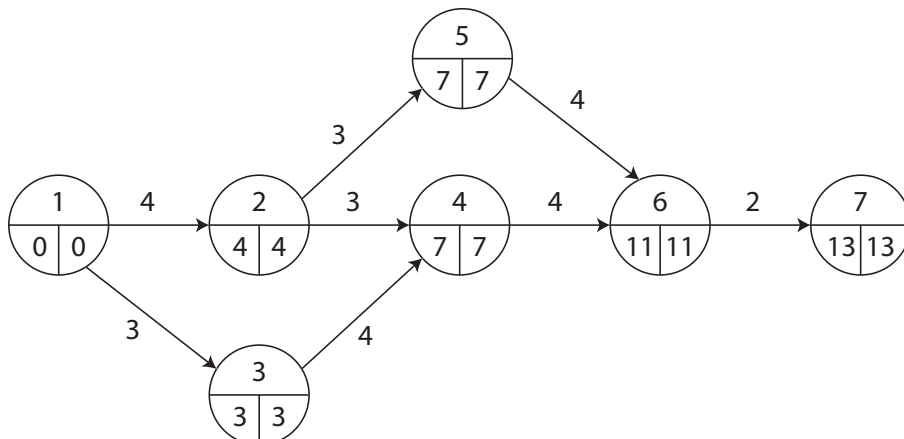
Next we crash activities 2 → 4 and 3 → 4 by 1 day each.

$$\text{Cost} = 8200 + 500 \times 1 + 550 \times 1 + 100 \times 14 = ₹ 10,650. \text{ Duration} = 14 \text{ days.}$$



We crash activities 1 – 3 and 2 – 4 by 1 day each.

$$\text{Cost} = 9250 + 700 \times 1 + 500 \times 1 + 100 \times 13 = ₹ 11,750 \text{ Duration} = 13 \text{ days.}$$



Now there are three critical paths:

1 – 2 – 5 – 6 – 7, 1 – 2 – 4 – 6 – 7, 1 – 3 – 4 – 6 – 7

Also, no further crashing is possible. Hence minimum duration of the project = 13 days with cost ₹ 11,750.

Q7

Ex. Book No.

Pg. No.

RTP Jun'18; RTP Dec'18

A Network is given below -

- **Name** the Paths and give their total duration.
- **Give** three different ways of reducing the above project duration by four weeks.

Activity	Duration	Activity	Duration
1 – 2	2 weeks	4 – 7	3 weeks
1 – 3	3 weeks	5 – 8	3 weeks
2 – 5	5 weeks	6 – 7	2 weeks
3 – 4	2 weeks	7 – 8	2 weeks
4 – 6	1 weeks	8 – 9	5 weeks

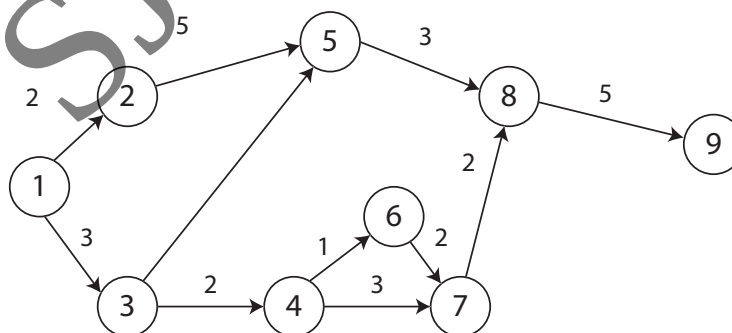
Reference

Crashing the Project

What's New

Network Diagram

Answer



Path	1-2-5-8-9	1-3-5-8-9	1-3-4-6-7-8-9	1-3-4-7-8-9
Path No.	I	II	III	IV
Duration	2+5+3+5 = 15 weeks	3+0+3+5 = 11 weeks	3+2+1+2+2+5 = 15 weeks	3+2+3+2+5 = 15 weeks

The different ways of reducing the Project Duration by four weeks are –

1. Reduce duration of Activity 8 - 9 by 4 weeks (presently 5 weeks), since this is a common activity on all Paths.
2. Reduce duration of Activity 1 - 3 (Common for Paths II, III and IV), and any one of Activities 1 - 2 and 2- 5 (on Path I) such that total time reduction is 4 weeks.
3. Reduce duration of Activity 3 - 4 or 7 - 8 (Common for Paths I II & IV), and Activity 5 -8 (Common for Paths I & II).

Q 8

Ex. Book No.

Pg. No.

MTP Dec'17

A company had planned its operations as follows:

Activity	Duration (days)
1-2	7
2-4	8
1-3	8
3-4	6
1-4	6
2-5	16
4-7	19
3-6	24
5-7	9
6-8	7
7-8	8

- Draw** the network and find the critical paths.
- After 15 days of working, the following progress is noted:
 - Activities 1-2, 1-3, and 1-4 completed as per original schedule.
 - Activity 2-4 is in progress and will be completed in 4 more days.
 - Activity 3-6 is in progress and will need 17 more days to complete.
 - The staff at activity 3-6 are specialised. They are directed to complete 3-6 and undertake an activity 6-7, which will require 7 days. This re-arrangement arose due to a modification in a specification.
 - Activity 6-8 will be completed in 4 days instead of the originally planned 7 days.
 - There is no change in the other activities.

Update the network diagram after 15 days of start of work based on the assumption given above. Indicate the revised critical paths along with their duration.

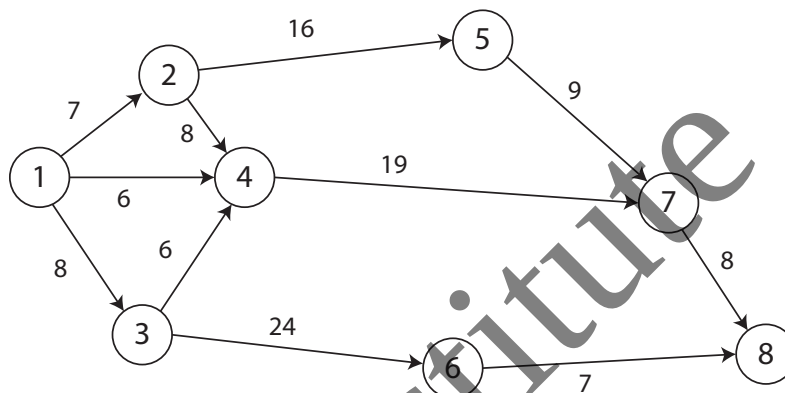
Reference

Updating the Network diagram

What's New

Answer

(i)



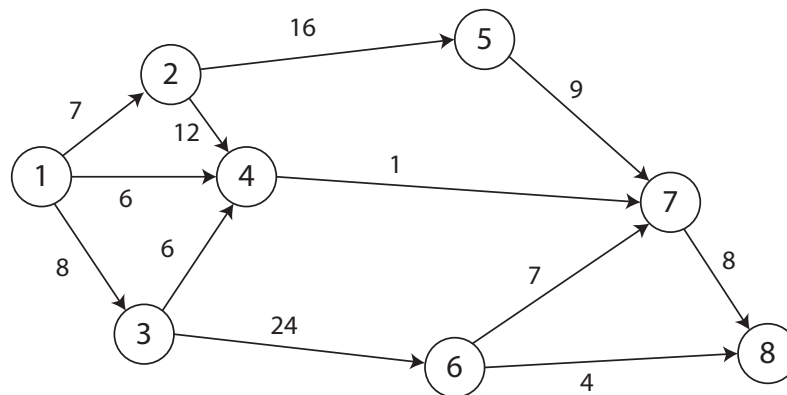
Paths	Duration
1-2-5-7-8	$7+16+9+8 = 40$
1-2-4-7-8	$7+8+19+8 = 42$
1-4-7-8	$6+19+8 = 33$
1-3-4-7-8	$8+6+19+8 = 41$
1-3-6-8	$8+24+7 = 39$

Critical Path 1-2-4-7-8 = 42 days

Revised duration of activities 2-4 and 3-6 after 15 days for updation.

Activity	Preceding Activity	Date of completion	Revised duration
2-4	1-2	$15+4 = 19$ days	$19-7 = 12$ days
3-6	1-3	$15+17 = 32$ days	$32-8 = 24$ days
6-7 (New-activity)	3-6		7 days
6-8	3-6		4 days

(ii)



Paths	Duration
1-2-5-7-8	$7+16+9+8 = 40$
1-2-4-7-8	$7+12+19+8 = 46$
1-4-7-8	$6+19+8 = 33$
1-3-4-7-8	$8+6+19+8 = 41$
1-3-6-7-8	$8+24+7+8 = 47$
1-3-6-8	$8+24+4 = 36$

Critical Path = 1-3-6-7-8 = 47 days

Q9**CMA Mat; MTP Jun'17**

A civil engineering firm has to bid for the construction of a dam. The activities and time estimates are given below:

Activity	DURATION		
	Optimistic	Most likely	Pessimistic
1 – 2	14	17	25
2 – 3	14	18	21
2 – 4	13	15	18
2 – 8	16	19	28
3 – 4 (dummy)			
3 – 5	15	18	27
4 – 6	13	17	21
5 – 7 (dummy)			
5 – 9	14	18	20
6 – 7 (dummy)			
6 – 8 (dummy)			
7 – 9	16	20	41
8 – 9	14	16	22

The policy of the firm with respect to submitting bids is to bid the minimum amount that will provide a 95% of probability of at best breaking even. The fixed costs for the project are 8 lakhs and the variable costs are ₹ 9,000 everyday spent working on the project. The duration is in days and the costs are in terms of rupees.

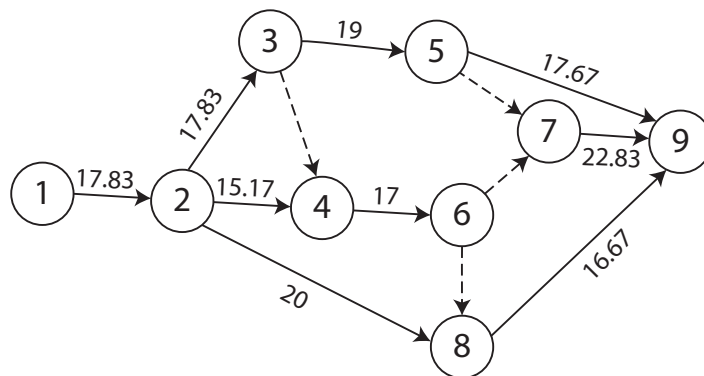
What amount should the firm bid under this policy? (You may perform the calculations on duration etc. upto two decimal places.) $A(Z = 1.645) = 0.45$

Reference	What's New
PERT	Bid Amount

Answer

The expected duration and variance of each activity is computed in the following table :

Activity	Optimistic (t_o)	Time most likely (t_m)	Pessimistic (t_p)	Expected duration $t_e = \frac{1}{6}(t_o + 4t_m + t_p)$	Variance $\left[\frac{1}{6}(t_p - t_o)\right]^2$
1 – 2	14	17	25	17.83	3.36
2 – 3	14	18	21	17.83	1.36
2 – 4	13	15	18	15.17	
2 – 8	16	19	28	20.00	
3 – 4	–	–	–	–	
3 – 5	15	18	27	19.00	4
4 – 6	13	17	21	17.00	
5 – 7	–	–	–	–	
5 – 9	14	18	20	17.67	
6 – 7	–	–	–	–	
6 – 8	–	–	–	–	
7 – 9	16	20	41	22.83	17.36
8 – 9	14	16	22	16.67	



The various paths and their lengths are as follows:

	Path	Duration
I.	1-2-3-5-7-9	77.49*
II.	1-2-3-5-9	72.33
III.	1-2-3-4-6-7-9	75.49
IV.	1-2-3-4-6-8-9	69.33
V.	1-2-8-9	54.50
VI.	1-2-4-6-8-9	66.67
VII.	1-2-4-6-7-9	72.83

Thus, the critical path is 1 – 2 – 3 – 5 – 7 – 9 with project duration of 77.49 days. Project variance is obtained by summing variances of critical activities, $\sigma^2 = 3.36 + 1.36 + 4 + 17.36 = 26.08$.

$\therefore$ **Standard duration of project length, $\sigma = \sqrt{26.08} = 5.11$**

To calculate the project duration which will have 95% chances of its completion, we find the value of Z corresponding to 95% area from normal distribution area table which is 1.645. Thus

$$P(X \leq T_s) = P\left(Z \leq \frac{T_s - 77.49}{5.11}\right) \approx 0.95$$

$$= \frac{T_s - 77.49}{5.11} = 1.645 \text{ or } T_s = 1.645 \times 5.11 + 77.49 = 86 \text{ days.}$$

Since the fixed cost of the project is ₹ 8 lakhs and the variable cost is ₹ 9000 per day, amount to bid = ₹ 8 lakhs + ₹ 9000 × 86 = ₹ 15,74,000.

Q10

CMA Mat

The activities involved in a PERT project are detailed in the following table:

Job(i- j)	DURATION TIME (DAYS)		
	Most optimistic time	Most likely time	Most pessimistic time
1 – 2	3	6	15
2 – 3	6	12	30
3 – 5	5	11	17
7 – 8	4	19	28
5 – 8	1	4	7
6 – 7	3	9	27
4 – 5	3	6	15
1 – 6	2	5	14
2 – 4	2	5	8

Draw a network diagram.

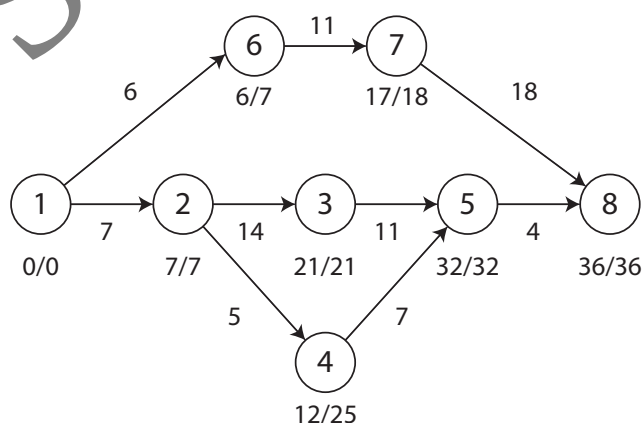


Reference
PERT

What's New
Network Diagram

Answer

Calculate feasible time = $\frac{\text{Optimistic Time} + 4 \text{ Likely Time} + \text{Pessimistic Time}}{6}$



Q11

Jun'18

A small project is composed of 8 activities whose estimated time are listed below:

Activity	1-2	2-3	2-4	3-5	4-6	5-6	5-7	6-7
Optimistic time (in weeks)	3	3	2	4	4	0	3	2
Most likely time (in weeks)	3	6	4	6	6	0	4	5
Pessimistic time (in weeks)	3	9	6	8	8	0	5	8

Required:

- Draw** the project network.
- Find** the expected duration and variance for each activity.
- Find** the critical path and expected project length.
- The probability that the project will be completed in 23 weeks. Given that:

Z Value:	1.00	1.91	1.92	1.93	1.94
Probability:	0.9713	0.9719	0.9726	0.9732	0.9738

[3 + 3 + 2 + 2 = 10]



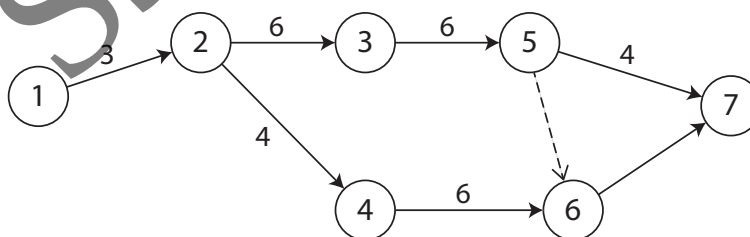
Reference
PERT

What's **New**

Pb of Completion

Answer

- The network diagram for the given data is shown below:



Network Diagram

- The earliest and latest expected time for each event is calculated by considering the expected time of each activity as shown in the following table:

Table: Calculation of expected times and variances

Activity	t_o	t_m	T_p	$t_e = \frac{1}{6}[t_o + 4t_m + t_p]^2$	$\sigma_e = \frac{1}{6}[t_p - t_o]^2$
1 – 2	3	3	3	3	0
2 – 3	3	6	9	6	1
2 – 4	2	4	6	4	4/9
3 – 5	4	6	8	6	4/9
4 – 6	4	6	8	6	4/9
5 – 6	0	0	0	0	0
5 – 7	3	4	5	4	1/9
6 – 7	2	5	8	5	1

The expected duration and variance for each activity is shown in the above table.

(iii) Critical path is : 1 — 2 — 3 — 5 — 6 — 7.

Expected project length. $T_e = 3 + 6 + 6 + 0 + 5 = 20$ weeks

(iv) Probability that the project will be completed in 23 weeks is given by:

$$P(0 \leq X \leq 23) = P(0 \leq Z \leq \frac{23-20}{\sqrt{2.444}}) = P(0 \leq Z \leq 1.920) = 0.9726 \text{ or } 97.26\%.$$



Learning Objective

THEORY

1. BASIC TERMS
2. ASSUMPTIONS
3. LIMITATIONS

TYPES OF PROBLEMS

1. PURE STRATEGY WITH SADDLE POINT & RULE OF DOMINANCE
2. MIXED STRATEGY – ODDS METHODS
3. MIXED STRATEGY – ODDS METHODS - RULE OF DOMINANCE
4. MIXED STRATEGY – GRAPHICAL METHOD ($2 \times n$)
5. MIXED STRATEGY – GRAPHICAL METHOD ($m \times 2$)

1. BASIC TERMS

- A. **Player** – A participant is called a Player.
- B. **Play** – A Play of the game is said to occur when each Player has chosen a course of action.
- C. **2 Person Game** – If the number of Players in a Game is two then it is called 2 Person Game. The term Person refers to an individual or a group aiming at a particular objective.
- D. **N Person Game** – If the number of Players in a Game is N (where $N > 2$) then it is called N Person Game.
- E. **Zero Sum Game** – If the sum of the amounts won by all winners is equal to that lost by all losers then the game is called Zero Sum Game. In other words, sum of the gains and losses in such a game is zero. When there are only two players participating in a game which has resulted in zero sum then it is called 2 Person Zero Sum Game. For a game with zero sum and N participants, we call it as N Person Zero Sum Game.

An example of 2 Person Zero Sum Game is the competition of two firms who are trying to increase their share of the market. Here gain of the market share of one will almost be equal to the other's loss of the market share.

Two Person Zero Sum Games are also called rectangular game because their payoff matrix is in the rectangular form.

- F. **Non Zero Sum Game** – If the sum of the gains or losses in a game is not equal to zero then it is called a Non Zero Sum Game. An example of such a game is the competition between two firms for increasing their respective market shares through intensive advertising campaigns. Due to the advertisement of their product, both the firms might gain market share which may not be of equal magnitude, but at the same time gain of one is not exactly equal to the loss of the other. Hence, sum of the amounts is not zero.
- G. **Strategy:** It is the predetermined rule by which a Player while playing decides the course of action from his own list of courses of action. There are two types of strategies – Pure and Mixed.
- H. **Pure Strategy** – If a Player knows exactly what the other Player is going to do, a deterministic situation is obtained. The objective is to maximize the gain. Thus, it is a decision in advance of all plays always to choose a particular course of action. A pure strategy is usually represented by a number with which the course of action is associated.
- I. **Mixed Strategy** – If a Player is guessing as to which activity is to be selected by the other on any particular occasion, a probabilistic situation is obtained. The objective in this case is to maximize the expected gain. Thus, it is a decision, in advance of all plays to choose a course of action for each play in accordance with some particular probability distribution. When a player decides in advance to use his available courses of action in some fixed proportion, he is said to use mixed strategy. In other words we

can say that the Mixed Strategy is a selection among Pure Strategies with some fixed probabilities.

- J. **Payoff** – The outcome of playing the game is known as Payoff. It is the quantitative measure of satisfaction a Player gets at the end of each play. For a business situation, the measure of satisfaction mentioned above could be Increase in profits, Expansion in actual market share etc. In other words, it is the net gain a course of action or strategy brings to a player for any counter course of action or strategy of the competitor.
- K. **Payoff Matrix** – This is a tabular representation showing the outcomes or payoffs corresponding to different strategies of the participating Players. Since a Game involves at least two Players, the table referred above always forms a matrix with some rows (m, say) and columns (n, say). Rules of a Payoff Matrix are –
- **Rows** denote the activities or courses of action available to Player A who is considered as the **maximising player**.
 - **Columns** denote the activities or courses of action available to Player B who is the **minimising player**.
 - Figure shown in the cell x_{ij} denotes payment to A when he chooses the activity i against B's choice of activity j.
 - For a Two Person Zero Sum Game, any cell entry of the Player B's payoff matrix will be negative of the corresponding cell entry in the Player A's payoff matrix, so that the sum of the payoffs of the two Players is ultimately zero.

The following table is an example of a Payoff Matrix of a Two Person Zero Sum Game which says that two firms are competing for business with the mentioned strategies so that one's gain is another's loss:

Strategies of Firm A	Strategies of Firm B		
	No advertising	Medium advertising	Heavy advertising
No advertising	10	5	-2
Heavy advertising	16	14	10

Here a positive payoff denotes gain to the maximising player i.e. Firm A (shown as Row) and loss to the minimising player i.e. Firm B (shown as Column). If Firm A chooses strategy "No advertising" and Firm B chooses strategy "Medium advertising" then gain of A will be 5 and loss of B will also be 5.

- L. **Optimal Strategy:** A course of action which puts the Player in the most preferred position, irrespective of the strategy of his competitors, is called Optimal Strategy. Not opting for this strategy will result in decreased payoff of the player.
- M. **Value of the Game:** It is the expected payoff of play when all the players of the game follow their optimal strategies. The game is called Fair if the value of the game is zero and Unfair if it is non-zero.

2. ASSUMPTIONS IN A GAME

1. The players act rationally and intelligently.
2. Each player has a finite set of strategies available to him.
3. The players attempt to maximise gains and minimise losses.
4. All relevant information is available to each player.
5. The players take individual decisions without direct communication with each other.
6. The players select their strategies simultaneously.
7. The payoff is fixed and determined in advance.

3. LIMITATIONS OF GAME THEORY

1. The assumption that the players have the knowledge about their own and the opponent's payoffs is unrealistic. He can only make a guess of his own and the opponent's strategies.
2. As the number of players increase in the game, the analysis of the strategies becomes increasingly complex and difficult. In practice there are many firms in an oligopoly situation where game theory is not useful.
3. The assumptions of Pure Strategy game show that the players are risk averse and have complete knowledge about each other's strategies. This is impractical.
4. Rather than each player in an oligopoly situation working under uncertain conditions, the players will allow each other to share the secrets of business in order to work out a collusion. So the mixed strategies are also not very useful.

Q1

Ex. Book No.

Pg. No.

Following Payoff Matrix is given Illustrate 2 Person Zero Sum Pure Strategy Game:

		Strategies of Player B	
		B ₁	B ₂
Strategies of Player A	A ₁	9	2
	A ₂	8	6
	A ₃	6	4

Reference

Pure Strategy Games with Saddle Point

What's New

Answer

In the table above, A is the maximising player with strategies represented along the rows and B is the minimising player with strategies represented along the columns.

Suppose the player A starts the game knowing fully well that whatever strategy he adopts B will select that particular counter strategy which will minimise the payoff to A. If A selects A₁ then B will definitely select B₂ so that A gets minimum possible gain i.e. 2 under the situation. Similarly if A chooses A₂ then B will go for B₂ and so on. Thus, A wants to maximise his gain which is possible by going for the maximum value among the Row minimums or the Maximin value. Similarly, B wants to minimise his loss which is the minimum among the Column maximums or the Minimax value.

We observe here that both **Maximin** and **Minimax** values are equal to **6**. Hence there exists a Saddle Point. Also this value corresponds to the cell A₂ B₂. That means the Optimal strategy for the Player A is A₂ and that for the Player B is B₂. Value of the Game is 6 for A and -6 for B which means the game is **Zero Sum**.

Q2

Ex. Book No.

Pg. No.

Solve the Game using Dominance Principle

$$\begin{bmatrix} 15 & 2 & 3 \\ 6 & 5 & 7 \\ -7 & 4 & 0 \end{bmatrix}$$

Reference

Pure Strategy Games with Saddle Point

What's New

Rule of dominance

Answer

Let the given Game is played by the Players A and B with A (the maximising player) having strategies A_1 , A_2 and A_3 represented along the rows and B (the minimising player) having strategies B_1 , B_2 and B_3 represented along the columns. So the given Payoff Matrix can be written as follows –

Strategies of A	Strategies of B		
	B_1	B_2	B_3
A_1	15	2	3
A_2	6	5	7
A_3	-7	4	0

All the elements of Row A_3 are less than the corresponding elements of Row A_2 . So A_3 is dominated by A_2 . Hence it is ignored and deleted. The new matrix is given below.

Strategies of A	Strategies of B		
	B_1	B_2	B_3
A_1	15	2	3
A_2	6	5	7

Here all the elements of B_3 are more than the corresponding elements of B_2 . Hence B_3 is dominated by B_2 and ignored to get the new matrix below.

Strategies of A	Strategies of B		Row Min.
	B_1	B_2	
A_1	15	2	2
A_2	6	5	5
Column Max.	15	5	

Maximum among the Row minimums = 5 = Maximin value and Minimum among the Column maximums = 5 = Minimax value. As, Maximin and Minimax values are equal, there exists a Saddle Point. It occurs at the cell $A_2 B_2$.

Hence optimal strategies of A and B are respectively A_2 and B_2 . Also value of the Game = 5

[NOTE – This is a problem of Pure Strategy and could have been solved without the use of Dominance Rules, but the question has specifically asked for the usage of Dominance Rules. So the same is used.]

Q3

Ex. Book No.

Pg. No.

Find the optimal strategies of the Players for the game having payoff matrix. **What** is the value of the Game?

Strategies	B1	B2	B3	B4
A1	1	7	3	4
A2	5	6	4	5
A3	7	2	0	3

Reference

Pure Strategy Games with Saddle Point

What's New

Rule of dominance

Answer

Optimal strategies A2 and B3. Value of the game = 4

Q4

Ex. Book No.

Pg. No.

Joy Givers and Milan Toys are the two toy manufacturers who always compete with each other to increase their respective market shares. For both the companies the Marketing team work with close coordination with the Design team and always come out with attractive toys which are normally in great demand. To meet the demand they have various strategic options like working for 8 hours a day, 12 hours a day, 16 hours a day, 24 hours a day, Subcontracting etc. which will ultimately increase the market share. Joy Givers have decided not to go for all the above mentioned options and set up the following payoff matrix in which the percentage increase in market share is given against different strategies of Milan Toys

STRATEGIES of Joy Givers	Milan Toys			
	Working 8 hrs/ day	Working 12 hrs/ day	Working 16 hrs/ day	Subcontracting
Working 12 hrs/day	8	10	9	14
Working 16 hrs/day	10	11	8	12
Working 24 hrs/day	13	12	14	13

Use **Principle of Dominance** to find the **Optimal Strategies** of the two manufacturers and the value of the Game.

Reference

Pure Strategy Games with Saddle Point

What's New

Rule of dominance – Compare with Average of Columns

Answer

Joy Givers is the Maximising player with strategies represented along the rows and Milan Toys is the Minimising Player with strategies represented along the columns. For ease of representation we consider the respective strategies of Joy Givers as J_1, J_2 & J_3 and those of Milan Toys as M_1, M_2 & M_3 .

STRATEGIES of Joy Givers	Milan Toys			
	M_1	M_2	M_3	M_4
J_1	8	10	9	14
J_2	10	11	8	12
J_3	13	12	14	13

All the elements of 4th Column are either greater than or equal to the corresponding elements of the 1st Column. So 4th Column's strategy (M_4) is dominated by the 1st Column's strategy (M_1). Hence M_4 is ignored. The new matrix is given below.

STRATEGIES Joy Givers	Milan Toys		
	M_1	M_2	M_3
J_1	8	10	9
J_2	10	11	8
J_3	13	12	14

All the elements of 1st Row are less than the corresponding elements of the 3rd Row. Thus, strategy of 1st Row i.e. J_1 is dominated by the strategy of the 3rd Row i.e. J_3 and ignored. The reduced matrix becomes –

STRATEGIES Joy Givers	Milan Toys		
	M_1	M_2	M_3
J_2	10	11	8
J_3	13	12	14

Apparently first two rules of dominance cannot be applied to either of the rows or columns of the above matrix, but if the average of the elements of the strategies M_2 and M_3 be taken then we get a matrix shown below.

STRATEGIES Joy Givers	Milan Toys	
	M_1	$[M_2 + M_3]/2$
J_2	10	$(11 + 8)/2 = 9.5$
J_3	13	$(12 + 14)/2 = 13$

So the elements of the strategy M_1 are either more or equal to the average of the corresponding elements of M_2 and M_3 . Hence M_1 is dominated by M_2 and M_3 . Thus, M_1 is deleted and the reduced matrix is as below.

STRATEGIES of Joy Givers	Milan Toys		Row Minimum
	M_2	M_3	
J_2	11	8	8
J_3	12*	14	12 = Maximin
Column Maximum	12 = Minimax	14	

So Maximin value = 12 = Minimax value. Hence there exists a Saddle Point at the junction J_3M_2

Thus, optimal strategy of Joy Giver is J_3 that is "Working 24 hours /day" and that for Milan Toys is M_2 that is "Working 12 hours/day". Value of the Game = 12 (which means a 12% increase in market share for Joy Givers)

Q5

Ex. Book No.

Pg. No.

Reduce the following game by Dominance Rules and Solve it.

	Strategies of Minimising Player					
		P	Q	R	S	T
Strategies of Maximising Player	I	1	3	2	7	4
	II	3	4	1	5	6
	III	6	5	7	6	5
	IV	2	0	6	3	1

Reference

Pure Strategy Games with Saddle Point

What's New

Rule of dominance – Compare with Average of Columns

Answer

Optimal strategy for the Maximising Player is III and that for the Minimising Player is Q. Value of the game = 5

Q6

Ex. Book No.

Pg. No.

Solve the Game with the Payoff Matrix $\begin{bmatrix} 1 & 5 \\ 4 & 2 \end{bmatrix}$

Reference

Mixed Strategy - Odds Method

What's New

Answer

Let the given Game is played by the Players A and B with A (the maximising player) having strategies A_1 and A_2 represented along the rows and B (the minimising player) having strategies B_1 and B_2 represented along the columns. So the given Payoff Matrix can be written as follows –

		Strategies of B		Row Min.
		B_1	B_2	
Strategies of A	A_1	1	5	1
	A_2	4	2	2 = Maximin
Column Max.		4 = Minimax	5	

Maximin value (2) $\neq$ Minimax value (4). Thus, Saddle Point does not exist. So this is a problem of Mixed Strategy with (2×2) Payoff Matrix.

We apply Odds Method to solve the problem. Odds are calculated as follows –

		Strategies of B		ODDs
		B_1	B_2	
Strategies of A	A_1	$1 = a_1$	$5 = a_2$	$b_1 - b_2 = 4 - 2 = 2$
	A_2	$4 = b_1$	$2 = b_2$	$a_1 - a_2 = 5 - 1 = 4$
ODDs		$a_2 - b_2 = 5 - 2 = 3$	$a_1 - b_1 = 1 - 4 = -3$	

[Note: Though $(a_1 - b_1) = 1 - 4 = -3$, but here it has been taken + 3 as per the concept of Odds]
Probabilities of A and B taking their different strategies are calculated as follows –

$$P(A_1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 2 \div [2 + 4] = 2/6 = 1/3$$

$$P(A_2) = (a_1 - a_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 4 \div [2 + 4] = 4/6 = 2/3$$

$$P(B_1) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)] = 3 \div [3 + 3] = 3/6 = 1/2$$

$$P(B_2) = (a_1 - b_1) \div [(a_2 - b_2) + (a_1 - b_1)] = 3 \div [3 + 3] = 3/6 = 1/2$$

$$\text{Value of the Game} = v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)] = [1 \times 2 + 4 \times 4] \div [2 + 4] = 18/6 = 3$$

So A chooses his strategies (A_1, A_2) with probabilities $(1/3, 2/3)$ & B chooses his strategies (B_1, B_2) with probabilities $(1/2, 1/2)$ and Value of the Game = 3

[Note: Calculated Value of the Game is the Expected Gain of A which is same as the Expected Loss of B]

Q7

Ex. Book No.

Pg. No.

Solve the game

$$\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$$

Reference

Mixed Strategy - Odds Method

What's New

Answer

Strategies of the Maximising Player = $(1/5, 4/5)$ & for the Minimising Player = $(3/5, 2/5)$. Value of the game = $17/5$

Q8

Ex. Book No.

Pg. No.

The Management of a company is negotiating with its Union for revision of hourly wages of its employees. The Management deployed a Consulting Firm who has prepared a payoff matrix for the purpose which indicates the additional hourly cost (in ₹) to the company. It is shown below: you being a part of the Consulting Firm have to assist the Management in selecting the best strategy.

What is the value of the game? **How** is it going to affect the company's cost?

Management's Strategies	Strategies of the Union			
	U_1	U_2	U_3	U_4
M_1	2.50	2.70	3.50	-0.20
M_2	2.00	1.60	0.80	0.80
M_3	1.40	1.20	1.50	1.30
M_4	3.00	1.40	1.90	0

Reference

Mixed Strategy - Odds Method

What's New

Rule of dominance

Answer

As the Management's objective is to minimise the cost, they can be considered as the Minimising Player and the Union as the Maximising Player in this problem of Game. Thus, to solve the problem we have to recast the given Payoff Matrix by transposing it as below:-

Strategies of the Union	Management's Strategies				Row Minimum
	M ₁	M ₂	M ₃	M ₄	
U ₁	2.50	2.00	1.40	3.00	1.40 = Maximin
U ₂	2.70	1.60	1.20	1.40	1.20
U ₃	3.50	0.80	1.50	1.90	0.80
U ₄	-0.20	0.80	1.30	0	-0.20
Column Maximum	3.50	2.00	1.50 = Minimax	3.00	

Maximin value (1.40) $\neq$ Minimax value (1.50). Thus, Saddle Point does not exist. So this is a problem of Mixed Strategy. Since the matrix is not a (2×2) Matrix, Dominance Rules are applied to reduce its size to make it a (2×2) Matrix.

As all the elements of the 3rd Row of the above matrix are either greater than or equal to the corresponding elements of the 4th Row, the 3rd Row can be considered to dominate the 4th. So the 4th Row is ignored and the new matrix is shown below.

Strategies of the Union	Management's Strategies			
	M ₁	M ₂	M ₃	M ₄
U ₁	2.50	2.00	1.40	3.00
U ₂	2.70	1.60	1.20	1.40
U ₃	3.50	0.80	1.50	1.90

Again all the elements of the 1st Column are greater than the corresponding elements of the 2nd Column, the 1st Column is dominated by the 2nd Column. So the 1st Column is ignored and the new matrix is shown below.

Strategies of the Union	Management's Strategies		
	M ₂	M ₃	M ₄
U ₁	2.00	1.40	3.00
U ₂	1.60	1.20	1.40
U ₃	0.80	1.50	1.90

All the elements of the 3rd Column (i.e. for Strategy M₄) of this matrix are more than the corresponding elements of 2nd Column (i.e. for Strategy M₃). Hence M₄ is dominated by M₃ and ignored. The new matrix is shown below.

Strategies of the Union	Management's Strategies	
	M_2	M_3
U_1	2.00	1.40
U_2	1.60	1.20
U_3	0.80	1.50

Again all the elements of the 1st Row (for strategy U_1) are greater than the corresponding elements of the 2nd Row (for strategy U_2). So U_2 is dominated by U_1 and ignored. The new matrix is shown below.

Strategies of the Union	Management's Strategies	
	M_2	M_3
U_1	2.00	1.40
U_3	0.80	1.50

This is a (2×2) Matrix. Now the problem of Game is solved by using Odds Method. Odds are calculated as below.

Strategies of the Union	Management's Strategies		ODDs
	M_2	M_3	
U_1	$2.00 = a_1$	$1.40 = a_2$	$b_1 - b_2 = 0.80 - 1.50 = 0.70$
U_3	$0.80 = b_1$	$1.50 = b_2$	$a_1 - a_2 = 2.00 - 1.40 = 0.60$
ODDs	$a_2 - b_2 = 1.40 - 1.50 = 0.10$	$a_1 - b_1 = 2.00 - 0.80 = 1.20$	Sum of the ODDs = 1.30

Probabilities of the Union and the Management taking their different strategies are calculated as follows – $P(U_1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 0.70 / [0.70 + 0.60] = 0.70/1.30 = 7/13$

$$P(U_3) = (a_1 - a_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 0.60 / [0.70 + 0.60] = 0.60/1.30 = 6/13$$

$$P(M_2) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)] = 0.10 / [0.10 + 1.20] = 0.10/1.30 = 1/13$$

$$P(M_3) = (a_1 - b_1) \div [(a_2 - b_2) + (a_1 - b_1)] = 1.20 / [0.10 + 1.20] = 1.20/1.30 = 12/13$$

$$\text{Value of the Game} = v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)] = [2.00 \times 0.70 + 0.80 \times 0.60] \div [0.70 + 0.60]$$

$$= [1.40 + 0.48] / 1.30 = 1.88/1.30 = 1.45$$

So the Union chooses its Strategies U_1 , U_2 , U_3 & U_4 with probabilities $(7/13, 0, 6/13, 0)$ and the Management chooses its Strategies M_1 , M_2 , M_3 & M_4 with probabilities $(0, 1/13, 12/13, 0)$.

Expected Gain to the Union is ₹ 1.45 and the corresponding Loss to the Management is ₹ 1.45.

Thus, the hourly cost of the company will increase by ₹ 1.45



Ex. Book No.

Pg. No.

Solve the Game represented by the payoff matrix : $\begin{bmatrix} 1 & 3 & 12 \\ 8 & 6 & 2 \end{bmatrix}$

Reference

Mixed Strategy - Graphical Method

What's **New**

(2 × n)

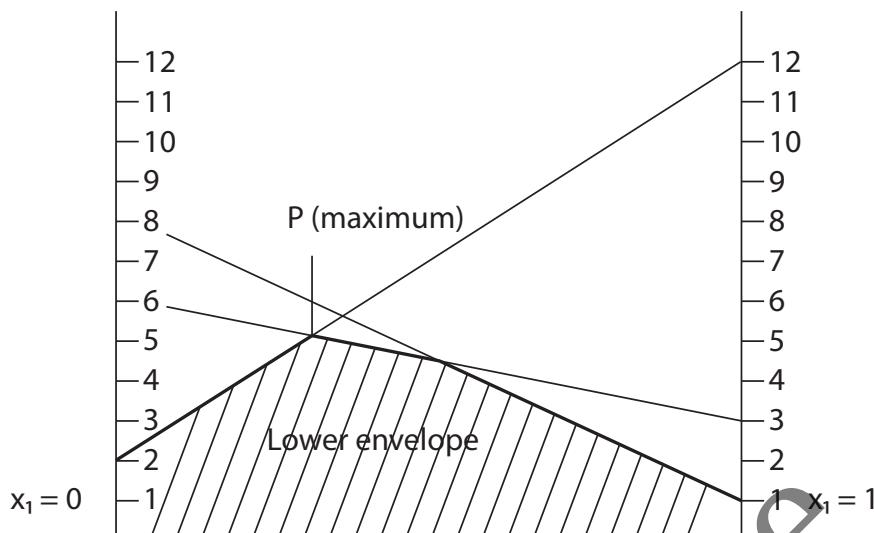
Answer

Let the given Game is played by the Players A and B with A (the maximising player) having strategies A_1 and A_2 represented along the rows and B (the minimising player) having strategies B_1, B_2 and B_3 represented along the columns. So the given Payoff Matrix can be written as follows:

Strategies of A	Strategies of B			Row Minimum
	B_1	B_2	B_3	
A_1	1	3	12	1
A_2	8	6	2	2 = Maximin
Column Maximum	8	6 = Minimax	12	

Maximin value (2) $\neq$ Minimax value (6). Thus, Saddle Point does not exist. So this is a problem of Mixed Strategy. Since the matrix is not a (2×2) Matrix, we check the possibility of applying Dominance Rules to reduce its size to (2×2) Matrix, but it is observed that the Dominance Rules are also not suitable for reducing the size of the given matrix. Hence we go for solving the problem using Graphical Method suitable for $(2 \times n)$ matrix because the given matrix is (2×3)

As shown below two vertical lines are drawn on a horizontal line 1 unit apart to represent the axes $x_1 = 0$ & $x_1 = 1$ and marked them to a scale. Now the values 1, 3 and 12 of the 1st Row of the given matrix are plotted as points on the axis $x_1 = 1$ and the values 8, 6 and 2 of the 2nd Row are plotted as points on the axis $x_1 = 0$. Then the pair of points 12 & 2, 3 & 6 and 1 & 8 are joined with the help of straight lines. These lines represent the expected payoff of Player A against the strategies B_3, B_2 & B_1 respectively of Player B. The lower envelope of the area bounded by these lines and the axes is shaded as shown. Highest point P of this envelope is the Maximin point. As P is the point of intersection of A's expected payoff lines against strategies B_2 & B_3 of B, we can say that B will opt for these two strategies and ignore strategy B_1 .



So the reduced payoff matrix is given as follows –

Strategies of A	Strategies of B		ODDs
	B ₂	B ₃	
A ₁	3 = a ₁	12 = a ₂	b ₁ - b ₂ = 6 - 2 = 4
A ₂	6 = b ₁	2 = b ₂	a ₁ - a ₂ = 3 - 12 = 9
ODDs	a ₂ - b ₂ = 12 - 2 = 10	a ₁ - b ₁ = 3 - 6 = 3	Sum of the ODDs = 13

Probabilities of A and B taking their different strategies are calculated as follows –

$$P(A_1) = (b_1 - b_2) \div [(b_1 - b_2) + (a_1 - a_2)] = 4 / [4 + 9] = 4/13, P(A_2) = 1 - P(A_1) = 1 - 4/13 = 9/13$$

$$P(B_2) = (a_2 - b_2) \div [(a_2 - b_2) + (a_1 - b_1)] = 10 / [10 + 3] = 10/13, P(B_3) = 1 - P(B_2) = 1 - 10/13 = 3/13$$

$$\text{Value of the Game} = v = [a_1(b_1 - b_2) + b_1(a_1 - a_2)] \div [(b_1 - b_2) + (a_1 - a_2)] = [3 \times 4 + 6 \times 9] \div [4 + 9] = 66/13$$

So A chooses the strategies with probabilities (4/13, 9/13) and B chooses the strategies with probabilities (10/13, 3/13).

Q10

Ex. Book No. Pg. No.

Solve the game $\begin{bmatrix} 3 & -4 \\ 2 & 5 \\ -2 & 8 \end{bmatrix}$ using Graphical method

Reference

Mixed Strategy - Graphical Method

What's New

(m × 2)

Answer

Optimal strategy of the Maximising Player (0.3, 0.7, 0) and for the Minimising Player (0.9, 0.1).
Value = 2.3

Q11

Ex. Book No.

Pg. No.

In the suburban area of a large city there are two stores Laxmi Bhandar and Goswami Stores who handle sundry goods. The total number of customers is equally divided between the two due to the fact that the price, quality of goods, services etc. of the two are at par. Assume that a gain of customers for Laxmi Bhandar is a loss to Goswami Stores and vice versa. Both the stores plan to run annual sales during the festival period of the year. Sales are advertised through Social Media, Cable TV local channel and Printed Leaflets. Based on the past experience, Laxmi Bhandar has prepared the following payoff matrix for the gain or loss in percentage of customers for its different strategies against various counter strategies of Goswami Stores.

Strategies of Laxmi Bhandar	Strategies of Goswami Stores		
	Printed leaflets	Cable TV	Social media
Printed leaflets	30	40	– 80
Cable TV	0	15	– 20
Social media	90	20	50

Determine the optimal strategies and worth of such strategies for the stores. **What** is meant by the cell entry – 80 in the above payoff matrix?

Reference

Mixed Strategy - Odds Method - Rule of dominance

What's New

Interpretation of cell entry

Answer

Optimal strategies of Laxmi Bhandar = (1/5, 0, 4/5) and for Goswami Stores = (0, 13/15, 2/15),
Value of the game = 24

Cell entry (– 80) means when Laxmi Bhandar will take the strategy of distributing Printed Leaflets against the counter strategy of Goswami Stores of Social Media advertisement then they will lose 80% of their customer which will be gained by Goswami Stores.


Q12

Ex. Book No.

Pg. No.

Two competing firms (A and B) produce consumer goods of different kind. Among the products one is considered as their bread and butter in terms of the revenue generated. Both the firms are very cautious about the market share for this particular product and keep on doing advertisement campaigns throughout the year to retain the existing customers and also to attract the new ones. For this the marketing teams of both work round the clock and that of A developed data corresponding to varying degrees of advertisement. Same is given below:

- If both the firms take same strategy to counter each other then their market share will be equal.
- Against firm A's strategy of "No marketing" if B goes for "Medium marketing" then A's share of the market will be 40%. For the same strategy of A the market share will be 28% if B takes the strategy "Large marketing"
- Against firm A's strategy of "Medium marketing" if B goes for "No marketing" then A's share of the market will be 70%. For the same strategy of A the market share will be 45% if B takes the strategy "Large marketing"
- Against firm A's strategy of "Large marketing" if B goes for "No marketing" then A's share of the market will be 75%. For the same strategy of A the market share will be 47.5% if B takes the strategy "Medium marketing"

Based on the above information **prepare** the Payoff Matrix. **Solve** the game problem to get the optimal strategies of the player A. **What** is the value of the game?


Reference
Mixed Strategy - Odds Method - Rule of dominance
What's New**Pay off Matrix**

Answer

The payoff matrix Showing A's market share is –

Strategies of A	Strategies of B		
	No advertising	Medium advertising	Large advertising
No advertising	50	40	28
Medium advertising	70	50	45
Large advertising	75	47.5	50

Probabilities of A's strategies are (0, 1/3, 2/3). Value of the game = $145/3 = 48.3$ Thus, A can expect to have 48.3% market share.

Q13

Ex. Book No.

Pg. No.

Using the data of the above problem **prepare** the Payoff Matrix for A when you are supplied with the following information.

- (a) Selling price of the product = ₹ 4 per unit
- (b) Variable cost of the product = ₹ 2.50 per unit
- (c) Annual cost for Medium advertising = ₹ 5000
- (d) Annual cost for Large advertising = ₹ 15000
- (e) Annual sales volume of the product for Firm A = 30000 units

What advertising policy should firm A pursue?



Reference

Pure Strategy

What's **New**
(MaxiMin)

Answer

Find out the Annual sales volume, for different combination of strategies of A and B. As an example, Annual Sales volume corresponding to A's strategy of "Medium advertisement" and B's strategy of "Large advertisement" is 45% of 30000 = 13500 units

Calculate Annual Profit to the Firm A using the formula below for various combination of strategies of A and B.

Annual Profit = (Selling price – Variable cost) × Annual Sales volume – Annual cost of advertising
Example of this calculation is:-

For A's strategy of "Medium advertisement" and B's strategy of "Large advertisement" the Annual Profit of Firm A is $(4 - 2.5) \times 13500 - 5000 = ₹ 15250/-$

When the Profit figures for all the combinations of strategies of A and B are calculated then the following payoff matrix is obtained.

Strategies of A	Strategies of B		
	No advertising	Medium advertising	Large advertising
No advertising	22500	18000	12600
Medium advertising	26500	17500	15250
Large advertising	18750	6375	7500

From the above matrix we find, against the various strategies of A, the minimum profit figures are as follows –

For **No advertising** – ₹ 12600

For **Medium advertising** – ₹ 15250

For **Large advertising** – ₹ 6375

Thus, to maximise the minimum profit, A should opt for Medium advertising and spend ₹ 5000 per annum.

SJC Institute

Business Application of Maxima and Minima



1. BASICS

[A] BASICS OF DIFFERENTIATION

[B] BASICS OF PARTIAL DIFFERENTIATION

[C] BASICS OF ECONOMICS

[D] BASICS OF DETERMINANTS

2. UNCONSTRAINED OPTIMISATION

[A] SINGLE INDEPENDENT VARIABLE

[B] MULTIPLE INDEPENDENT VARIABLES

[C] EQUILIBRIUM OF FIRMS

3. CONSTRAINED OPTIMISATION

(i) Single Equality Constraints

4. COBBS DOUGLAS FUNCTION

1. BASICS

A. BASICS OF DIFFERENTIATION

1. If $y = f(x) = x^n$ then $dy/dx = f'(x) = nx^{n-1}$
2. If $y = f(x) = e^x$ then $dy/dx = f'(x) = e^x$
3. If $y = f(x) = e^{mx}$ then $dy/dx = f'(x) = me^{mx}$, where 'm' is a constant
4. If $y = f(x) = \log_e x$ or $\ln x$ then $dy/dx = f'(x) = 1/x$
5. If $y = f(x) = k$ then $dy/dx = f'(x) = 0$, where k is a constant
6. If $y = f(x) = p(x) \pm q(x)$ then $dy/dx = f'(x) = p'(x) \pm q'(x)$
7. $\frac{d}{dx}(u \times v) = u \times \frac{dv}{dx} + v \times \frac{du}{dx}$
8. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \times \frac{du}{dx} - u \times \frac{dv}{dx}}{v^2}$

B. BASICS OF PARTIAL DIFFERENTIATION

In case of Partial differentiation, all the above formulae / rules hold good with the exception that all the terms present in the given function are considered as constant except the ones with the participating variable of partial differentiation. Some examples are -

- (a) If $U = xy$ then $\partial U / \partial x = y$ and $\partial U / \partial y = x$
- (b) If $U = x^a y^b$ then $\partial U / \partial x = ax^{a-1}y^b$ and $\partial U / \partial y = bx^a y^{b-1}$ [where 'a' and 'b' are constant]
- (c) If $U = px + qy$ then $\partial U / \partial x = p$ and $\partial U / \partial y = q$ [where 'p' and 'q' are constant]
- (d) If $U = a\sqrt{x} + b\sqrt{y}$ then $\partial U / \partial x = a/2.(x)^{-1/2}$ and $\partial U / \partial y = b/2.(y)^{-1/2}$ [where 'a' and 'b' are constant].

C. BASICS OF ECONOMICS - TR, AR, MR, TC, AC, MC

Revenue: The Revenue is defined as the money earned by selling certain quantity of output. More precisely it should be called the "Sales Revenue" and it must not be mixed up with other similar concepts of earning money like income, profit etc.

Revenue involves three inter-related concepts – **Total Revenue (TR), Average Revenue (AR) & Marginal Revenue (MR).**

Total Revenue is the product of Price (P) and the Quantity Sold (x). Thus $TR = P \times x$

Average Revenue is the Revenue earned per unit sale.

So $AR = TR / x$ Or, $AR = Px / x$ Or, $AR = P = \text{Price}$

Hence Average Revenue can be termed as the Price or vice versa. Thus Average Revenue curve is the same as the Demand curve.

Marginal Revenue is defined as the revenue earned by selling an additional unit of output. In other words, it represents the rate of change of Total Revenue with respect to Output which is nothing but the derivative of Total Revenue with respect to Output.

$$\text{Hence } MR = \frac{d}{dx} (TR)$$

Mathematically Revenue is expressed as a function of Output. That is $R = f(x)$

Cost of Production: Like Revenue Cost, too has three basic concepts – **Total Cost (TC), Average Cost (AC) and Marginal Cost (MC)**

Total Cost has two components: Fixed and Variable. So $TC = TFC + TVC$

Average Cost is given as $AC = TC / x$ Or, $AC = TFC / x + TVC / x$ Or, $AC = AFC + AVC$

Marginal Cost is the cost of producing an additional unit of output.

$$\text{Mathematically, } MC = \frac{d}{dx} (TC) = \frac{d}{dx} (TFC + TVC)$$

Cost of Production is expressed as a function of Output in order to know its behaviour at different levels of output produced or capacity utilization. So $C = f(x)$ is the mathematical way of representation of Cost.

Profit: It is the residual after deducting Cost from the Revenue. It is represented as $\pi = f(x)$

Total Profit = $T \pi = TR - TC$ Or, $T \pi = TR - TC$ Or, $T \pi = x (AR) - x (AC)$ Or, $T \pi = x (AR - AC)$

Average Profit = $A \pi = T \pi / x$ Or, $A \pi = (TR - TC) / x$ Or, $A \pi = TR / x - TC / x$ Or, $A \pi = AR - AC$

$$\text{Marginal Profit} = M \pi = \frac{d}{dx} (T \pi) \text{ Or, } M \pi = \frac{d}{dx} (TR - TC) \text{ Or, } M \pi = \frac{d}{dx} (TR) - \frac{d}{dx} (TC)$$

$$\text{Or, } M \pi = MR - MC$$

D. BASICS OF DETERMINANTS

2. UNCONSTRAINED OPTIMISATION

A. SINGLE INDEPENDENT VARIABLE (DIFFERENTIATION)

When the objective function is given as an Algebraic Function and no constraints are imposed then **Calculus approach of derivatives** is used to optimize the function. The conditions of optimization are two fold and given as

1. **Necessary condition:** The 1st Order Derivative should be Zero, that is $\frac{dy}{dx} = 0$

This is applicable for both the situations of Maximization as well as Minimization.

2. **Sufficient condition:** $\frac{d^2y}{dx^2} < 0$ for Maxima and $\frac{d^2y}{dx^2} > 0$ for Minima at the value of x obtained from (1)

The value/s of x obtained from the Necessary Condition (1) written above, is/are called Critical Value/s. The sign of 2nd Order Derivative is checked by putting the Critical value/s in it and subsequently decision regarding Maxima or Minima is taken as per Sufficient Condition (2) above.

It can be mentioned that when $\frac{d^2y}{dx^2} = 0$ then there exist neither a Maxima nor a Minima. Such a point is known as **Point of Inflexion**.

B. MULTIPLE INDEPENDENT VARIABLES (PARTIAL DIFFERENTIATION)

For the situations where the Objective Function involves more than one Independent variable (say two) and the change in the dependent variable is the joint impact of changes in both the variables then the approach towards optimization of the Objective Function, though takes help of differential calculus, but is not exactly same as that used for the case of single Independent variable.

In fact the measurement of the independent impact of one variable is not possible without assuming that the other variable remains unchanged. As example we consider the case of Sales of a commodity which is a multivariate function of Price and Advertising Expenditure. Now the impact of change in Price over Sales cannot be measured if we do not assume the Advertising Expenditure to remain same. Partial Derivative of a function explains the same logic mathematically.

The conditions of optimization in this case is given as follows –

1. The values of the 1st Order Partial Derivatives should be Zero. That is $\frac{\partial f}{\partial x}$ or $f_x = 0$ and $\frac{\partial f}{\partial y}$ or $f_y = 0$, when the Objective Function is Bivariate. If the number of

variables are more, the Partial Derivatives of the function with respect to those variables should also be zero.

From the above equations the Critical Values of x and y are determined. In other words the coordinates of the Critical Point (a,b) are determined, where a = Value of x and b = Value of y for a Bivariate function.

2. Next find out all possible 2nd Order Partial Derivatives. For a Bivariate Function these are

$\frac{\partial^2 f}{\partial x^2}$ or f_{xx} , $\frac{\partial^2 f}{\partial y^2}$ or f_{yy} , $\frac{\partial^2 f}{\partial y \partial x}$ or f_{xy} and $\frac{\partial^2 f}{\partial x \partial y}$ or f_{yx} . Calculate the numerical values

of these by putting $x = a$ and $y = b$

3. Assume: $A = f_{xx}(a,b)$, $B = f_{xy}(a,b)$ and $C = f_{yy}(a,b)$ and find the value of $D = AC - B^2$
- If $D > 0$ and $A, C > 0$ then there is a local Minima at (a,b)
 - If $D > 0$ and $A, C < 0$ then there is a local Maxima at (a,b)
 - If $D < 0$ then (a,b) is a Saddle Point
 - If $D = 0$ then the test fails.

[Note: The expression $D = AC - B^2$ can also be represented in the form of a determinant as follows –

$$D = \begin{vmatrix} A & B \\ B & C \end{vmatrix} \text{ Or, } D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}. \text{ As } f_{xy} \text{ is equal to } f_{yx} \text{ always, we represent both as } B. \text{ This}$$

determinant actually corresponds to a (2×2) Matrix called Hessian Matrix. So

Hessian Matrix of order 2 is given as $\begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix}$ and the same is applicable when

there are two variables in the problem. For problems with three variables, we will come across Hessian Matrix of order 3.]

C. CONDITION FOR FIRM'S EQUILIBRIUM

It has already been discussed that a Firm is said to be in Equilibrium when it maximizes its Profit. As per the conditions of mathematical approach it requires that

$$\frac{d}{dx}(T\pi) = 0 \text{ and } \frac{d^2}{dx^2}(T\pi) < 0$$

Now, $T\pi = TR - TC$

Differentiating both sides with respect to x we get

$$\frac{d}{dx}(T\pi) = \frac{d}{dx}(TR) - \frac{d}{dx}(TC)$$

Or,

$$\frac{d}{dx}(T\pi) = MR - MC$$

But, $(T\pi) = 0$ Or, $MR - MC = 0$ Or, **MR = MC** is the **Condition for a Firm's Equilibrium**

This is the **necessary condition** of a Firm's Equilibrium.

For the sufficient condition of the Firm's Equilibrium we should have

$$\frac{d^2}{dx^2}(T\pi) < 0 \text{ Or, } \frac{d^2}{dx^2}(TR - TC) < 0$$

$$\text{Or, } \frac{d^2}{dx^2}(TR) - \frac{d^2}{dx^2}(TC) < 0$$

$$\text{Or, } \frac{d^2}{dx^2}(\text{TR}) < \frac{d^2}{dx^2}(\text{TC})$$

$$\text{Or, } \frac{d}{dx} \left[\frac{d(\text{TR})}{dx} \right] < \frac{d}{dx} \left[\frac{d(\text{TC})}{dx} \right]$$

$$\text{Or, } \frac{d}{dx}[\text{MR}] < \frac{d}{dx}[\text{MC}]$$

The conditions written above are meant for single variable type situations of optimization. In case the number of variables is more than one then the Condition of Firm's Equilibrium for the following situations are –

1. **For Multi-plant Monopolist Firm producing the same product in all the Plants**

The **necessary condition** is $\text{MR} = \text{MC}_1 = \text{MC}_2$, where MC_1 and MC_2 = Respective Marginal Costs of Production in Plants 1 & 2 (for the sake of simplicity the discussion has been confined to two Plants. When there are more than two Plants, then also the condition is applicable)

The sufficient condition is given as –

$$\frac{\partial^2}{\partial x^2}(\text{TR}) < \frac{\partial^2}{\partial x_1^2}(\text{TC}_1) \text{ and } \frac{\partial^2}{\partial x^2}(\text{TR}) < \frac{\partial^2}{\partial x_2^2}(\text{TC}_2)$$

[x_1 & x_2 are the quantities produced in the Plants 1 and 2 respectively. Also $x = x_1 + x_2$]

2. **For Price discriminating Monopolist Firm selling the same product at different prices in different markets**

The **necessary condition** is $\text{MC} = \text{MR}_1 = \text{MR}_2$, where MR_1 and MR_2 are the respective Marginal Revenues for the two different market segments having Demand Functions given as p_1 and p_2

The sufficient condition is given as –

$$\frac{\partial^2}{\partial x_1^2}(\text{TR}_1) < \frac{\partial^2}{\partial x^2}(\text{TC}) \text{ and } \frac{\partial^2}{\partial x_2^2}(\text{TR}_2) < \frac{\partial^2}{\partial x^2}(\text{TC})$$

[x_1 & x_2 are the quantities sold at the prices p_1 and p_2 respectively. Also $x = x_1 + x_2$]

[N.B – Monopolist Firms operate in a Monopoly market structure in which there is a single seller, there are no close substitutes for the commodity it produces and there are barriers to entry.]

3. CONSTRAINED OPTIMISATION

Some of the situations which come under the purview of such type of optimization can be –

- A consumer has to choose how much to buy of each product such that it satisfies the budget constraint. In other words it is a case of Maximization of Utility subject to budget constraint.
- A firm would look to minimize its cost of production subject to a given output level which can also be termed as Minimization of Cost subject to output constraint.
- A manufacturer would try to maximize its production with whatever quantity of a scarce material available with him/her. That means a problem of Maximization of Outputs subject to resource constraint.

In all the above situations Lagrangian Multiplier Method can be successfully used for optimization. To explain the steps involved, we take help of the following example.

A consumer has a choice of two commodities X & Y. Prices of these are p and q respectively. The person has limited money (M) and wants to procure maximum possible quantities of X & Y with the amount he / she has.

The problem can be rewritten as –

Maximize Utility function $U = f(X, Y)$ subject to the constraint $pX + qY = M$

Steps to be followed by Lagrangian Multiplier Method are –

1. Transform the Constraint equation to a form with 0 on the R.H.S. In this case it is $pX + qY - M = 0$
2. Multiply L.H.S of the transformed Constraint Equation by Lagrange's Multiplier (λ). In this case it takes the form $\lambda.(pX + qY - M)$
3. From the original Objective Function subtract the one obtained in the previous step to form Lagrangian function given by $L(X, Y, \lambda) = f(X, Y) - \lambda.(pX + qY - M)$

[It can be noted that $L(X, Y, \lambda) = f(X, Y)$ when the constraint holds i.e $pX + qY = M$. Hence maximization of Lagrangian function ultimately maximises the original objective function $f(X, Y)$]

4. Find the critical values of the unknowns (X, Y and λ) using the 1st Order conditions i.e all the partial derivatives are equal to zero.

$$\text{Thus, } \partial L / \partial X = 0 \text{ Or, } f_x - \lambda p = 0$$

$$\partial L / \partial Y = 0 \text{ Or, } f_y - \lambda q = 0$$

$$\text{and } \partial L / \partial \lambda = 0 \text{ Or, } pX + qY - M = 0$$

Solving the above three equations one can get the values of three unknowns X, Y and λ .

5. Now find a Bordered Hessian Matrix (HB) given as –

$$HB = \begin{bmatrix} 0 & g_x & g_y \\ g_x & L_{xx} & L_{xy} \\ g_y & L_{yx} & L_{yy} \end{bmatrix}$$

It can be noted that g_x and g_y are the partial derivatives with respect to x and y respectively for the function given by $g(X,Y) = pX + qY - M$ i.e. the given constraint expressed as a function.

6. Find the value of the Determinant corresponding to the matrix HB i.e evaluate $\text{Det. } HB$
7. If **Det.HB** > 0 then the critical values of X and Y obtained in step (4) corresponds to a **Maxima**.

If **Det.HB** < 0 then the critical values of X and Y obtained in step (4) corresponds to a **Minima**.

4. COBBS DOUGLAS PRODUCTION FUNCTION

Total Cost in the Long Run and the Isocost Line

For a long-run, two-input production function, $Q = f(L,K)$, the total cost of production is the sum of the cost of the labor input, L , and the capital input, K

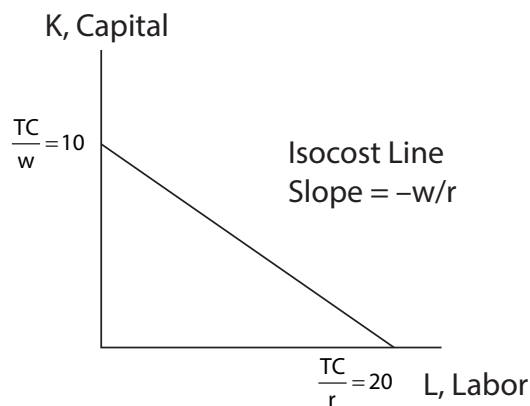
- The cost of labor is called the wage rate, w
- The cost of capital is called the rental rate, r
- The cost of the labor input is the wage rate multiplied by the amount of labor employed, wL
- The cost of capital is the rental rate multiplied by the amount of capital, rK

The total cost (C), therefore, is

$$C(Q) = wL + rK$$

If we hold total cost $C(Q)$ constant, we can use this equation to find **isocost lines**. An isocost line is a graph of every possible combination of inputs that yields the same cost of production. By picking a cost, and given wage rates, w , and rental rates, r , we can find all the combinations of L and K that solve the equation and graph the isocost line.

Consider the example of a pencil-making factory, where both capital in the form of pencil-making machines and labor to run those machines are utilized. Suppose the wage rate of labor for the pencil maker is \$20 per hour and the rental rate of capital is \$10 per hour. If the total cost of production is \$200, the firm could be employing ten hours of labor and no capital, twenty hours of capital and no labor, five hours of labor and ten hours of capital, or any other combination of capital and labor for which the total cost is \$200. Following Figure illustrates this particular isocost line.

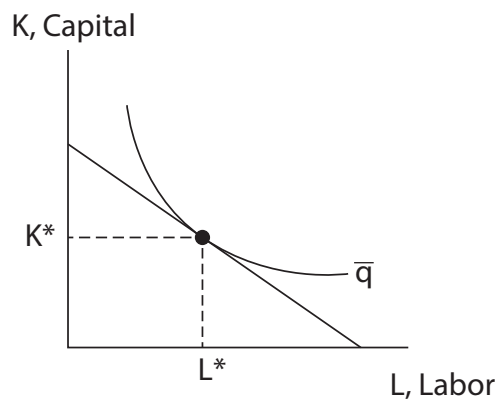


An isocost line

This figure represents the isocost line where total cost equals \$200. But we can draw an isocost line that is associated with any total cost level. Notice that any combination of labor hours and capital that is less expensive than this particular isocost line will end up on a lower isocost line. For example, two hours of labor and five hours of capital will cost \$90. Any combination of hours of labor and capital that are more expensive than this particular isocost line will end up on a higher isocost line. For example, twenty hours of labor and thirty hours of capital will cost \$700.

Note that the slope of the isocost line is the ratio of the input prices, $-w/r$. This tells us how much of one input (capital) we have to give up to get one more unit of the other input (labor) and maintain the same level of total cost. For example, if both labor and capital cost \$10 an hour, the ratio would be $-10/10$ or -1 . This is intuitive—if they cost the same amount, to get one more hour of labor, you need to give up one hour of capital. In our pencil-maker example, labor is \$20 per hour, and capital is \$10 per hour, so the ratio is -2 : to get one more hour of labor input, you must give up two hours of capital in order to maintain the same total cost or remain on the same isocost line.

The solution to the long-run cost minimization problem is illustrated in following figure. The plant manager's problem is to produce a given level of output at the lowest cost possible. A given level of output corresponds to a particular isoquant, so the cost minimization problem is to pick the point on the isoquant that is the lowest cost of production. This is the same as saying the point that places the firm on the lowest isocost line. We can see this by examining following figure and noting that the point on the isoquant that corresponds to the lowest isocost line is the one where the isocost is tangent to the isoquant.



Solution to the long-run cost minimization problem

From above figure, we can see that the optimal solution to the cost minimization problem is where the isocost and isoquant are tangent: the point at which they have the same slope. We just learned that the slope of the isocost is $-w/r$, and we learned that the slope of the isoquant is the marginal rate of technical substitution (MRTS), which is the ratio of the marginal product of labor and capital:

$$\text{MRTS} = -\frac{\text{MP}_L}{\text{MP}_K}$$

So the solution to the long-run cost minimization problem is

$$\text{MRTS} = \frac{w}{r},$$

or

$$\frac{\text{MP}_L}{\text{MP}_K} = \frac{w}{r} \quad \dots (1)$$

This can be rearranged to help with intuition:

$$\frac{\text{MP}_L}{w} = \frac{\text{MP}_K}{r} \quad \dots (2)$$

Equation (2) says that at the cost-minimizing mix of inputs the marginal products per dollar must be equal. This conclusion makes sense if you think about what would happen if equation (1) did not hold. Suppose instead that the marginal product of capital per dollar was more than the marginal product of labor per dollar:

$$\frac{\text{MP}_L}{w} < \frac{\text{MP}_K}{r}$$

This inequality tells us that this current use of labor and capital cannot be an optimal solution to the cost minimization problem. To understand why, consider the effect of taking a dollar spent on labor input away, thereby lowering the amount of labor input

(raising the MP_L – remember the law of diminishing marginal returns), and spending that dollar instead on capital and increasing the capital input (lowering the MP_K). We know from the inequality that if we do this, overall output must increase because the additional output from the extra dollar spent on capital has to be greater than the lost output from the diminished labor. Therefore, the net effect is an increase in overall output. The same argument applies if the inequality were reversed.

An Example of Minimizing Costs in the Long Run

Consider a specific example of a gourmet root beer producer whose labor cost is \$20 an hour and whose capital cost is \$5 an hour. Suppose the production function for a barrel of root beer, Q , is $Q = 10L^{1/2}K^{1/2}$. If the output target is one thousand barrels of root beer, they could, for example, utilize one hundred hours of labor, L , and one hundred hours of capital, K , to yield $10(10)(10) = 1,000$ barrels of root beer. But is this the most cost-efficient way to do it? More generally, what is the most cost-effective mix of labor and capital to produce one thousand barrels of root beer?

To determine this, we must start with the marginal products of labor and capital, which for this production function are the following:

$$MP_L = 5L^{-1/2}K^{1/2} \quad \dots (3)$$

$$MP_K = 5L^{1/2}K^{-1/2} \quad \dots (4)$$

And thus the MRTS, $-\frac{MP_L}{MP_K}$ is $-\frac{K}{L}$

The ratio $-\frac{w}{r}$ in this case is $-\frac{20}{5}$, or -4 .

So the condition that characterizes the cost-minimizing level of input utilization is $\frac{K}{L} = 4$,

or $K = 4L$. That is, for every hour of labor employed, L , the firm should utilize four hours of capital. This makes sense when you think about the fact that labor is four times as expensive as capital. Now, what are the specific amounts? To find them, we substitute our ratio into the production function set at one thousand barrels:

$$1,000 = 10L^{1/2}K^{1/2} \quad \dots (5)$$

If $K = 4L$, then $1,000 = 10L^{1/2}(4L)^{1/2}$, or $L = \frac{100}{\sqrt{4}}$, which equals fifty.

If $L = 50$, then $K = 200$. So using fifty hours of labor and two hundred hours of capital is the most cost-effective way to produce one thousand barrels of root beer for this firm.

Q1

Ex. Book No.

Pg. No.

CMA Mat

The demand (rides per day) of Roller Coaster Ride in an Entertainment Park in one of the metro cities is given by the equation $q = -450p + 41500$, where p = Price per ride in ₹.

What price should have been charged to maximize the Total Revenue?



Reference

**Single Independent Variable
(Differentiation)**

What's **New**

Maximise Total Revenue

Answer

Total Revenue is algebraically expressed as a function of price as follows

$R(p)$ = Price per ride $\times$ Demand Or, $R(p) = p \times q$ Or, $R(p) = p(-450p + 41500)$ Or, $R(p) = 41500p - 450p^2$

Differentiating both sides with respect to 'p' we get

$$\frac{d}{dp} [R(p)] = 41500 - 900p \quad \dots (i)$$

As per the necessary condition of optimization, $\frac{d}{dp} [R(p)] = 0$ Or, $41500 - 900p = 0$ Or, $p = 46.11$

To ascertain whether the value of p obtained corresponds to a maxima, we have to take help of sufficient condition written above.

Again differentiating both sides of (i) with respect to 'p' we get, $\frac{d^2}{dp^2} [R(p)] = -900 < 0$

So there exist a Maxima at $p = 46.11$

Thus the price to be charged to maximize the Total Revenue is ₹46.11

Q2

Ex. Book No.

Pg. No.

CMA Mat

Assume the Cost (₹) of manufacturing x numbers of a product per day is $C(x) = 14400 + 550x + 0.01x^2$. **How many** of the product should be manufactured per day so that the Average Cost is minimum? Also **find** the values of the Average Cost and the Total Cost at this level of production.



Reference

**Single Independent Variable
(Differentiation)**

What's **New**

Minimise Average Cost

Answer

1. Cost function is given to be $C(x) = 14400 + 550x + 0.01x^2$

So Average Cost function = $C(x) / x$ Or, $AC(x) = (14400 + 550x + 0.01x^2) / x$ Or, $AC(x) = 14400/x + 550 + 0.01x$

This is the Objective function which has to be minimized.

2. Differentiating both sides of the above function with respect to 'x' we get

$$\frac{d}{dx} [AC(x)] = -14,400/x^2 + 0.01 \dots (i)$$

As per the necessary condition of optimization, $\frac{d}{dx} [AC(x)] = 0$

$$\text{Or, } -14,400/x^2 + 0.01 \text{ Or, } 0.01x^2 = 14,400$$

$$\text{Or, } x^2 = 14,400 / 0.01 \text{ Or, } x = \sqrt{14,40,000} \text{ Or, } x = 1,200$$

But x being the quantity cannot be negative. Hence $x = 1,200$

3. To ascertain whether this value of x corresponds to minima, we have to take help of the sufficient condition mentioned above.

Again differentiating both sides of (i) with respect to 'x' we get, $\frac{d^2}{dx^2} [AC(x)] = 28800/x^3$

For $x = 1200$, the value of 2nd order Derivative is $\frac{d^2}{dx^2} [AC(1200)] = 28800/(1200)^3 = 1.67 \times 10^{-5} > 0$

So there exist a Minima to the Objective Function at $x = 1200$

4. Hence 1200 units should be produced per day to minimize the Average Cost.

At this level of production, Average Cost = $[AC(x)]_{\text{at } x=1200} = 14400/1200 + 550 + 0.01 \times 1200$
= ₹574 per unit

Also at this level of production, Total Cost = $[C(x)]_{\text{at } x=1200} = 14400 + 550 \times 1200 + 0.01 \times 1200^2 = ₹ 6,88,800/-$

Q3

Ex. Book No.

Pg. No.

CMA Mat

A firm has the Cost function $C = x^3/3 - 7x^2 + 111x + 50$ and Demand function $x = 100 - p$.
Determine the Equilibrium Output, Price and Profit earned.

Reference

Equilibrium of Firms (Single Independent Variable)

What's New

Equilibrium Output

Answer

1. Demand function is $x = 100 - p$ Or, $p = 100 - x$

So, Total Revenue = $TR = p \cdot x$ Or, $TR = (100 - x)x$ Or, $TR = 100x - x^2$

Also Profit = Total Revenue - Cost Or, $\pi = TR - C$ Or, $\pi = (100x - x^2) - (x^3/3 - 7x^2 + 111x + 50)$

$$\text{Or, } \pi = -x^3/3 + 6x^2 - 11x - 50$$

2. Differentiating both sides with respect to x we have $\frac{d}{dx}(\pi) = -x^2 + 12x - 11 \dots (1)$

As per the necessary condition of maximization we have $\frac{d}{dx}(\pi) = 0$ Or, $-x^2 + 12x - 11 = 0$
 Or, $(x - 1)(x - 11) = 0$

So the critical values are $x = 1$ and $x = 11$

3. Now differentiating both sides of (1) we have $\frac{d^2}{dx^2}(\pi) = -2x + 12$

When $x = 1$ then $\frac{d^2}{dx^2}(\pi) = -2.1 + 12 = 10 > 0$

So by the sufficient condition of 2nd Order Derivative test there is a minima at $x = 1$

When $x = 11$ then $\frac{d^2}{dx^2}(\pi) = -2.11 + 12 = -10 < 0$

So by the sufficient condition of 2nd Order Derivative test there is a maxima at $x = 11$

Thus Profit (π) is Maximum when $x = 11$ units. This is the required Equilibrium Output.

4. Equilibrium Price = $p_{\text{Equilibrium}} = [100 - x]_{\text{at } x=11} = 100 - 11 = ₹89$

Equilibrium Profit = $(\pi)_{\text{Max}} = [-x^3/3 + 6x^2 - 11x - 50]_{\text{at } x=11} = -(11)^3/3 + 6(11)^2 - 11.11 - 50 = ₹111.33$

[Note - The equilibrium output can be determined by using the relation $MR = MC$. Subsequently this value of output can be substituted in the Demand and Profit functions to obtain Equilibrium Price and Profit.]

Q4

Ex. Book No.

Pg. No.

CMA Mat

If the relation between Total Cost (y) and Output (x) is $y = 3x \left(\frac{x+7}{x+5} \right) + 5$, **prove** that the Marginal Cost falls continuously as the output increases.

Reference

Multiple Independent Variable (Partial Differentiation)

What's New

Slope of MC

Q5

Ex. Book No.

Pg. No.

CMA Mat

A company produces two products x and y. The total Profit (in ₹ '000) earned by the company is expressed algebraically by the function $P = 100x - x^2 - 2xy + 200y - 3y^2$. **Find** the Profit maximizing quantities of the products. Also find out the maximum Profit.

Reference

Multiple Independent Variables (Partial Differentiation)

What's New

Maximise Profit (Local Maxima)

Answer

- Profit function is given as :- $\Pi = 100x - x^2 - 2xy + 200y - 3y^2$
Differentiating the function partially with respect to x we get,
 $\Pi_x = 100 - 2x - 2y \quad \dots(I)$
Also differentiating the function partially with respect to y we get
 $\Pi_y = -2x + 200 - 6y \quad \dots(II)$
- To determine the Critical Point we have $\Pi_x = 0$ and $\Pi_y = 0$
So, $100 - 2x - 2y = 0$ Or, $x + y = 50 \quad \dots(1)$ and
 $-2x + 200 - 6y = 0$ Or, $x + 3y = 100 \quad \dots(2)$
(2) - (1) gives, $2y = 50$ Or, $y = 25$
Putting $y = 25$ in (1) we get $x = 25$
Thus Critical Point is (25, 25)
- To check whether this point is a local Maxima, we have to find out the values of the 2nd Order Partial Derivatives at this point.
Again differentiating (I) partially with respect to x we get Π_{xx} Or, $A = -2$ (Let) Or, $A < 0$
Similarly differentiating (II) partially with respect to y we get Π_{yy} Or, $C = -6$ (Let) Or, $C < 0$
Also differentiating (I) partially with respect to y we get $\Pi_{xy} = -2$ Or, $B = -2$ (Let)
So $D = AC - B^2 = (-2) \times (-6) - (-2)^2 = 8 > 0$
Hence $D > 0$ and $A, C < 0$
Thus there is a local Maxima at the already determined Critical Point (25, 25).
- Required Profit maximizing quantities of the products are $x = 25$ units and $y = 25$ units.
Also, Maximum Profit = Value of the function Π at $x = 25$ & $y = 25 = 100 \times 25 - 25^2 - 2 \times 25 \times 25 + 200 \times 25 - 3 \times 25^2$
 $= ₹ 3750$ (₹000)

Q 6

Ex. Book No.

Pg. No.

CMA Mat

Find the Critical Points for the function $f(x,y) = x^2y - 2xy^2 + 3xy + 4$. Examine the presence of Local Maxima and Minima among the Critical Points.



Reference

**Multiple Independent Variable (Partial
Differentiation)**

What's New

Maxima & Minima

Answer

1. For the Critical Points of $f(x,y) = x^2y - 2xy^2 + 3xy + 4$, at first we have to find out the 1st order Partial Derivatives.

Partial differentiation of the given function $f(x,y)$ with respect to x gives

$$f_x = 2xy - 2y^2 + 3y \quad \dots(1)$$

Partial differentiation of the given function $f(x,y)$ with respect to y gives

$$f_y = x^2 - 4xy + 3x \quad \dots(2)$$

2. As per the 1st Order Partial Derivative rule we have,

$$f_x = 0 \text{ Or, } 2xy - 2y^2 + 3y = 0 \text{ Or, } y(2x - 2y + 3) = 0 \quad \dots (3) \text{ \&}$$

$$f_y = 0 \text{ Or, } x^2 - 4xy + 3x = 0 \text{ Or, } x(x - 4y + 3) = 0 \quad \dots (4)$$

$$\text{From (3) \& (4) we get } x = 0, y = 0 \text{ and } 2x - 2y + 3 = 0 \quad \dots (5)$$

$$\text{as well as } x - 4y + 3 = 0 \quad \dots (6)$$

$$(5) - 2 \times (6) \text{ gives, } y = 1/2 \text{ and } x = -1$$

Again putting $x = 0$ in (5) we get $y = 3/2$ and putting $y = 0$ in (6) we get $x = -3$

So the **Critical Points** of the function are $(0,0)$, $(-3, 0)$, $(0, 3/2)$ and $(-1, 1/2)$

3. Now the 2nd Order Partial Derivatives are found out by differentiating (1) and (2) partially as follows –

Partially differentiating (1) w.r.t ' x ' gives $f_{xx} = 2y$ and partially differentiating w.r.t ' y ' gives $f_{xy} = 2x - 4y + 3$

Partially differentiating (2) w.r.t ' y ' gives $f_{yy} = -4x$

4. Let, $A = f_{xx}$, $B = f_{xy}$ and $C = f_{yy}$.

For the Critical Point $(0,0)$ we have $A = (2y) = 2.0 = 0$, $B = (2x - 4y + 3) = 2.0 - 4.0 + 3 = 3$ & $C = (-4x) = -4.0 = 0$

$D = AC - B^2 = 0.0 - 3^2 = -9 < 0$ So there is a Saddle Point at $(0,0)$

For the Critical Point $(-3, 0)$ we have $A = 0$, $B = 2.(-3) - 4.0 + 3 = -3$ & $C = -4.(-3) = 12$

$D = AC - B^2 = 0.(12) - (-3)^2 = -9 < 0$ So there is a Saddle Point at $(-3, 0)$

For the Critical Point $(0, 3/2)$ we have $A = 2.3/2 = 3 > 0$, $B = 2.0 - 4.3/2 + 3 = -3$ & $C = 0$

$D = AC - B^2 = 3.0 - (-3)^2 = -9 < 0$ So there is a Saddle Point at $(0, 3/2)$

For the Critical Point $(-1, 1/2)$ we have $A = 2.1/2 = 1 > 0$, $B = 2.(-1) - 4.1/2 + 3 = -1$ & $C = -4.(-1) = 4 > 0$

$D = AC - B^2 = 1.4 - (-1)^2 = 3 > 0$ Thus $D > 0$ as well as A & $C > 0$. So there is a local Minima at $(-1, 1/2)$.

Q7

Ex. Book No.

Pg. No.

CMA Mat

A manufacturer produces a liquid commodity at two different Plants located in the two regions of the country. The selling price (in ₹/litre) of the product is given by the equation $p = 200 - 0.8x$, where $x = x_1 + x_2$ = Total production of the two Plants together. The Cost functions of the 2 Plants are given as $C_1 = 0.3(x_1)^2 + 60x_1 + 5000$ and $C_2 = 0.5(x_2)^2 + 30x_2 + 8000$. **Determine** the quantities produced by the two Plants which will put the manufacturer into an equilibrium condition.

Reference

Equilibrium of Firms (Multiple Variable)

What's New

Multi plant Cost Monopolist Equilibrium

Answer

1. Total Revenue = $TR = p \cdot x$ Or, $TR = (200 - 0.8x)x$ Or, $TR = 200x - 0.8x^2$

Marginal Revenue (MR) = $d(TR)/dx = \frac{d}{dx} (200x - 0.8x^2) = 200 - 1.6x$... (1)

Marginal Cost for the 1st Plant (MC_1) = $dC_1 / \frac{d}{dx_1} = [0.3(x_1)^2 + 60x_1 + 5000] = 0.6x_1 + 60$... (2)

Marginal Cost for the 2nd Plant (MC_2) = $dC_2 / \frac{d}{dx_2} = [0.5(x_2)^2 + 30x_2 + 8000] = 1.0x_2 + 30$... (3)

2. As per the Necessary Condition of Equilibrium of a Multi Plant Firm we have $MR = MC_1 = MC_2$

So from (1) & (2) we get $200 - 1.6x = 0.6x_1 + 60$ Or, $200 - 1.6(x_1 + x_2) = 0.6x_1 + 60$ Or, $1.1x_1 + 0.8x_2 = 70$... (4)

Also from (2) & (3) we get, $0.6x_1 + 60 = 1.0x_2 + 30$ Or, $x_2 = 0.6x_1 + 30$... (5)

Substituting $x_2 = 0.6x_1 + 30$ from (5) in (4) we get, $1.1x_1 + 0.8(0.6x_1 + 30) = 70$ Or, $x_1 = 29.1$

Substituting $x_1 = 29.1$ in (5) we get, $x_2 = 47.5$

Now $\frac{\partial^2}{\partial x^2} (TR) = \frac{\partial}{\partial x} (MR) = (200 - 1.6x) = -1.6$

$$\text{Also } \frac{\partial^2}{\partial x_1^2}(TC_1) = \frac{\partial}{\partial x_1}(MC_1) = \frac{\partial}{\partial x_1}(0.6x_1 + 60) = 0.6$$

$$\text{Again } \frac{\partial^2}{\partial x_2^2}(TC_2) = \frac{\partial}{\partial x_2}(MC_2) = \frac{\partial}{\partial x_2}(1.0x_2 + 30) = 1.0$$

$$\text{Hence from above we get } \frac{\partial^2}{\partial x^2}(TR) < \frac{\partial}{\partial x_1^2}(TC_1) \text{ and } \frac{\partial}{\partial x^2}(TR) < \frac{\partial^2}{\partial x_2^2}(TC_2)$$

Thus the sufficient condition for Equilibrium is satisfied.

Required equilibrium outputs are **29.1 litres** and **47.5 litres** from the Plants 1 and 2 respectively.

Q 8

Ex. Book No.

Pg. No.

CMA Mat

A discriminating Monopolist is able to separate its customers into two markets with respective Demand functions as $x_1 = 16 - 0.2p_1$ and $x_2 = 9 - 0.05p_2$. Total Cost function is $TC = 20 + 20x$, where $x = x_1 + x_2$. **Determine** the Equilibrium Price of the product in the two markets. Also determine the Equilibrium Profit.

Reference

Equilibrium of Firms

What's New

Price discriminating Monopolist Equilibrium

Answer

$$1. \text{ Demand function of the 1st market is } x_1 = 16 - 0.2p_1 \text{ Or, } p_1 = 80 - 5x_1 \quad \dots (1)$$

$$\text{Also demand function of the 2nd market is } x_2 = 9 - 0.05p_2 \text{ Or, } p_2 = 180 - 20x_2 \quad \dots (2)$$

$$\text{From the 1st market, Revenue earned} = TR_1 = p_1x_1 = (80 - 5x_1).x = 80x_1 - 5(x_1)^2 \quad \dots (3)$$

$$\text{From the 2nd market, Revenue earned} = TR_2 = p_2x_2 = (180 - 20x_2).x = 180x_2 - 20(x_2)^2 \quad \dots (4)$$

$$2. \text{ So Marginal Revenue for the 1st case} = MR_1 = \frac{d}{dx_1}(TR_1) = \frac{d}{dx_1}[80x_1 - 5(x_1)^2] = 80 - 10x_1 \quad \dots (5)$$

$$\text{Also Marginal Revenue for the 2nd case} = MR_2 = \frac{d}{dx_2}(TR_2) = \frac{d}{dx_2}[180x_2 - 20(x_2)^2] = 180 - 40x_2 \quad \dots (6)$$

Total Cost Function is $TC = 20 + 20x$

$$\text{So Marginal Cost (MC)} = \frac{d}{dx}(TC) = \frac{d}{dx}(20 + 20x) = 20 \quad \dots (7)$$

3. As per the necessary condition of equilibrium of a Price Discriminating Monopolist Firm we have $MC = MR_1 = MR_2$

From (5) and (7) we have, $80 - 10x_1 = 20$ Or, $x_1 = 6$

From (6) and (7) we have, $180 - 40x_2 = 20$ Or, $x_2 = 4$

$$\frac{\partial^2}{\partial x_1^2}(TR_1) = \frac{\partial}{\partial x_1}(MR_1) = \frac{\partial}{\partial x_1}(80 - 10x_1) = -10$$

$$\frac{\partial^2}{\partial x_2^2}(TR_2) = \frac{\partial}{\partial x_2}(MR_2) = \frac{\partial}{\partial x_2}(180 - 40x_2) = -40$$

$$\left(\frac{\partial^2}{\partial x^2}(TC)\right) = \frac{\partial}{\partial x}(MC) = \frac{\partial}{\partial x}(20 + 20x) = 20$$

So from above we get $\frac{\partial^2}{\partial x_1^2}(TR_1) < \frac{\partial^2}{\partial x^2}(TC)$ and $\frac{\partial^2}{\partial x_2^2}(TR_2) < \frac{\partial^2}{\partial x^2}(TC)$

Thus the sufficient condition for equilibrium is satisfied.

4. Hence the equilibrium price for the 1st market = $p_1 = (80 - 5x_1)_{\text{At } x_1=6} = 80 - 5.6 = ₹ 50/-$

Also the equilibrium price for the 2nd market = $p_2 = (180 - 20x_2)_{\text{At } x_2=4} = 180 - 20.4 = ₹ 100/-$

Now Total Revenue of the Firm = $TR = TR_1 + TR_2 = 80x_1 - 5(x_1)^2 + 180x_2 - 20(x_2)^2$

Also Total Cost = $TC = 20 + 20x = 20 + 20(x_1 + x_2)$

Thus Profit Function is given as $TT = TR - TC$ Or, $TT = [80x_1 - 5(x_1)^2 + 180x_2 - 20(x_2)^2] - [20 + 20(x_1 + x_2)]$

Equilibrium Profit = $(TT)_{\text{Equilibrium}} = 80.6 - 5.6^2 + 180.4 - 20.4^2 - 20 - 20.(6 + 4) = ₹ 480$

Q9

Ex. Book No.

Pg. No.

CMA Mat

A manufacturing company selling two wheelers (x) and three wheelers (y) has the following Demand functions $p_x = 40 - 0.02x - 0.01y$ and $p_y = 80 - 0.06y - 0.01x$. Find the Revenue maximizing levels of output and price of the Two Wheelers as well as Three Wheelers. **What** is the maximum Total Revenue? The prices are in ₹ '000. **Ensure** the Total Revenue obtained is maximum by 2nd Order derivative test.

Reference

Multi Product Equilibrium

What's New

Maximise Revenue

Answer

Revenue maximising output of two and three wheelers are respectively 727 and 545 numbers and the corresponding prices are ₹20,000/- and ₹40,000/- Maximum Total Revenue = ₹36,340,000/-

Q10

Ex. Book No.

Pg. No.

CMA Mat

A manufacturer estimates his Annual Sales (in units) as a function of the expenditure made for Social Media and TV Advertising as $Z = 50000x + 40000y - 10x^2 - 20y^2 - 10xy$, where Z denotes Number of TVs sold per year and x & y denote Amounts spent on TV & Social Media Advertising in ₹ '000. **Determine** how much amounts should be spent on the two types of Advertising in order to maximize the number of TVs sold.

Reference

Multi Product EquilibriumWhat's **New****Maximise Revenue****Answer**

Amounts to be spent on TV and Social Media Advertising are respectively ₹ 22,85,720/- and ₹ 4,28,570/-

Q11

Ex. Book No.

Pg. No.

CMA Mat

A monopolist offers 2 products which have the demand functions given as $q_1 = 14 - p_1/4$ & $q_2 = 24 - p_2/2$. The monopolist's joint cost function is $C(q_1, q_2) = (q_1)^2 + 5q_1q_2 + (q_2)^2$. **Determine** the quantities to be sold in order to maximize the Total Profit.

Reference

Multi Product EquilibriumWhat's **New****Monopolist Maximise Profit****Answer**

The quantities to be sold are $q_1 = 2.75$ and $q_2 = 5.7$.

Q12

Ex. Book No.

Pg. No.

CMA Mat

Suppose a firm produces TV Sets at two different locations which produced x_1 and x_2 sets respectively. The joint cost function is given as $C = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$. If the firm has to supply 1000 sets of TV then **find** the number of sets to be produced in the two plants so that the joint cost is minimum.

Reference

Single Equality Constraint

What's New

Lagrange's Multiplier

Answer

- The firm has to supply 100 sets in total. So $x_1 + x_2 = 1000$

Thus the above equation has become a constraint under which the plants are to work. So the problem becomes Minimize $C = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$

Subject to $x_1 + x_2 = 1000$ Or, $x_1 + x_2 - 1000 = 0$ Or, $g(x_1, x_2) = 0$ [Let]

- Using Lagrange's Multiplier (λ) we can get the Lagrangian Function $L(x_1, x_2, \lambda) = C - \lambda \cdot g(x_1, x_2)$

Substituting $C = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000$ and $g(x_1, x_2) = x_1 + x_2 - 1000$ we rewrite the Lagrangian Function as $L(x_1, x_2, \lambda) = 0.1(x_1)^2 + 0.2(x_2)^2 + 0.2x_1x_2 + 180x_1 + 60x_2 + 25000 - \lambda(x_1 + x_2 - 1000)$

- Differentiating the function partially with respect to x_1 we get $= 0.2x_1 + 0.2x_2 + 180 - \lambda$

Differentiating the function partially with respect to x_2 we get $= 0.4x_2 + 0.2x_1 + 60 - \lambda$

Differentiating the function partially with respect to λ we get $= -(x_1 + x_2 - 1000)$

- As per the 1st Order conditions of Lagrangian Method we have

$$L_{x_1} = 0 \text{ Or, } 0.2x_1 + 0.2x_2 + 180 - \lambda = 0 \quad \dots (1)$$

$$L_{x_2} = 0 \text{ Or, } 0.4x_2 + 0.2x_1 + 60 - \lambda = 0 \quad \dots (2)$$

$$L_{\lambda} = 0 \text{ Or, } -(x_1 + x_2 - 1000) = 0 \text{ Or, } x_1 + x_2 = 1000 \quad \dots (3)$$

From (1) we have, $0.2(x_1 + x_2) + 180 - \lambda = 0$ Or, $0.2(1000) + 180 - \lambda = 0$ [Since from (3), $x_1 + x_2 = 1000$]

$$\text{Or, } \lambda = 380 \quad \dots (4)$$

From (2) we have, $0.2x_2 + 0.2(x_1 + x_2) + 60 - 380 = 0$ [Putting the value of λ from (4)]

$$\text{Or, } 0.2x_2 + 0.2(1000) - 320 = 0 \text{ [Since from (3), } x_1 + x_2 = 1000 \text{]}$$

$$\text{Or, } 0.2x_2 = 120 \text{ Or, } x_2 = 600 \text{ sets}$$

Putting $x_2 = 600$ in (3) we get $x_1 = 400$ sets

We have $g(x_1, x_2) = x_1 + x_2 - 1000$

Differentiating partially with respect to x_1 as well as x_2 we get $= 1$ and $= 1$

Again $= 0.2x_1 + 0.2x_2 + 180 - \lambda$ gives $= 0.2$ by partially differentiating with respect to x_1

Also $= 0.2x_1 + 0.2x_2 + 180 - \lambda$ gives $= 0.2$ by partially differentiating with respect to x_2

and $= 0.4x_2 + 0.2x_1 + 60 - \lambda$ gives $= 0.4$ by partially differentiating with respect to x_2

5. So the Bordered Hessian Matrix (HB) is given as $HB = \begin{bmatrix} 0 & g_{x1} & g_{x2} \\ g_{x1} & L_{x1x1} & L_{x1x2} \\ g_{x2} & L_{x1x2} & L_{x2x2} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0.2 & 0.2 \\ 1 & 0.2 & 0.4 \end{bmatrix}$

Value of the determinant corresponding to Matrix HB is obtained by expanding with respect the first row as below

$$\text{Det. HB} = 0 \cdot \begin{vmatrix} 0.2 & 0.2 \\ 0.2 & 0.4 \end{vmatrix} - 1 \cdot \begin{vmatrix} 1 & 0.2 \\ 1 & 0.4 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 0.2 \\ 1 & 0.4 \end{vmatrix} = 0 - 1 \cdot (1 \times 0.4 - 0.2 \times 1) + 1 \cdot (1 \times 0.2 - 0.2 \times 1) \\ = -0.2 < 0$$

As Det. HB < 0, there exist a minima corresponding to the critical values of $x_1 = 400$ sets & $x_2 = 600$ sets

6. Hence Cost minimisation can be possible by producing 400 TV Sets in 1st Plant & 600 Sets in 2nd Plant.

Q13

Ex. Book No.

Pg. No.

CMA Mat

The Production function of a Firm is given as $Q = f(K, L) = K^{0.5} L^{0.25}$ and the prices of Capital (K) and Labour (L) are respectively w and r. Derive Cost function of the Firm. Find the cost minimizing combination of Capital and Labour.

Reference

Cobb Douglas Production Function

What's New

Cost function

Answer

Cost function is – $C = rk + wl$ & Cost minimizing combination of Labour and Capital is given as

$$K = \sqrt[3]{(2w/r)Q^4} \text{ and } L = \sqrt[3]{(r^2/4w^2)Q^4}$$

Business Forecasting Models



Learning Objective

1. QUANTITATIVE METHODS & QUALITATIVE METHODS

2. REGRESSION ANALYSIS

- (a) Simple Linear Regression (SLR)
- (b) Multiple Linear Regression (MLR)

3. TIME SERIES ANALYSIS

(a) Components of Time Series data

- (i) Secular Trend or Trend (T)
- (ii) Seasonal Variation (S)
- (iii) Cyclical Fluctuation (C)
- (iv) Irregular or Random Movement (I)

(b) Measurement of Trend

- (i) Method of least square
- (ii) Moving Average method
- (iii) Exponential Smoothing Method

(c) Measurement of Seasonal Variation

- (i) Method of Simple Average
- (ii) Moving Average Method
- (iii) Trend Ratio Method
- (iv) Link Relative Method
- (v) Deseasonalisation

1. METHODS OF FORECASTING - QUALITATIVE METHODS

Input – Output Model

Under this model a forecast of output is based on given inputs if the coefficients of input – output relationship are known. Similarly, input requirements can be forecast on the basis of final output with a given input – output relationship. Due to this mechanics of forecasting the model is named as Input – Output Model.

Assumptions on which the model is established are as follows –

1. The economy consists of a number of interacting industries.
2. Each industry produce only one item and use only one process of production.
3. To produce an item, the industry requires as input the goods made by other industries, labour and perhaps imports. An industry may use some of its own goods. Such use may be taken as sale to itself.
4. The output of any industry becomes either the input to another industry or the final demand.
5. In any productive process all inputs are used in fixed proportions and increase in input is in proportion with the level of output. Production takes place through processes with constant technical coefficients. Technical Coefficient shows the number of units of any industry's output needed to produce one unit of another industry's output.
6. All transactions may be taken in terms of money values because it is a suitable common unit for aggregating inputs and outputs of industries.

Uniqueness of Input – Output Model lies in the fact that the same is the only forecasting technique which is based on inter industry flow of goods and services, given technical data on input usage by various industries. However the product quality, technology and industrial organisation undergo changes over time causing inter industry relationship to change, too. The degree of accuracy of the forecasts through this model, therefore depends on the extent to which the technical coefficients truly project the latest inter industry flow relationship.

Historical Analogy Method

Under this method forecast in regard to a particular phenomenon is based on some analogous conditions elsewhere in the past. For example, the forecast of demand for a product in India can be based on the demand for the same in some developed country in the past if it is found that the present conditions in India are very much like the same prevailing in that country in the past.

In fact such a method is more useful for indicating qualitative changes in society. It is said social analogies have helped in indicating the trends of changes in the norms of corporate behaviour in terms of attitude of the worker against inequality etc. find similarities in various countries at different stages of the history of industrial growth. But it is difficult to quantify most of these phenomena and therefore this method does not have much relevance for statistical analysis.

Jury or Executive Opinion Method

This method attempts to pool the knowledge, experience and judgement of managers inside the organisation, by asking about their opinion on the likely sales of a product in future. This is particularly applicable for a new product which does not have and past data to facilitate forecasting.

Survey Technique

First hand data on the behaviour of certain desired variables can be obtained through selective surveys by means of questionnaires and interviews. The data so collected can be processed and analysed for purposes of testing some predetermined hypotheses and making predictions and future estimates of the behaviour of the variables under study.

Barometric Technique

The behaviour of certain economic or business variables can have an important effect on some other variables. For example, a shortfall in supply of a commodity may lead to a rise in its price. Here the shortfall is an indicator or a barometer of the likely increase in price. Indicators can be of three types – Lead, Lag and Concurrent. Issuance of Industrial License is a Lead Indicator of future industrial activity. Similarly demand for Household Furniture in a housing complex is a Lag Indicator because it becomes active only after the construction of houses are completed i.e after a certain time lag. Concurrent Indicators move together i.e no lead or lag time is involved.

Delphi Technique

This technique is used to make more realistic judgemental forecasts by minimizing bias. In this method a panel of experts (including decision makers, staff personnel and respondents) is asked sequential questions. It is a step by step procedure and final forecast is obtained by the common opinion of all the experts.

2. IMPORTANT NOTES ON SIMPLE LINEAR REGRESSION

1. SLR deals with only two variables x and y of which x is conventionally considered as the Independent one, but on a number of occasions y is considered as Independent, too. Thus there can be two types of SLR models known as – Regression line y on x (where x is Independent) and Regression line x on y (where y is Independent).
2. General form of Regression line y on x is $y = a + bx$. Using statistical measures this equation is also written as $y - \bar{y} = b_{yx}(x - \bar{x})$ where $\bar{x}$ and $\bar{y}$ are Mean values of x & y and $b_{yx} =$ Regression Coefficient y on $x = r(\sigma_y / \sigma_x)$
 $r =$ Correlation Coefficient and σ_y & σ_x are S.Ds of y and x respectively
3. Similarly General form of Regression line x on y is $x = a + by$. This equation is also expressed as $x - \bar{x} = b_{xy}(y - \bar{y})$ where $\bar{x}$ and $\bar{y}$ are Mean values of x & y and $b_{xy} =$ Regression Coefficient x on $y = r(\sigma_x / \sigma_y)$
 $r =$ Correlation Coefficient and σ_y & σ_x are S.Ds of y and x respectively
4. Two Regression lines intersect at the point $(\bar{x}, \bar{y})$.

5. Sign of b_{yx} and b_{xy} should be same always – either both positive or both negative.
6. $b_{yx} \cdot b_{xy} = r^2$ Or, $r = \pm \sqrt{b_{yx} \cdot b_{xy}}$ [r will take the sign same as b_{yx} and b_{xy}]
7. Slope of Regression line y on x is b_{yx} and that of x on y is $1/b_{xy}$
8. Two Regression lines coincide if and only if there is a perfect correlation between x and y i.e $r = \pm 1$. In such situation slopes of the two lines will be equal i.e $b_{yx} = 1/b_{xy}$ Or, $b_{yx} \cdot b_{xy} = 1$
9. Two Regression lines will be perpendicular to each other if $b_{yx} \cdot 1/b_{xy} = -1$ Or, $b_{yx} = -b_{xy}$

[Correlation Coefficient = $r = \text{Cov}(x,y) / \sigma_x \cdot \sigma_y$ where $\text{Cov}(x,y) = \Sigma xy / N - (\Sigma x / N) \cdot (\Sigma y / N)$

$$\& \sigma_x = \sqrt{\left[\Sigma x^2 / N - (\Sigma x / N)^2 \right]}$$

SJC Institute

Q1

Ex. Book No.

Pg. No.

CMA Mat

There are two variables that need to be studied – Exports of raw cotton and Imports of manufactured goods into India. Following dataset for 7 years is provided. **What** kind of regression model should be used here? **What** are the results of this regression? Interpret the model estimators.

	₹ in Crores						
Exports	42	44	58	55	89	98	60
Imports	56	49	53	58	67	76	58

Reference	What's New
Simple Linear Regression	

Answer

As only two variables are involved, Simple Linear Regression or SLR model should be used.

Let the Regression model be $y = a + bx$ where x = Exports and y = Imports

Using the concept of Least Squares the values of a and b can be calculated. For that the Normal Equations are -

$\Sigma y = a.N + b. \Sigma x$ (where N = No. of pairs of observations) and $\Sigma xy = a.\Sigma x + b.\Sigma x^2$

Sl. No.	x in ₹ Crores	y in ₹ Crores	x^2	xy
1	42	56	1764	2352
2	44	49	1936	2156
3	58	53	3364	3074
4	55	58	3025	3190
5	89	67	7921	5963
6	98	76	9604	7448
7	60	58	3600	3480
Total	446	417	31214	27663

Putting the values of Σy , Σx and N in the 1st Normal Equation we get $417 = a.7 + b. 446 \dots (1)$

Similarly putting the values of Σxy , Σx and Σx^2 in the 2nd Normal Equation we get

$27663 = a.446 + b.31214 \dots (2)$

From (1) we get $a = (417 - 446b)/7$

Substituting the above expression for a in (2) we get $27663 = 446.(417 - 446b)/7 + 31214b$

Or, $27663 = 26568.86 - 28416.57b + 31214b$

Or, $1094.14 = 2797.43b$ Or, $b = 0.39$

Putting $b = 0.39$ in the expression for a we have $a = (417 - 446 \times 0.39)/7 = 34.72$

Substituting the values of a and b in the assumed model we get, $y = 34.72 + 0.39x$

Thus the given data can be fit to the SLR Model $y = 34.72 + 0.39x$

From this Estimators Slope and y intercept can be interpreted as follows –

Slope = $b = 0.39$ indicates the amount of change in y (Imports) per unit change in the value of x (Exports)

y intercept = value of y when $x = 0$ i.e 34.72 indicates that even for no Exports there will be Imports worth 34.72 Crores of Rupees.

Q2

Ex. Book No. Pg. No.

CMA Mat

While calculating the Coefficient of Correlation between two variables x and y , the following results were obtained: $N = 25$, $\Sigma x = 125$, $\Sigma y = 100$, $\Sigma x^2 = 650$, $\Sigma y^2 = 460$, $\Sigma xy = 508$. It was however discovered later on that two pairs of observation (x, y) were copied as $(6, 14)$ and $(8, 6)$ instead of $(8, 12)$ and $(6, 8)$ respectively. **Determine** the correct equations of the two Regression Lines.



Reference

Simple Linear Regression

What's New

Answer

$9x - 5y - 25 = 0$ and $4x - 5y = 0$

Q3

Ex. Book No. Pg. No.

CMA Mat

In order to find out the effect of Educational Qualification and Experience on the Earnings of the workers of a CNC Machine Shop, a study has been conducted on 10 workers and the dataset below is provided to you.

Worker	1	2	3	4	5	6	7	8	9	10
Education	11th Std	11th Std	12th Std	12th Std	1st Yr. Bach.	2nd Yr. Bach.	2nd Yr. Bach.	1st Yr. Masters	1st Yr. Masters	1st Yr. Masters
Experience (Years)	10	6	10	5	5	6	5	8	7	2
Salary (₹ per month)	30000	27000	20000	25000	29000	35000	38000	40000	45000	28000

Conduct a Multiple Regression Analysis considering the following numerical equivalents for Educational Qualification – 11th Std. = 11, 12th Std. = 12, 1st Yr Bachelors = 13, 2nd Yr. Bachelors = 14, 1st Yr. Masters = 16 Also **interpret** the meaning of the coefficients of the variables and the constant term.



Reference

Multiple Regression Analysis

What's New

Answer

Let the Multiple Regression Model be $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

For calculation of β_0 , β_1 and β_2 following formulae to be used.

$\beta_0 = \bar{y} - \beta_1 \bar{x}_1 - \beta_2 \bar{x}_2$ where $\bar{y}$ = Mean value of y , $\bar{x}_1$ = Mean value of x_1 and $\bar{x}_2$ = Mean value of x_2

$\beta_1 = [\Sigma(X_2)^2 \cdot \Sigma(X_1 y) - \Sigma(X_1 X_2) \cdot \Sigma(X_2 y)] / [\Sigma(X_1)^2 \cdot \Sigma(X_2)^2 - (\Sigma X_1 X_2)^2]$

$\beta_2 = [\Sigma(X_1)^2 \cdot \Sigma(X_2 y) - \Sigma(X_1 X_2) \cdot \Sigma(X_1 y)] / [\Sigma(X_1)^2 \cdot \Sigma(X_2)^2 - (\Sigma X_1 X_2)^2]$

where $\Sigma(X_1)^2 = \Sigma(x_1)^2 - (\Sigma x_1)^2/N$, $\Sigma(X_2)^2 = \Sigma(x_2)^2 - (\Sigma x_2)^2/N$

$\Sigma X_1 y = \Sigma x_1 y - (\Sigma x_1 \cdot \Sigma y)/N$, $\Sigma X_2 y = \Sigma x_2 y - (\Sigma x_2 \cdot \Sigma y)/N$ and $\Sigma X_1 X_2 = \Sigma x_1 x_2 - (\Sigma x_1 \cdot \Sigma x_2)/N$

N = Number of observation sets = 10 (In this case)

Worker	Salary (₹ y)	Education (x_1)	Experience (x_2 years)	(x_1) ²	(x_2) ²	$x_1 y$	$x_2 y$	$x_1 x_2$
1	30000	11	10	121	100	330000	300000	110
2	27000	11	6	121	36	297000	162000	66
3	20000	12	10	144	100	240000	200000	120
4	25000	12	5	144	25	300000	125000	60
5	29000	13	5	169	25	377000	145000	65
6	35000	14	6	196	36	490000	210000	84
7	38000	14	5	196	25	532000	190000	70
8	40000	16	8	256	64	640000	320000	128
9	45000	16	7	256	49	720000	315000	112
10	28000	16	2	256	4	448000	56000	32
Total	317000	135	64	1859	464	4374000	2023000	847

Putting the values of $\Sigma(x_1)^2$ and Σx_1 in the above mentioned formula we have

$$\Sigma(X_1)^2 = 1859 - (135)^2/10 = 36.5$$

Putting the values of $\Sigma(x_2)^2$ and Σx_2 in the above mentioned formula we have

$$\Sigma(X_2)^2 = 464 - (64)^2/10 = 54.4$$

Putting the values of $\Sigma(x_1y)$, Σx_1 & Σy in the above mentioned formula we have

$$\Sigma(X_1y) = 4374000 - (135)(317000)/10 = 94500$$

Putting the values of $\Sigma(x_2y)$, Σx_2 & Σy in the above mentioned formula we have

$$\Sigma(X_2y) = 2023000 - (64)(317000)/10 = -5800$$

Putting the values of $\Sigma(x_1x_2)$, Σx_1 & Σx_2 in the above mentioned formula we have

$$\Sigma(X_1X_2) = 847 - (135)(64)/10 = -17$$

Substituting the values calculated above for $\Sigma(X_2)^2$, $\Sigma(X_1y)$, $\Sigma(X_1X_2)$, $\Sigma(X_2y)$ and $\Sigma(X_1)^2$ in the formula of β_1 we get

$$\beta_1 = [54.5 \times 94500 - (-17) \times (-5800)] / [36.5 \times 54.4 - (-17)^2] = 2977.51$$

Substituting the values calculated above for $\Sigma(X_1)^2$, $\Sigma(X_1y)$, $\Sigma(X_1X_2)$, $\Sigma(X_2y)$ and $\Sigma(X_1)^2$ in the formula of β_2 we get

$$\beta_2 = [36.5 \times (-5800) - (-17) \times 94500] / [36.5 \times 54.4 - (-17)^2] = 822.11$$

Also we have $\bar{y} = \Sigma y / N = 317000/10 = 31700$, $\bar{x}_1 = \Sigma x_1 / N = 135/10 = 13.5$ & $\bar{x}_2 = \Sigma x_2 / N = 64/10 = 6.4$

$$\text{So } \beta_0 = \bar{y} - \beta_1\bar{x}_1 - \beta_2\bar{x}_2 = 31700 - 2977.51 \times 13.5 - 822.11 \times 6.4 = -13757.75$$

Required Multiple Regression Model is $y = -13757.75 + 2977.51x_1 + 822.11x_2$

Meaning of β_1 : 1 unit increase in Education (x_1) will cause increase in Salary (y) by ₹ 2977.51 when $x_2 = \text{Const.}$

Meaning of β_2 : 1 unit increase in Experience (x_2) will cause an increase in Salary (y) by ₹ 822.11 when Educational Qualification (x_1) is held constant.

Meaning of β_0 : When both the predictors Educational Qualification (x_1) and Experience (x_2) are zero then the value of y is (-13757.75)

Q4

Ex. Book No.

Pg. No.

CMA Mat

The CNC Machine Shop mentioned in the problem above is going to bid for few very critical machining items required for a Nuclear Power project. For that they have to recruit a dedicated programmer for the machines. Besides they need to procure two new CNC Vertical Machining centres which have to be operated by qualified operators. The profile they are looking for the programmer is Graduate Engineer with 5 years of experience and that for the operators of the new machining centres is 2nd year Bachelors' degree holder with 8 years of working experience.

If 1000 nos. of the critical items are to be supplied within a period of 3 years then **how much** extra the company should bid for each item to accommodate the recruitments of a programmer and two operators. Use the dataset of the previous illustration. [Assume educational qualification of a Graduate Engineer = 20 and that of an operator with 2nd year Bachelors' degree = 14]

Reference	What's New
Multiple Regression Analysis	

Answer

As the dataset of the previous illustration is to be used, the Regression model should be as find out there.

Therefore, $y = -13757.75 + 2977.51x_1 + 822.11x_2$ with $y = \text{Salary/month}$, $x_1 = \text{Education}$ & $x_2 = \text{Experience}$

Estimated monthly salary of the Graduate Engineer having $x_1 = 20$ and $x_2 = 5$ years is

$$y = -13757.75 + 2977.51 \times 20 + 822.11 \times 5 = ₹ 49903$$

Similarly estimated monthly salary of each of the Operator having $x_1 = 14$ and $x_2 = 8$ years is

$$y = -13757.75 + 2977.51 \times 14 + 822.11 \times 8 = ₹ 34504.27$$

Hence total estimated monthly outlay of the company for accommodating the programmer and 2 operators is $49903 + 2 \times 34504.27 = ₹ 118911.54$

As the supply has to be completed within 3 years, the total estimated outlay = $118911.54 \times 3 \times 12 = ₹ 4280815.44$

So for 1000 items, the company has to bid $4280815.44/1000 = ₹ 4280.81$ extra per piece to accommodate the new recruits.

Q5 Ex. Book No. Pg. No.

CMA Mat

In the following dataset consider y as the Response Variable and x_1 and x_2 as the Predictor Variables to find the Linear Regression model. **Interpret** the estimators of the model.

y	140	155	159	179	192	200	212	215
x₁	60	62	67	70	71	72	75	78
x₂	22	25	24	20	15	14	14	11

Reference	What's New
Multiple Regression Analysis	

Answer

$$y = -6.867 + 3.148 x_1 - 1.656 x_2$$

Q 6

Ex. Book No.

Pg. No.

CMA Mat

The following table **relates** to the tourist arrivals in India during 1990 to 1996.

Year	1990	1991	1992	1993	1994	1995	1996
Tourist arrivals (lakhs)	18	20	23	25	24	28	30

Fit a Straight Line trend by the Method of Least Squares and estimate the number of tourists that would arrive in the year 2000.

Reference	What's New
Straight line trend	Odd no. of Years

Answer

Let the best fit Trend line to the given data be $y = a + bx$ (Origin at the year 1993 and x unit = 1 year)

Normal equations are

$$\Sigma y = a.n + b.\Sigma x \quad \dots (1) \text{ and}$$

$$\Sigma xy = a.\Sigma x + b.\Sigma x^2 \quad \dots (2)$$

where n = No. of years = 7 (here)

Calculations for fitting Straight Line Trend

Year	Tourist arrivals (y in lakhs)	x	x ²	xy
1990	18	-3	9	-54
1991	20	-2	4	-40
1992	23	-1	1	-23
1993	25	0	0	0
1994	24	1	1	24
1995	28	2	4	56
1996	30	3	9	90
Total	168	0	28	53

Putting the values of Σy , Σx and n in equation (1) we get $168 = a.7 + b.0$ Or, $a = 24$

Also putting the values of Σxy , Σx and Σx^2 in equation (2) we get, $53 = a.0 + b.28$ Or, $b = 1.893$

So the required equation of Straight Line Trend is $y = 24 + 1.893x$ (Origin = 1993, x unit = 1 year)

For the year 2000, $x = 7$. So the estimated number of tourists in the year 2000 = $24 + 1.893 \times 7 = 37.25$ lakhs



Ex. Book No.

Pg. No.

CMA Mat

From the following past data of Sales (in lakhs Rupees) of a company **estimate** the same for the year 2005.

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Sales	15.3	14.6	16.8	17.3	17.2	20.9	22.3	20	23.1	24.5

Assume the trend line to be linear. **What** is the monthly rate of increase of Sales?

Reference	What's New
Straight line trend	Even no. of Years

Answer

Let the best fit Linear Trend line to the given data be $y = a + bx$

(Origin at the middle of the years 1994 & 1995 and x unit = 6 months)

Normal equations are $\Sigma y = a.n + b.\Sigma x$... (1) where n = No. of years = 10 (here)

$$\Sigma xy = a.\Sigma x + b.\Sigma x^2 \dots (2)$$

Using the values (from calculations below) of Σy , Σx and n in equation (1) we get

$$192 = a.10 + b.0$$

$$\text{Or, } a = 19.2$$

Also using the values (from calculations below) of Σxy , Σx and Σx^2 and putting in the equation (2) we get,

$$177 = a.0 + b.330$$

$$\text{Or, } b = 0.536$$

Calculations for fitting Straight Line Trend

Year	Sales (y in ₹ Millions)	x	x ²	xy
1990	15.3	-9	81	- 137.7
1991	14.6	-7	49	- 102.2
1992	16.8	-5	25	- 84
1993	17.3	-3	9	- 51.9
1994	17.2	-1	1	- 17.2
1995	20.9	1	1	20.9
1996	22.3	3	9	66.9
1997	20.0	5	25	100

1998	23.1	7	49	161.7
1999	24.5	9	81	220.5
Total	192	0	330	177

So the required equation of Straight Line Trend is $y = 19.2 + 0.536x$

(Origin = At the middle of 1994 & 1995, x unit = 6 months)

For the year 2005, $x = 21$. So the estimated sales for the year 2005 = $19.2 + 0.536 \times 21 = 30.456$ Million ₹.

Yearly rate of increase in Sales = $b = 0.536$. So monthly rate of increase in Sales = $b/12 = 0.0467$ Million ₹.

Q 8

Ex. Book No.

Pg. No.

CMA Mat

Find trend values of the following year wise data of Goods carried by a fleet of trucks of a Transport Company having pan India network using the Moving Average Method. [Assume a 4 yearly cycle]

Year	1975	1976	1977	1978
Goods carried (Tons)	2204	2500	2360	2680
Year	1979	1980	1981	1982
Goods carried (Tons)	2424	2634	2904	3098
Year	1983	1984	1985	1986
Goods carried (Tons)	3172	2952	3248	3172

Reference

Moving Average Method - Simple Average

What's New

4 Yearly

Answer

Calculations for 4 Yearly Moving Average Trend values

Year	Goods carried (Tons)	4 Yearly Moving Total	4 Yearly Moving Average (Not centred)	2 item Moving Total (Centred)	4 Yearly Moving Average (Centred)
(1)	(2)	(3)	(4) = (3) 4	(5)	(6) = (5) 2
1975	2204	-	-	-	-
1976	2500	-	-	-	-
		9744	2436		
1977	2360			4927	2463.50

		9964	2491		
1978	2680			5015.5	2507.75
		10098	2524.5		
1979	2424			5185	2592.50
		10642	2660.5		
1980	2634			5425.5	2712.75
		11060	2765		
1981	2904			5717	2858.50
		11808	2952		
1982	3098			5983.5	2991.75
		12126	3031.5		
1983	3172			6149	3074.5
		12470	3117.5		
1984	2952			6253.5	3126.75
		12544	3136		
1985	3248	-	-	-	-
1986	3172	-	-	-	-

Q9 Ex. Book No. Pg. No.

CMA Mat

For the following series of observations **verify** that the 4 yearly Centred Moving Average is equivalent to the 5 yearly Weighted Moving Average with weights 1, 2, 2, 2, 1 respectively.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Sales ('0000 ₹)	2	6	1	5	3	7	2	6	4	8	3

Reference
Moving Average Method - Weighted Average

What's New
4 Yearly

Q10 Ex. Book No. Pg. No.

CMA Mat

Calculate the Seasonal Indices for the following quarterly data in certain units. Appropriate method for finding the Indices has to be decided by you with due explanation.

Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
1992	39	21	52	81
1993	45	23	63	76

1994	44	26	69	75
1995	53	23	64	84

Reference Seasonal Indices - Simple Average Method	What's New Multiplicative

Answer

The values in any quarter do not reveal any definite tendency to change. Thus there is no appreciable trend in the given dataset. So it is decided to use Method of Simple Average (Quarterly) to find out the Seasonal Indices. Also a Multiplicative Model is assumed for the data.

Calculations for Seasonal Index

Year	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	Total
1992	39	21	52	81	-
1993	45	23	63	76	-
1994	44	26	69	75	-
1995	53	23	64	84	-
Total	181	93	248	316	838
Arithmetic Mean	45.25	23.25	62	79	209.5
Seasonal Index	86.4	44.4	118.4	150.8	400

Calculations

Arithmetic Mean for any Quarter = Total for that quarter / 4, Grand Average = Total of the Arithmetic Means / 4

Seasonal Index for any Quarter = (Arithmetic Mean of that Quarter / Grand Average) x 100

Q11

Ex. Book No.

Pg. No.

CMA Mat

Calculate Seasonal Fluctuation from the following Time Series data obtained from a Mini Steel Plant

Year	Quarterly Output of Steel in ₹000 Tons			
	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
2005	65	58	56	61
2006	68	63	63	67
2007	70	59	56	52
2008	60	55	51	58

Use 4 Quarter Moving Average Method. Consider Additive Model for the Time Series data.

Deseasonalise the given dataset.

Reference

Seasonal Indices - Moving Average Method & Deseasonalisation

What's New

Additive

Answer

Calculations for 4 Quarter Moving Averages & Deviations from Trend

Year	Quarter	Output in ₹000 Tons	4 Quarter Moving Total (Not centred)	2 Period Moving Total (Centred)	4 Quarter Moving Average (Centred)	Deviation from Trend
(1)	(2)	(3)	(4)	(5)	(6) = (5) ÷ 8	(7) = (6) – (3)
2005	1st	65	-	-	-	-
	2nd	58	-	-	-	-
			240			
	3rd	56		483	60.38	- 4.38
			243			
	4th	61		491	61.38	- 0.38
			248			
2006	1st	68		503	62.88	5.12
			255			
	2nd	63		516	64.50	- 1.50
			261			
	3rd	63		524	65.50	- 2.50
			263			
	4th	67		522	65.25	1.75
			259			
2007	1st	70		511	63.88	6.12
			252			
	2nd	59		489	61.12	- 2.12
			237			
	3rd	56		464	58.00	- 2.00
			227			
	4th	52		450	56.25	- 4.25
			223			
2008	1st	60		441	55.12	4.88
			218			

	2nd	55		442	55.25	- 0.25
			224			
	3rd	51	-	-	-	-
	4th	58	-	-	-	-

Calculations

1st entry of Column (4) = $65+58+56+61 = 240$,

2nd entry of Column (4) = $58+56+61+68 = 243$

3rd entry of Column (4) = $56+61+68+63 = 248$ and so on.

1st entry of Column (5) = $240+243 = 483$

2nd entry of Column (5) = $243+248 = 491$ and so on

Calculations for Seasonal Fluctuations

Year	Deviation from Trend for				Total
	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter	
2005	-	-	- 4.38	- 0.38	-
2006	5.12	- 1.50	- 2.50	1.75	-
2007	6.12	- 2.12	- 2.00	- 4.25	-
2008	4.88	- 0.25	-	-	-
Total	16.12	- 3.87	- 8.88	- 2.88	0.49
Arithmetic Mean	5.37	- 1.29	- 2.96	- 0.96	0.16
Adjustment	- 0.04	- 0.04	- 0.04	- 0.04	- 0.16
Seasonal Fluctuation	5.33	- 1.33	- 3.00	- 1.00	0

Calculations

Arithmetic Mean = (Quarterly total value of Deviation from trend) /3,

Grand Average = (Total of Arithmetic Mean values) /4 = $0.16/4 = 0.04$

Adjustment for each quarter = - (Grand Average)

Seasonal Fluctuation for any Quarter = Arithmetic Mean for that Quarter + Adjustment

[As explained in the theory, Adjustment has been done to get a total of zero value for the Seasonal Fluctuations]

Deseasonalising given Time Series data

Year	Quarter	Output (yt in ₹000 Tons)	Seasonal Fluctuation (S)	Deseasonalised Data ($y_t - S$)
2005	1st	65	5.33	59.67
	2nd	58	- 1.33	59.33
	3rd	56	- 3.00	59.00
	4th	61	- 1.00	62.00

2006	1st	68	5.33	62.67
	2nd	63	- 1.33	64.33
	3rd	63	- 3.00	66.00
	4th	67	- 1.00	68.00
2007	1st	70	5.33	64.67
	2nd	59	- 1.33	60.33
	3rd	56	- 3.00	59.00
	4th	52	- 1.00	53.00
2008	1st	60	5.33	54.67
	2nd	55	- 1.33	56.33
	3rd	51	- 3.00	54.00
	4th	58	- 1.00	59.00

[N.B – Deseasonalisation means elimination of Seasonal Variation from the Time Series data. For data in Additive Model it is done by subtracting Seasonal Variation from the original data. When Multiplicative Model is used then it is done by dividing the original data by Seasonal Effect which is Seasonal Index/100]

Q12 Ex. Book No. Pg. No.

CMA Mat

On the basis of quarterly Sales (₹ in Lakhs) of a certain commodity for the period 2001 to 2005 the following calculations were made.

Trend:- Straight Line trend equation is $y = 25 + 0.6t$, Origin – 1st Quarter of 2001, t unit – 1 Quarter, y – Sales

Seasonal Variations:

Quarter	I	II	III	IV
Seasonal Index	90	95	110	105

Estimate the Quarterly Sales figures for the year 2010

Reference Seasonal Indices	What's New Back Calculation

Answer

Given Trend equation is $y = 25 + 0.6t$ [Origin at the 1st Quarter of 2001, t unit = 1 Quarter]

For the 1st Quarter of the year 2010, the value of $t = 36$, For the 2nd Quarter of the same year, $t = 37$

For the 3rd Quarter of the same year, $t = 38$ and for the 4th Quarter, $t = 39$

Putting these values of t in the Trend equation we find –

Trend for the 1st Quarter of 2010 = $25 + 0.6 \times 36 = 46.6$,

Trend for the 2nd Quarter of 2010 = $25 + 0.6 \times 37 = 47.2$

Trend for the 3rd Quarter of 2010 = $25 + 0.6 \times 38 = 47.8$ &

Trend for the 4th Quarter of 2010 = $25 + 0.6 \times 39 = 48.4$

Quarter of the year 2010	Trend (T) in ₹ Lakhs	Seasonal Index	Seasonal Effect (S) = Seasonal Index/100	Estimated Sales in ₹ Lakhs (TxS)
I	46.6	90	0.90	41.94
II	47.2	95	0.95	44.84
III	47.8	110	1.10	52.58
IV	48.4	105	1.05	50.82

So the estimated Sales is calculated by multiplying Trend and Seasonal values. But for the cases where the data follows Additive Model, the estimated Value should be calculated by adding Trend and Seasonal Variation.

Q13 Ex. Book No. Pg. No.

CMA Mat

The number of quarterly traffic accidents in a Metro city during 2018-2020 are as below –

Year / Quarter	1st	2nd	3rd	4th
2018	165	135	140	180
2019	152	121	127	163
2020	140	100	105	158

Find the Seasonal Indices by Trend Ratio Method, assuming a linear trend for the data.

Reference Seasonal Indices Trend Ratio Method	What's New Additive

Answer

105 for 1st quarter, 83 for 2nd Quarter, 89 for 3rd Quarter and 123 for 4th Quarter.

Q14

Ex. Book No.

Pg. No.

CMA Mat

M/S B.P. Leathers, a shoe manufacturer has modern outlook and they depend heavily on Business Forecasting methodology to plan their business activities like manufacturing, marketing, finance etc. At the beginning of the year 2022 they have forecasted data of demand of their shoes for the beginning of the month of March as 1000 pairs. But the actual demand turned out to be 900 pairs. Using a Smoothing Coefficient of 0.1 **forecast** the demand at the beginning of the 2nd week of March 2022.

Also **forecast** the demands using Exponential Smoothing technique at the beginning of each week till mid April 2022 when the actual demands are as follows –

At the beginning of the 2nd week of March – 1010 pairs, At the beginning of the 3rd week of March – 1032 pairs, At the beginning of the 4th week of March – 976 pairs, At the beginning of the 1st week of April – 934 pairs, At the beginning of 2nd week of April – 1008 pairs & At the end of the 2nd week of April – 1020 pairs.

Reference

Exponential Smoothing

What's New

Answer

As per the concept of Exponential Smoothing we have $u_t = u_{t-1} + \alpha e_t$ where $e_t = y_t - u_{t-1}$ = Forecast Error & $\alpha = 0.1$

Calculations for Exponential Smoothing

Beginning of	Demand of shoe (y_t in Pairs)	Previous Forecast (u_{t-1})	Forecast Error ($e_t = y_t - u_{t-1}$)	Correction (αe_t) ($\alpha = 0.1$)	New Forecast ($u_t = u_{t-1} + \alpha e_t$)
March 1st week	900	1000	- 100	- 10	990
March 2nd week	1010	990	20	2	992
March 3rd week	1032	992	40	4	996
March 4th week	976	996	- 20	- 2	994
April 1st week	934	994	- 60	- 6	988
April 2nd week	1008	988	20	2	990
April 2nd week end or Mid April	1020	990	30	3.0	993

[Note – Except the 1st entry of 3rd column, all the other entries are taken from the last column].

Q15

Ex. Book No.

Pg. No.

CMA Mat

A company dealing in logistics business has a business wing named Ship Unloading. During the years 2020 and 2021 they have the following quarterly figures (in tonnage) of material unloaded from Ships.

Year	2020				2021			
Quarter	I	II	III	IV	I	II	III	IV
Material Unloaded	180	168	159	175	190	205	180	182

If the forecast for the first quarter of 2020 is 175 tons then **what** is the forecast figure for the first quarter of 2022? Use a Smoothing coefficient of 0.1



Reference

Exponential Smoothing

What's New

Answer

178.6 Tons

Introduction to Tools for Data Analytics

1. WHAT IS DATA ANALYTICS

'Data Analytics' is a broad term that encompasses many diverse types of Data Analysis. Any type of data or information can be subjected to the techniques of data analytics to get insight that can be used to improve things.

These techniques can reveal trends and metrics that would otherwise be lost in the mass of information. The output of data analytics can then be used to optimize processes to increase the overall efficiency of a system, including that of a business. Implementing it into business models means companies can help reduce costs by identifying more efficient ways of doing business and by storing large amounts of data. Actually this helps to make better business decisions by analysing customer trends and level of satisfaction which can lead to development of new and better products and services.

2. ROLE OF DATA ANALYTICS

Gather hidden insights – Hidden insights from data are gathered and then analysed with respect to the requirements of the business.

Generate reports – Reports are generated and passed on to the respective teams and individuals to deal with further actions for a high rise in business.

Perform Market Analysis – Market analysis is performed to understand the strengths and weaknesses of the competitors.

Improve business requirements – Analysis of data facilitates improvement in business by meeting the actual requirements of the customer.

3. STEPS IN ANALYTICS

Step 1

The first step is to determine the data requirements or how the data is grouped. Data may be separated by age, demography, income, gender etc. Data values may be numerical or be divided by category.

Step 2

The second step in Data Analytics is the process of collecting it. This can be done through a variety of sources – computers, on line sources, cameras, satellites or through personnel.

Step 3

Once the data is collected, it must be organized so that it can be analysed. This may take place on a Spread-sheet or other form of software that can take statistical data.

Step 4

The data is then cleaned up before analysis. This means it is scrubbed and checked to ensure there is no duplication or error and that it is not incomplete. This step helps correct any error before being analysed. This process is also known as Data Wrangling.

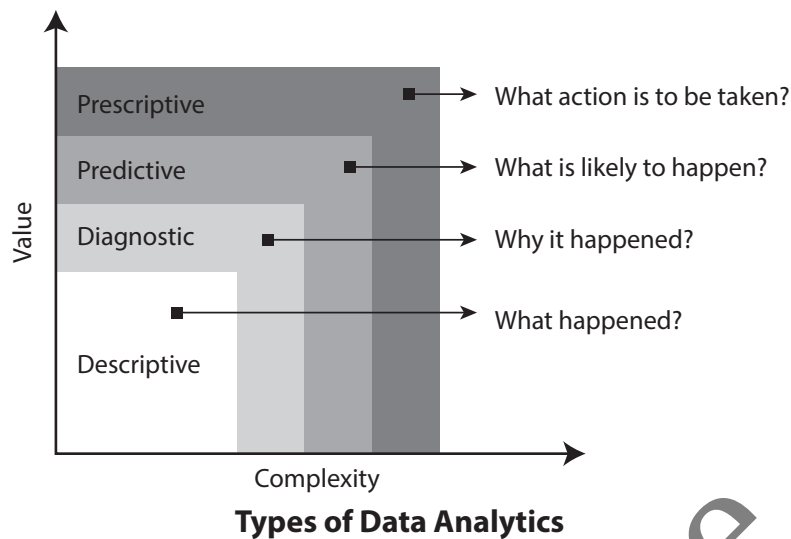
Step 5

Analyse data and generate report.

4. TYPES OF DATA ANALYTICS

There are four types of Data Analytics depending on the type of data available and the type of knowledge one is looking for from the available data. These are –

1. **Descriptive Analytics:** This is the most basic type of Analytics. It looks at data to examine, understand and describe something that has already happened. It provides quantitative information on “What happened” by analysing the data with the help of statistical techniques like calculation of Mean, Median, Mode etc. Daily report of Cost of Production of an item being produced continuously in an automatic machine is an example.
2. **Diagnostic Analytics:** It goes deeper than Descriptive Analytics by seeking to understand the “Why” behind what happened. While the production report may show an increase in the Cost of Production of the item, but the reason behind the same is not mentioned. Diagnostic Analytics will look into the patterns and deviations to find the root cause of increase in the cost. Diagnostic Analytics involve correlation of two different datasets.
3. **Predictive Analytics:** This relies on historical data, past trends and assumptions to answer questions about “What will happen” in future. This is of immense importance as far as the situations of future prediction (like Sales forecasting) are concerned. Predictive Analytics correlates the results of Descriptive and Diagnostic Analytics with external datasets.
4. **Prescriptive Analytics:** This identifies the specific actions an individual or organisation should take to reach future targets or goals. In other words it comes up with recommendations of “What actions should be taken”. In fact different possible scenarios are taken into account and the corresponding outcomes of each is considered of which the best possible one is recommended. Huge computing power is necessary for going through such type of analytics. This type of Analytics skill is observed in Artificial Intelligence (AI) and Machine Learning (ML).



Besides the above four, there can be other type known as –

Exploratory and Confirmatory Data Analytics

As the name implies, Exploratory Data Analytics explores the dataset to find answer as to which describes the data well. In this case no hypothesis is stated before analysing the data. It starts by framing questions about what to do with the data. Thereafter the data is manipulated to the best possible extent to find the answers to the questions. This involves understanding data structure, identifying erroneous or missing data, establishing a margin of error and figuring out the hypothesis.

Confirmatory Data Analytics deals with some stated hypothesis. Same is tested for the dataset using statistical tools and inference is drawn about its correctness. An example of hypothesis can be – “I predict that a person’s likelihood of recommending our product is directly proportional to their reported satisfaction with the product”.

Now if the trend analysis is carried out on the available data then from that the hypothesis is either proved or disproved. But the availability of the stated hypothesis gives a clear cut direction towards which the data analysis is done. In a way it is obviously advantageous compared to the Exploratory Data Analytics.

5. DATA ANALYTICS TOOLS

- (i) Data Transformation Tools
- (ii) Data Analytics Tools
 - (a) Spread-sheets
 - (b) Business Intelligence (BI) tools
 - (c) Financial data Analytics tools
 - (d) Programming Languages
 - (e) Tools for Statistical Data Analysis
 - (e) Industry Specific tools
- (iii) Visualisation Tools

Part II

Decision Making with Probability

SJC Institute

Decision Making using Probability



Learning Objective

1. EXPECTED VALUE USING PROBABILITY
2. PROBABILITY TO ACHIEVE BES OR TARGETED SALE
3. EXPECTED VALUE USING JOINT PROBABILITY

SJC Institute



Ex. Book No.

Pg. No.

CMA Mat

A company has a choice among three products A, B and C for which the following estimates are available:

Estimated profits based on demand forecast (₹ '000)

	Market X	Market Y	Market Z
Product A	380	100	30
Product B	300	280	220
Product C	220	400	320

Probabilities are: X = 0.60, Y = 0.20, Z = 0.20

Which project should be undertaken by the company?

Reference	What's New
Expected Value-profit	

Answer

In order to answer the question, it is desirable to take the help of a pay-off matrix which in turn demands the identification of the elements, e.g., profits, events (demand), probabilities, actions (products A, B or C), outcomes represented by Expected Values (EVs). (Profit × Probability)

		Profit (₹ '000)	Probability	Expected Value (₹ '000)
Product A	X	380	0.6	228
	Y	100	0.2	20
	Z	30	0.2	6
	Total			254
Product B	X	300	0.6	180
	Y	280	0.2	56
	Z	220	0.2	44
	Total			280
Product C	X	220	0.6	132
	Y	400	0.2	80
	Z	320	0.2	64
	Total			276

From the above matrix it is evident that Product B having the maximum EV of ₹ 2,80,000 should be selected.

Q 2

Ex. Book No.

Pg. No.

CMA Mat

You are given the following estimates for next year's budgeted sales and costs of a single product produced by Bee Ltd.:

Selling Price	₹ 12 per unit	
Sales Demand:	Units	Probability
	3200	0.50
	4000	0.30
	5000	0.20
Variable Cost per unit	(₹)	Probability
	5.00	0.30
	6.00	0.50
	7.00	0.20
Fixed Cost for the period		₹ 20,000

Required:

- Expected Value of sales for the period.
- Expected variable cost and contribution for the period.
- Expected profit or loss for the budgeted period.

Reference**Expected Value-Vales, Vc, Profit****What's New****Answer**

(1)	Expected Value of Sales:		(₹)
	Expected Sales × Profitability		
	$3,200 \times 0.5 = 1,600$		
	$4,000 \times 0.3 = 1,200$		
	$5,000 \times 0.2 = 1,000$	3,800 @ ₹12	45,600
(2)	Expected Variable Cost:		
	Unit Variable Costs × Probability		
	$₹ 5 \times 0.3 = 1.5$		
	$₹ 6 \times 0.5 = 3.0$		
	$₹ 7 \times 0.2 = 1.4$	5.9 × 3,800	22,420
(3)	Expected Contribution		23,180

(4)	Expected Profit:		
	Expected Contribution		23,180
	(-) Fixed Cost		20,000
	Expected Profit		₹ 3,180

Q3

Ex. Book No.

Pg. No.

CMA Mat

A company has estimated the following demand level of its product:

Sales Volume (units)	10000	12000	14000	16000	18000
Probability	0.10	0.15	0.25	0.30	0.20

It has assumed that the sales price of ₹ 6 per unit, marginal cost of ₹ 3.50 per unit, and fixed costs of ₹ 34,000.

What is the probability that:

- The company will break-even in the period?
- The company will make a profit of at least ₹ 10,000?



Reference

Probability of break-even, making a profit

What's New

Sales based

Answer

- Probability of Break-even for the period**

In order to break-even, the company must earn enough total contribution to cover its fixed costs. The contribution is ₹ 2.50 per unit (i.e., 6 - 3.5).

$$\begin{aligned}\text{Break-even Sales} &= (\text{Fixed Cost} \div \text{Contribution per Unit}) \\ &= (34,000 \div 2.50) = 13,600 \text{ units}\end{aligned}$$

$$\text{Contribution required/ Contribution per unit} = ₹ 34,000 / ₹ 2.50 = 13600 \text{ units}$$

The probability that sales will equal or exceed 13,600 units is the probability that sales will be 14,000, 16,000 or 18,000 units which is $(0.25 + 0.30 + 0.20) = 0.75$ or 75%.

- Probability of earning Profit of ₹10,000**

$$\begin{aligned}\text{Contribution Needed} &= (\text{Profit Needed} + \text{Fixed Cost}) \\ &= (10,000 + 34,000) = ₹ 44,000\end{aligned}$$

$$\begin{aligned}\text{Desired Sales} &= (\text{Contribution Needed} \div \text{Contribution per Unit}) \\ &= (44,000 \div 2.50) = 17,600 \text{ units}\end{aligned}$$

The probability that sales will equal or exceed 17,600 units is the probability that sales will be 18,000 units which is 0.20 or 20%

Q4

Ex. Book No.

Pg. No.

CMA Mat

A company has estimated the unit variable cost of a Product to be ₹ 10, and the selling price is ₹ 15 per unit. Budgeted sales for the year are 20,000 units. Estimated fixed costs are as follows:

Fixed Cost p.a. (₹)	50,000	60,000	70,000	80,000	90,000
Probability	0.1	0.3	0.3	0.2	0.1

What is the probability that the company will equal or exceed its target profit of ₹ 25,000 for the year?



Reference

Probability of profit

What's New

Fixed cost based
Answer

The different outcomes for fixed cost are mutually exclusive events. If fixed costs are ₹ 50,000 for example, they can't be anything else as well.

Budgeted sales = 20,000 units

Budgeted Contribution per unit = $15 - 10 = ₹ 5$

Budgeted total contribution (20,000 × ₹5)	1,00,000
Target profit	25,000
Maximum fixed costs if target is to be achieved	75,000

Higher Profit would mean lower FC, other things remaining constant.

So, Pb that Co. will equal or Exceed its target profit of ₹ 25,000 = Pb that FC will be ₹ 75,000 or less.

The probability that fixed costs will be ₹ 75,000 or less is:

= P (50,000 or 60,000 or 70,000)

= P (50,000) + P (60,000) + P (70,000)

= 0.1 + 0.3 + 0.3

= 0.7 or 70%

Q5

Ex. Book No.

Pg. No.

CMA Mat

The Managing Director of Y Ltd. has evolved some decision making to the operating division of the firm. He is anxious to extend this process but first wishes to be assured that decisions are being taken properly in accordance with group policy. As a check on existing practice, he has asked for an investigation to be made into a recent decision to increase the price of the sole product of Z division to ₹ 14.50 per unit but to rising costs. The following information and estimates were available for the management of Z division:

Last year 75,000 units were sold at ₹ 12 each with total units cost of ₹ 9 of which ₹ 6 were variable costs. For the year ahead the following cost and demand estimates have been made:

Variable costs:

Pessimistic	Probability 0.15	₹ 7.00 per unit
Most likely	Probability 0.65	₹ 6.50 per unit
Optimistic	Probability 0.20	₹ 6.20 per unit

Total fixed costs:

Pessimistic	Probability 0.3	Increase by 50%
Most likely	Probability 0.5	Increase by 25%
Optimistic	Probability 0.2	Increase by 10%

Demand estimates at various prices

Particulars	Probability	₹ 13.50 per unit	₹ 14.50 unit
Pessimistic	0.30	45,000	35,000
Most likely	0.50	60,000	55,000
Optimistic	0.20	70,000	68,000

(Unit variable costs, fixed costs and demand estimates are statistically independent)

For this type of decision the group has decided that the option should be chosen which has the highest expected outcome with at least an 80% chance of breaking even.

You are **required**:

- to assess **whether** the decision was made in accordance with group guidelines,
- to obtain **what** is the group attitude to risk as evidenced by the guidelines



Reference

Joint probability effect - break even and risk attitude

What's New

Answer

Situation	Demand	Proba- bility	Contribution per unit	Proba- bility	Total Contribution	Joint Probability	Cumulative Joint Probability
Selling Price ₹ 13.50							
Pessimistic	45,000	0.3	6.50	0.15	2,92,500	0.045	0.045
			7.00	0.65	3,15,000	0.195	0.240
			7.30	0.20	3,28,500	0.060	0.300
Most Likely	60,000	0.5	6.50	0.15	3,90,000	0.075	0.375
			7.00	0.65	4,20,000	0.325	0.700
			7.30	0.20	4,38,000	0.100	0.800
Optimistic	70,000	0.2	6.50	0.15	4,55,000	0.030	0.830
			7.00	0.65	4,90,000	0.130	0.960
			7.30	0.20	5,11,000	0.040	1.000
Selling Price ₹ 14.50							
Pessimistic	35,000	0.3	7.50	0.15	2,62,500	0.045	0.045
			8.00	0.65	2,80,000	0.195	0.240
			8.30	0.20	2,90,500	0.060	0.300
Most Likely	55,000	0.5	7.50	0.15	4,12,500	0.075	0.375
			8.00	0.65	4,40,000	0.325	0.700
			8.30	0.20	4,56,500	0.100	0.800
Optimistic	68,000	0.2	7.50	0.15	5,10,000	0.030	0.830
			8.00	0.65	5,44,000	0.130	0.960
			8.30	0.20	5,64,400	0.040	1.000

Last year's fixed costs = 75,000 units $\times$ ₹ 3 = ₹ 2,25,000

Estimated Fixed Costs	(₹)
₹ 2,25,000 $\times$ 1.10 $\times$ 0.2	49,500
₹ 2,25,000 $\times$ 1.25 $\times$ 0.5	1,40,625
₹ 2,25,000 $\times$ 1.50 $\times$ 0.3	1,01,250
	2,91,375

To break-even the contribution must be equal to or greater than ₹ 2,91,375. It is noticed from the above tables that at selling price of ₹ 13.50 there is 100% chance to break-even. However, at selling price of ₹ 14.50 there is 70% (0.075 + 0.325 + 0.1 + 0.03 + 0.13 + 0.04) chance of break-even. The selling price of ₹ 14.50, therefore, contravenes group guidelines.

Attitude to Risk: The group seeks to minimize the downside risk whilst maximizing its return. It is to some extent risk averse, but it is prepared to take some risk i.e., 20% risk of loss. It is always sought maximize its returns, ignoring the probability of failure, it would be risk neutral.

NOTES

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Part III Theories

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Kaizen Costing



Learning Objective

1. PROCESS OF KAIZEN COSTING
2. KAIZEN COSTING VS VALUE ENGINEERING
3. DIFFERENCE BETWEEN STANDARD COSTING AND KAIZEN COSTING

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1. MEANING OF KAIZEN COSTING

Introduction: The initial VE review may not be complete and perfect in all cost aspects. There may be further chances of waste reduction, cost and time reduction and product improvement. Such continuous cost reduction technique is called as Kaizen Costing.

The review of product costs under the target costing methodology is not reserved just for the period up to the completion of design work on a new product. There are always opportunities to control costs after the design phase is completed, though these opportunities are fewer than during the design phase.

Meaning: Kaizen Costing refers to the ongoing continuous improvement program that focuses on the reduction of waste in the production process, thereby further lowering costs below the initial targets specified during the design phase. It is a Japanese term for a number of cost reduction steps that can be used subsequent to issuing a new product design to the factory floor.

2. PROCESS OF KAIZEN COSTING

Activities in Kaizen Costing include elimination of waste in production, assembly, and distribution processes, as well as the elimination of unnecessary work steps in any of these areas. Thus Kaizen Costing is intended to repeat many of the value engineering steps, continuously and constantly refining the process, thereby eliminating out extra costs at each stage.

3. KAIZEN COSTING VS VALUE ENGINEERING:

Cost reductions resulting from Kaizen Costing are much smaller than those achieved with value engineering. But these are still significant since competitive pressures are likely to force down the price of a product over time, and any possible cost savings allow a Company to still attain its targeted profit margins.

4. DIFFERENCES BETWEEN STANDARD COSTING AND KAIZEN COSTING [DEC'18]

Basis of difference	Standard costing	Kaizen Costing
Concepts	It is used for cost control	It is used for cost reduction .
	It is assumed that current manufacturing conditions remain unchanged.	It assumes continuous improvement.
	The cost focus is on standard costs based on static conditions	The cost focus is on actual costs assuming dynamic conditions
	The aim is to meet cost performance standards	The aim is to achieve cost reduction targets.

Techniques	Standards are set every six or twelve months	Cost reduction targets are set and applied monthly
	Costs are controlled using variance analysis based on standard and actual costs .	Costs are reduced by implementing continuous improvement (kaizen) to attain the target profit or to reduce the gap between target and estimated profit.
	Management should investigate and respond when standards are not met .	Management should investigate and respond when target kaizen amount are not attained .
Employees	They are often viewed as the cause of problems	They are viewed as the source of, and are empowered to find, the solutions.

5. 'STANDARD COSTING IS USED FOR COST CONTROL WHEREAS KAIZEN COSTING IS USED FOR COST REDUCTION.' JUSTIFY THIS STATEMENT. [MTP JUN'22]

Under Standard costing, it is assumed that **current manufacturing conditions remain unchanged**. The cost focus is on standard costs based on **static conditions**. The aim is to meet cost performance standards, Costs are controlled using variance analysis based on standard and actual costs. whereas under Kaizen Costing, it assumes **continuous improvement**. The cost focus is on actual costs assuming dynamic conditions. The aim is to achieve cost reduction targets. Costs are reduced by implementing continuous improvement (kaizen) to attain the target profit or to reduce the gap between target and estimated profit.

Value Analysis & Value Engineering



Learning Objective

A. VALUE ANALYSIS

- (1) Value Analysis
- (2) Defining Cost and Value
- (3) Focus of Value Analysis
- (4) Types of Value Analysis Exercises
- (5) How to use Value Analysis

B. VALUE ENGINEERING

- (1) Value Engineering
- (2) Issues
- (3) Steps

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1. VALUE ANALYSIS

The **Value Analysis (VA)** technique was developed after the **Second World War in America at General Electric during the late 1940s**. Since this time the basic VA approach has evolved and been supplemented with new techniques that have become available and have been integrated with the formal VA process. Today, VA is enjoying a renewed popularity as competitive pressures are forcing companies to re-examine their product ranges in an attempt to offer higher levels of customization without incurring high cost penalties. In parallel, many major corporations are using the VA process with their suppliers to extend the benefits of the approach throughout the supply chain. Businesses, big and small, will therefore benefit from understanding and applying the VA process.

It is likely that those companies that do not take the time to develop this capability will face an uncertain future as the lessons and problems of the past are redesigned into the products of the future.

Definition of Value Analysis

Value Analysis (VA) or Value Engineering (VE) is a function-oriented, structured, multi-disciplinary team approach to solving problems or identifying improvements. The goal of any VA Study is to:

- **Improve value by** sustaining or improving performance attributes (of the project, product, and/or service being studied)
- while at the same time **reducing overall cost** (including life cycle operations and maintenance expenses).

Value Analysis can be defined as a process of systematic review that is applied to **existing product designs** in order to compare the function of the product required by a customer to meet their requirements **at the lowest cost** consistent with the specified performance and reliability needed.

This is a rather complicated definition and it is worth reducing the definition to key points and elements:

- (i) **Value Analysis (and Value Engineering)** is a **systematic, formal and organized process** of analysis and evaluation. It is not haphazard or informal and it is a management activity that requires planning, control and co-ordination.
- (ii) The analysis **concerns the function of a product to meet the demands or application needed by a customer**. To meet this functional requirement the review process must include an understanding of the purpose to which the product is used.
- (iii) Understanding the use of a product implies **that specifications can be established to assess the level of fit** between the product and the value derived by the customer or consumer.
- (iv) To succeed, the **formal management** process must meet these functional specification and performance criteria consistently in order to **give value to the customer**.
- (v) In order to **yield a benefit to the company**, the **formal review process** must result in a process of design improvements that serve to lower the production costs of that product whilst maintaining this **level of value through function**.

2. DEFINING COST AND VALUE

Any attempt to improve the value of a product must consider two elements, the first concerns the use of the product (known as **Use value**) and the second source of value comes from ownership (**Esteem value**). This can be shown as the difference between a luxury car and a basic small car that each has the same engine. From a use point of view both cars conduct the same function – they both offer safe economical travel (Use value) – but the luxury car has a greater esteem value. The difference between a gold-plated ball pen and a disposable pen is another example. However, **use value** and the **price** paid for a product are rarely the same, the difference is actually the esteem value, so even though the disposable pen is priced at X the use value may be far less.

It is important for all managers to understand the nature of costs in the factory and for any given product. While there is no direct relationship between 'Cost' (for the factory) and customer 'Value' in use and esteem, this education process is important. A shocking figure, that is often used as a general measure, is that typically 80% of the manufacturing costs of a product will be determined once the design drawing has been released for manufacturing.

The costs of production are therefore '**frozen**' and determined at this point. These costs include the materials used, the technology employed, the time required to manufacture the product and such like. Therefore, the design process creates many constraints for the business and fixes a high degree of the total product cost. It is therefore a process that **demands periodic review in order to recover any 'avoidable' costs that can be removed throughout the life of the product** (by correcting weaknesses or exploiting new processes, materials or methods) and lowering the costs of production whilst maintaining its Use value to the customer.

Basically, there are three key costs of a product:

- **Cost of the parts purchased:** These are costs associated with the supply of parts and materials.
- **Cost of direct labour used to convert products.**
- **Cost of factory overheads that recover the expenses of production.**

Although there are three elements of total cost accumulation it is traditionally the case that cost reduction activities have focused on the labour element of a product. Activities such as work-study, incentive payments and automation have compressed labour costs and as a result there is little to be gained, for most companies, in attempting to reduce this further. Instead, comparatively **greater gains and opportunities lie in the redesign and review of the products themselves to remove unnecessary materials and overhead costs**. This approach to the '**total costs**' of a product involves taking a much broader look at the way costs in the factory accumulate and the relationship between costs and value generation. **These new sources of costs and evaluations would therefore include such sources as:**

- Cost of manufacture
- Cost of assembly
- Cost of poor quality
- Cost of warranty

A detailed understanding of how costs are rapidly accumulated throughout the process of design to the despatch of the product is key to exploiting the process of VA. **All VA activities are aimed at the reduction of avoidable and unnecessary costs, without compromising customer value, and therefore the VA process should target the largest sources of potential cost reduction** rather than being an indiscriminate or unsystematic process (such as focusing on labour alone). It is therefore preferable to take the holistic approach to understanding costs and losses in the 'entire system' of design and conversion of value in order to determine how to achieve customer service 'functionality' at a minimal cost per unit.

3. THE FOCUS OF VALUE ANALYSIS

The key focus of the VA approach is therefore the **management of 'functionality'** to yield value for the customer. Let us emphasize this point a little. Not that long ago, consumers of electric kettles were offered a variety of different types of **metal-based boiling device**. The value of a kettle is derived through heating water and therefore its functionality can be determined (temperature, capacity, reliability, safety etc.). Now faced with the **same functionality** (to boil water), designers would probably look towards a **kettle made of plastic**.

Plastic has the same functionality as metal in terms of containing and boiling water. The action to boil water is conducted by the same part - known as the element. However the switch from metal to plastic does not impair this value and functionality with the customer – **they still want to boil water** - but it does result in a cost saving for the manufacturing company. If a company that traditionally made metal kettles did not review its design process then it would be severely disadvantaged when attempting to compete against the lower cost plastic alternative. This is a simple example used only to provide an illustration of the VA concept but it does demonstrate the point of maintaining value whilst reducing costs.

If a company seeks to reduce the costs of producing a product then it must **seek out costs that are unnecessary or items of the product that provide no functional value to the customer**. If you adopt this approach then the VA process is concerned with removing a specific type of cost. This cost is one that can be removed without negatively affecting the function, quality, reliability, maintainability or benefit required by the customer. As such, the target for all VA activities is to find these costs as opposed to simply re-engineering a product design with no real purpose to the re-engineering exercise. The VA approach is therefore formal and systematic because it is directed towards highlighting and dealing with these 'recoverable costs' of production. The objective is to **create value for money** as opposed to creating new products that do not provide customer satisfaction but are relatively **inexpensive**.

The rules governing the application of the VA approach are therefore simple:

- **No cost can be removed if it compromises the quality** of the product or its reliability, as this would lower customer value, create complaints and inevitably lead to the withdrawal of the product or lost sales.
- **Saleability** is another issue that cannot be compromised, as this is an aspect of the product that makes it attractive to the market and gives it appeal value.

- Any activity that **reduces the maintainability** of the product increases the cost of ownership to the customer and can lower the value attached to the product.

4. TYPES OF VALUE ANALYSIS EXERCISES

VA for Existing Products

One of the best approaches to VA is simply to select an existing product that is sold in relatively large volumes. This product, or product family, will tend to have a great deal of the basic information, and documented history, which can be used quickly as opposed to a newly introduced product where such a history is not available. An existing product unites all the different managers in a business, each with an opinion and list of complaints concerning the ability to convert the design into a 'saleable' product. Therefore any team that is created for the purpose of VA will understand their own problems but not necessarily the cause of these problems across the entire business. These opinions regarding poor performance (and documented evidence of failures) are vital to the discussions and understanding of how the product attracts costs as it is converted from a drawing to a finished product. These discussions therefore allow learning to take place and allow all managers to understand the limitations to the scope of product redesign and re-engineering activities. These **issues** include:

- The **inability to change existing product designs** due to the need to redesign tooling and the expense of such an initiative.
- The **project team may have a finite duration** before the project is concluded and therefore time will dictate what can be achieved.
- The high levels of purchased costs may imply a **need to engage with suppliers** in the VA process. This initiative will be constrained by a number of issues such as the timing of the project, the availability of resources from the supplier, the location of the suppliers, and other constraints.

VA for New Products – Value Engineering

For new products, the team will need to modify the VA approach and will operate in an environment that is less certain and has poor levels of available information upon which to make decisions. In this case, the analysis and systematic process of review for new products is known as Value Engineering (VE). The VE approach is similar to that of Value Analysis but **requires a much greater level of investment by the organization** in terms of the skilled, experienced and proficient human resources seconded to the group.

VA for Product Families- Horizontal Deployment

The final form of VA is results when there is scope for the 'horizontal deployment' of the results of a VA exercise **with a single product or family of products**. Under conditions where the value analysis project team finds **commonalties** with many products manufactured by the company, then it is possible to extend the benefits to all these other products concurrently. In this manner, all affected products can be changed quickly to bring major commercial benefits and to introduce the improvement on a '**factory-wide basis**'. This is particularly the case when supplying companies offer improvements that affect all the products to which their materials

or parts are used. The horizontal deployment activity has many advantages both in terms of financial savings and also the relatively short amount of time required to introduce the required changes to the product design.

Competitive VA

VA techniques are not simply the prerogative of the business that designed the product. Instead VA is often **used as a competitive weapon** and applied to the **analysis of competitor products in order to calculate the costs of other company's products**. This is often termed 'strip down' but is effectively **the reverse value analysis**. Here the VA team are applied to understanding the design and conversion costs of a competitor product. The results of the analysis is to understand how competitor products are made, what weaknesses exist, and at what costs of production together with an understanding of what innovations have been incorporated by the competitor company.

It is recommended that the best initial approach, for companies with no real experience of VA, is to **select a single product** that is currently in production **and has a long life ahead**. This approach offers the ability to gain experience, to learn as a team, and to test the tools and techniques with a product that has known characteristics and failings. In the short term it is most important to develop the skills of VA, including understanding the right questions to ask, and finally to develop a skeleton but formal process for all VA groups to follow and refine.

5. HOW TO USE VALUE ANALYSIS

Keys to Success

There are many keys to the success of a VA programme and it is wise to consider these issues before commencing the project, as errors in the project plan are difficult to correct, without causing frustration, once the VA project has started. One of the most important initial steps in developing the VA process is to **create a formal team** of individuals to conduct the exercise. These individuals must be drawn from different parts of the business that affect the costs associated with design, manufacturing, supply and other relevant functions. In addition, the team must be **focused on a product or product family in order to begin the exercise**.

Further key success factors include:

- **Gain approval of senior management** to conduct a Value Analysis exercise. Senior management support, endorsement and mandate for the VA project provides legitimacy and importance to the project within the business. This approval process also removes many of the obstacles that can prevent progress from being made by the team.
- **Enlist a senior manager as a champion** of the project to report back directly to the board of directors and also **to act as the programme leader**.
- Once a programme team has been developed it is important to **select an operational leader to co-ordinate the efforts, monitor progress** and to **support** the project champion. This leader will remain with the VA team throughout the life of the project and will be the central linking pin between the team and the senior management champion.

- **Establish the reporting procedure** for the team and the timing of the project. This project plan needs to be formal and displayed as a means of controlling and evaluating achievements against time.
- **Present the VA concept and objectives of the team to all the middle and senior managers** in the business. Widespread communication of the VA project is important so that other employees, particularly managers (who may not be involved directly with the process) understand the need to support the project either directly by assigning staff or indirectly through the provision of data.
- **Maintain a list of those business functions that should receive a regular communication** of progress even though they may not be directly involved with the project. This process allows other individuals in the business to be informed about the progress and findings of the group. This form of promotion is important as it maintains a momentum and communicates the findings of the team as widely as possible.
- **Provide an office space and co-locate the team members where practical and possible to do so.** The ability to locate a VA improvement group in one area of the business is important and assists the communication within the group. A convenient area can also be used to dismantle the product and also the walls of the area can be used to record, on paper charts, the issues that have been discovered by the team (and the associated actions that must be undertaken).
- **Select the product for the first study.** Ideally the existing product, or family of products, will be one that is established, sells in volume and has a relatively long life expectancy.

As such any improvement in the cost performance of the product will provide a large financial saving to the business.

- **Write down the objectives of the project** and the key project review points. Estimate the targets to be achieved by the project. These objectives provide a reference point and framework for the exercise. The objectives also focus attention on the outputs and achievements required by the company.
- **Select and inform any personnel** who will act in a part time or temporary role during the project. This process is used to schedule the availability of key specialist human resources to support the team throughout the duration of the project.
- **Train the team in both the process** of VA and also in basic team building activities. It is important that all members understand the nature of the project and its importance. The **initial team building exercises** are also a good way of understanding the attitude of all members to the project – especially those with reservations or a negative attitude to what can be achieved. As with most team exercises there is a requirement to allow the team to build and bond as a unit. It is often difficult for individuals, drawn from throughout the factory, to understand the language that is used throughout the business and also to understand the 'design to market' process when their own role impacts on a small section of this large and complex process.

6. VALUE ENGINEERING [MTP DEC'17; MTP DEC'19; MTP DEC'21; DEC'17; JUN'18]

Value Engineering is an organized/systematic approach directed at analyzing the function of systems, equipment, facilities, services, and supplies for the purpose of achieving their essential functions at the lowest life-cycle cost consistent with required performance, reliability, quality, and safety. Society of Japanese Value Engineering defines VE as:

“A systematic approach to analyzing functional requirements of products or services for the purposes of achieving the essential functions at the lowest total cost”.

Value Engineering is an effective **problem solving technique**. Value engineering is essentially a process which uses function analysis, team- work and creativity to improve value. Value Engineering is not just **“good engineering.”** It is not a suggestion program and **it is not routine** project or plan review. It is not typical cost reduction in that it doesn't “cheapen” the product or service, nor does it “cut corners.”

Value Engineering simply answers the question **“what else will accomplish the purpose of the product, service, or process we are studying?”**. VE technique is applicable to all type of sectors. Initially, VE technique was introduced in manufacturing industries. This technique is then expanded to all type of business or economic sector, which includes construction, service, government, agriculture, education and healthcare.

7. WHAT ARE THE ISSUES CONSIDERED DURING A VE REVIEW?

The Issues are as follows :

- Elimination of **unnecessary functions** from the production process.
- Elimination of **unnecessary product qualities**
- **Design minimisation**
- **Substitution of parts**
- **Search for better way of doing things.**

8. WHAT ARE THE STEPS TO BE FOLLOWED FOR DOING VALUE ENGINEERING?

The following are the steps to be used for carrying out the Value Engineering exercise :

- (i) Selection of the **Product Plan**.
- (ii) Gathering **Product Information**
- (iii) **Functional** Analysis
- (iv) **Creativity Phase** and **preparing the work-sheet**
- (v) **Evaluation Sheet**

- (vi) **Cost** Analysis
- (vii) **Result** and **Conclusion**
- (viii) **Implementation.**

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Business Process Reengineering



Learning Objective

1. CHARACTERISTICS AND PRINCIPLES OF RE-ENGINEERING PROCESS
2. SEVEN PRINCIPLES OF BPR
3. STEPS OF BUSINESS PROCESS RE-ENGINEERING SUGGESTED BY VAKOLA ET AL

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1. MEANING OF BUSINESS PROCESS RE-ENGINEERING [DEC'18; MTP JUN'17; MTP JUN'18; MTP DEC'19]

Business Process Re-engineering (BPR) refers to the **fundamental rethinking** and **redesign** of business processes to achieve **improvement in critical measures** of performance such as cost, **quality, service, speed** and **customer satisfaction**. In contrast the concept of Kaizen, which involves small, incremental steps towards gradual improvement, re-engineering involves a giant leap. It is the complete redesign of a process with an emphasis on finding creative new way to accomplish an objective. It has been described as taking a blank piece of paper and starting from scratch to redesign a business process. Rather than searching continually for minute improvement, re- engineering involves a **radical shift** in thinking about how an objective should be met. Re-engineering prescribes radical, quick and significant change. Admittedly, it can entail high risks, but it can also bring big rewards. These benefits are **most dramatic** when new models are discovered for conducting business.

2. CHARACTERISTICS AND PRINCIPLES OF RE-ENGINEERING PROCESS

- (i) Several jobs are combined into one
- (ii) Often workers make decisions
- (iii) The steps in the process are performed in a logical order
- (iv) Work is performed, where it makes most sense
- (v) Quality is built in.
- (vi) Manager provides a single point of contact
- (vii) Centralized and decentralized operations are combined.

3. SEVEN PRINCIPLES OF BPR [MTP JUN'19]

- (a) Processes should be designed to achieve a desired outcome rather than focusing on existing tasks.
- (b) Personnel who use the output from a process should perform the process
- (c) Information processing should be included in the work, which produces the information
- (d) Geographically dispersed resources should be treated, as if they are centralized
- (e) Parallel activities should be linked rather than integrated
- (f) Doers should be allowed to be self-managing
- (g) Information should be captured once at source.

4. EIGHT STEPS OF BUSINESS PROCESS RE-ENGINEERING SUGGESTED BY VAKOLA ET AL. (1998). [MTP JUN'19]

The eight proposed stages, as SUGGESTED BY Vakola et al. (1998) are as follows:

- Develop **Vision** and **Objectives**
- Understand **existing processes**
- **Identify** Process for **Re-design**
- Identify **Change Levers**
- Implement the new process
- Make new **process operational**
- **Evaluate** the new process
- **Monitor** ongoing continuous improvement.

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Lean Accounting



Learning Objective

1. PRINCIPLES, PRACTICES AND TOOLS OF LEAN ACCOUNTING
2. CONTRASTING LEAN ACCOUNTING AND TRADITIONAL STANDARD COSTING
3. PURPOSE OF PREPARATION OF "LEAN ACCOUNTING"

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1. DEFINE LEAN ACCOUNTING [DEC'17; MTP JUN'17; MTP JUN'18; MTP DEC'18; DEC'21]

What we now call **lean manufacturing** was developed by **Toyota** and other Japanese companies. Toyota executives claim that the famed Toyota Production System was inspired by what they learned during visits to the Ford Motor Company in the 1920s and developed by Toyota leaders such as Taiichi Ohno and consultant Shigeo Shingo after World War II. As pioneer American and European companies embraced lean manufacturing methods in the late 1980s, they discovered that lean thinking must be applied **to every aspect of the company including the financial and management accounting processes**.

Lean Accounting is the general term used **for the changes required to a company's accounting, control, measurement, and management processes to support lean manufacturing and lean thinking**. Most companies embarking on lean manufacturing soon find that their accounting processes and management methods are at odds with the lean changes they are making. The reason for this is that traditional accounting and management methods were designed to support traditional manufacturing; they are based upon mass production thinking. Lean manufacturing **breaks the rules of mass production**, and so the traditional accounting and management methods are (at best) unsuitable and usually actively hostile to the lean changes the company is making.

2. PRINCIPLES, PRACTICES AND TOOLS OF LEAN ACCOUNTING

Principles	Practices	Tools of lean accounting
A. Lean & simple business accounting	1. Continuously eliminate waste from the transactions processes, reports, and other accounting methods	(a) Value stream mapping; current & future state (b) Kaizen(leancontinuousimprovement) (c) PDCA problem solving
B. Accounting processes that support lean transformation	1. Management control & continuous improvement	(a) Performance Measurement Linkage Chart; linking metrics for cell/process, value streams, plant & corporate reporting to the business strategy, target costs, and lean improvement (b) Value stream performance boards containing break-through and continuous improvement projects (c) Box scores showing value stream performance
	2. Cost management	(a) Value stream costing (b) Value stream income statements

	3. Customer & supplier value and cost management	(a) Target costing
C. Clear & timely communication of information	1. Financial reporting	(a) "Plain English" financial statements (b) Simple, largely cash-based accounting
	2. Visual reporting of financial & nonfinancial performance measurements	(a) Primary reporting using visual performance boards; division, plant, value stream, cell/ process in production, product design, sales/ marketing, administration, etc.
	3. Decision-making	(a) Incremental cost & profitability analysis using value stream costing and box scores
D. Planning from a lean perspective	1. Planning & budgeting	(a) Hoshin policy deployment (b) Sales, operations, & financial planning (SOFP)
	2. Impact of lean improvement	(a) Value stream cost and capacity analysis (b) Current state & future state value stream maps (c) Box scores showing operational, financial, and capacity changes from lean improvement. Plan for financial benefit from the lean changes
	3. Capital planning	(a) Incremental impact of capital expenditure on value stream box-score. Often used with 3P approaches
	4. Invest in people	(a) Performance measurements tracking continuous improvement participation, employee satisfaction, & cross-training (b) Profit sharing
E. Strengthen internal accounting control	1. Internal control based on lean operational controls	(a) Transaction elimination matrix (b) Process maps showing controls and SOX risks

	2. Inventory valuation	(a) Simple methods to value inventory without the requirement for perpetual inventory records and product costs can be used when the inventory is low and under visual control.

3. CONTRASTING LEAN ACCOUNTING AND TRADITIONAL STANDARD COSTING

Lean Accounting	Standard Costing	Why is this important for Lean?
Quick, simple, and timely	Complex and wasteful processes	FLOW: Frequent and simple value stream income statements create much better control.
Clear and easy to understand	Difficult for people to understand	EMPOWERMENT: Information people can understand empowers those people for Lean improvement and growth.
Provides information for effective decisions.	Leads to bad decisions	VALUE: Readily understandable costs lead to better decisions and improved cash-flow and profits.
Supports value stream measurements and box scores	Supports measurements that undermine Lean endeavors	PURSUE PERFECTION: Measurements motivate people, Value Stream costs motivate lean improvement. Standard variance reports are anti-Lean.
Supports a value stream (total process) approach	Supports a departmental view of production	VALUE STREAMS: Focus on value streams lead to improved value, flow, and costs. Department focus opposes Lean.
Enables value stream financial control and improvement.	Narrows the focus of financial control and improvement.	PURSUE PERFECTION: Value Stream cost information creates better financial control and drives improvement.
Enables inventory valuation, easily	Enables inventory valuation, in a complex manner	FLOW: Standard costs were designed to value inventory. Lean accounting values inventory easier and better.
Enables value based pricing	Enables Cost+ Pricing	VALUE: Value streams focus on customer value leading to value pricing. Traditional companies use cost.

4. PURPOSE OF PREPARATION OF 'LEAN ACCOUNTING'

There are **positive and negative reasons for using Lean Accounting**. The **positive reasons** include the issues addressed in the **"Vision for Lean Accounting"**. Lean Accounting provides:

- (i) Provide **accurate, timely and understandable** information to motivate the lean transformation throughout the organization, to eliminate waste from accounting processes while maintaining thorough financial control,
- (ii) Fully comply with **Generally Accepted Accounting Principles (GAAP)**,
- (iii) Support the lean culture by motivating investment in people. Lean Accounting is nothing more than **tracking inflow to outflow as measure of profitability**, which is governed by GAAP.

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Decision Making Techniques

The cost of a product or process can be ascertained using different elements of cost using any of the following two techniques viz.,

1. Absorption Costing
2. Marginal Costing

1. ABSORPTION COSTING

Under this method, the cost of the product is determined after considering the total cost i.e., both fixed and variable costs. Thus this technique is also called **traditional or total costing**. The variable costs are directly charged to the products whereas the fixed costs are apportioned over different products on a suitable basis, manufactured during a period. Thus under absorption costing, all costs are identified with the manufactured products.

2. LIMITATIONS OF ABSORPTION COSTING

1. **Being dependent on levels of output** which vary from period to period, costs are vitiated due to the existence of fixed overhead. This renders them useless for purposes of comparison and control. (If, however, overhead recovery rate is based on normal capacity, this situation will not arise).
2. **Carryover of a portion of fixed costs**, i.e., period costs to subsequent accounting periods as part of the cost of inventory is a unsound practice because costs pertaining to a period should not be allowed to be vitiated by the inclusion of costs pertaining to the previous period.
3. **Profits and losses in the accounts** are related not only to sales but also to production, including the production which is unsold. This is contrary to the principle that profits are made not at the stage when products are manufactured but only when they are sold.
4. **There is no uniformity in the methods** of application of overhead in absorption costing. These problems have, no doubt, to be faced in the case of marginal costing also but to a less extent because of the exclusion of fixed costs, as different assumptions made in the matter of application of fixed overhead will not arise in the case of marginal costing.
5. Absorption costing is **not always suitable for decision making** solutions to various types of problems of management decision making, where the absorption cost method would be practically ineffective, such as selection of production volume and optimum capacity utilisation, selection of production mix, whether to buy or manufacture, choice

of alternatives and evaluation of performance can be had with the help of marginal cost analysis. Sometimes, the conclusion drawn from absorption cost data in this regard may be misleading and lead to losses.

3. MARGINAL COSTING

Marginal costing is **“the ascertainment of marginal costs and of the effect on profit of changes in volume or type of output by differentiating between fixed costs and variable costs.”** Several other terms in use like direct costing, contributory costing, variable costing, comparative costing, differential costing and incremental costing are used more or less synonymously with marginal costing.

It is a process whereby costs are classified into fixed and variable and with such a division so many managerial decisions are taken. The **essential feature of marginal costing is division of total costs into fixed and variable**, without which this could not have existed. Variable costs vary with volume of production or output, whereas fixed costs remains unchanged irrespective of changes in the volume of output. It is to be understood that **unit variable cost remains same at different levels of output and total variable cost changes in direct proportion with the number of units**. On the other hand, **total fixed cost remains same disregard of changes in units, while there is inverse relationship between the fixed cost per unit and the number of units**.

4. FEATURES OF MARGINAL COSTING

The main features of Marginal Costing may be summed up as follows:

1. Appropriate and accurate division of total cost into fixed and variable by picking out variable portion of semi variable costs also.
2. Valuation of stocks such as finished goods, work-in-progress is valued at variable cost only.
3. The fixed costs are written off soon after they are incurred and do not find place in product cost or inventories.
4. Prices are based on Marginal Cost and Marginal Contribution.
5. It combines the techniques of cost recording and cost reporting.

5. ADVANTAGES OR MERITS OR APPLICATIONS OF MARGINAL COSTING

1. Marginal costing system is **simple to operate than absorption costing because they do not involve the problems of overhead apportionment and recovery**.
2. Marginal costing **avoids, the difficulties of having to explain the purpose and basis of overhead absorption to management that accompany absorption costing**. Fluctua-

tions in profit are easier to explain because they result from cost volume interactions and not from changes in inventory valuation.

3. It is **easier to make decisions on** the basis of marginal cost presentations, e.g., marginal costing shows which products are making a contribution and which are failing to cover their avoidable (i.e., variable) costs. Under absorption costing the relevant information is difficult to gather, and there is the added danger that management may be misled by reliance on unit costs that contain an element of fixed cost.
4. Marginal costing is **essentially useful to management as a technique in cost analysis** and cost presentation. It enables the presentation of data in a manner useful to different levels of management for the purpose of controlling costs. Therefore, it is an important technique in cost control.
5. **Future profit planning** of the business enterprises can well be carried out by marginal costing. The contribution ratio and marginal cost ratios are very useful to ascertain the changes in selling price, variable cost etc. Thus, marginal costing is greatly helpful in profit planning.
6. When a business concern consists of several units and produces several products and evaluation of performance of such components can well be made with the help of marginal costing.
7. It is **helpful in forecasting**.
8. **When there are different products, the determination of number of units of each product, called Optimum Product Mix**, is made with the help of marginal costing.
9. Similarly, **optimum sales mix i.e., sales of each and every product to get maximum profit** can also be determined with the help of marginal costing.
10. Apart from the above, numerous managerial decisions can be taken with the help of **marginal costing, some of which, may be as follows:**
 - (a) Make or buy decisions,
 - (b) Exploring foreign markets,
 - (c) Accept an order or not,
 - (d) Determination of selling price in different conditions,
 - (e) Replace one product with some other product,
 - (f) Optimum utilisation of labour or machine hours,
 - (g) Evaluation of alternative choices,
 - (h) Subcontract some of the production processes or not,
 - (i) Expand the business or not,
 - (j) Diversification,
 - (k) Shutdown or continue,

6. LIMITATIONS OF MARGINAL COSTING

- (a) The **separation of costs** into fixed and variable presents technical difficulties and no variable cost is completely variable nor is a fixed cost completely fixed.
- (b) Under the marginal cost system, **stock of finished goods and work-in-progress are understated**. After all, fixed costs are incurred in order to manufacture products and as such, these should form a part of the cost of the products. It is, therefore, not correct to eliminate fixed costs from finished stock and work-in-progress.
- (c) **The exclusion of fixed overhead from the inventories affects the Profit and Loss Account and produces an unrealistic and conservative Balance Sheet**, unless adjustments are made in the financial accounts at the end of the period.
- (d) In marginal costing system, **marginal contribution and profits increase or decrease with changes in sales volume**. Where sales are seasonal, profits fluctuate from period to period. Monthly operating statements under the marginal costing system will not, therefore, be as realistic or useful as in absorption costing.
- (e) During the earlier stages of a period of recession, **the low profits or increase in losses, as revealed in a magnified way in the marginal costs statements**, may unduly create panic and compel the management to take action that may lead to further depression of the market.
- (f) Marginal costing **does not give full information**. For example, increased production and sales may be due to extensive use of existing equipments (by working overtime or in shifts), or by an expansion of the resources, or by the replacement of labour force by machines. The marginal contribution fails to reveal these.
- (g) Though for **short-term assessment of profitability marginal costs may be useful, long term profit is correctly determined on full costs basis only**.
- (h) Although marginal costing eliminates the difficulties involved in the apportionment and under and over-absorption of fixed overhead, the **problem still remains so far as the variable overhead is concerned**.
- (i) With **increased automation and technological developments, the impact on fixed costs on products is much more than that of variable costs**. A system which ignores fixed costs is therefore, less effective because a major portion of the cost, such as not taken care of.
- (j) Marginal costing **does not provide any standard for the evaluation of performance**. A system of budgetary control and standard costing provides more effective control than that obtained by marginal costing.

7. DIFFERENCES BETWEEN ABSORPTION COSTING AND MARGINAL COSTING

	Absorption Costing	Marginal Costing
1.	Both fixed and variable costs are considered for product costing and inventory valuation.	Only variable costs are considered for product costing and inventory valuation.
2.	Fixed costs are charged to the cost of production. Each product bears a reasonable share of fixed cost and thus the profitability of a product is influenced by the apportionment of fixed costs.	Fixed costs are regarded as period costs. The profitability of different products is judged by their P/V ratio.
3.	Cost data are presented in conventional pattern. Net profit of each product is determined after subtracting fixed cost along with their variable cost.	Cost data are presented to highlight the total contribution of each product.
4.	The difference in the magnitude of opening stock and closing stock affects the unit cost of production due to the impact of related fixed cost.	The difference in the magnitude of opening stock and closing stock does not affect the unit cost of production.
5.	In case of absorption costing the cost per unit reduces, as the production increases as it is fixed cost which reduces, whereas, the variable cost remains the same per unit.	In case of marginal costing the cost per unit remains the same, irrespective of the production as it is valued at variable cost.

8. DIFFERENCE IN PROFIT UNDER MARGINAL AND ABSORPTION COSTING

- (i) **No opening and closing stock:** In this case, profit/loss under absorption and marginal costing will be equal.
- (ii) **When opening stock is equal to closing stock:** In this case, profit/loss under two approaches will be equal provided the fixed cost element in both the stocks is same amount.
- (iii) **When closing stock is more than opening stock:** In other words, when production during a period is more than sales, then profit as per absorption approach will be more than that by marginal approach. The reason behind this difference is that a part of fixed overhead included in closing stock value is carried forward to next accounting period.
- (iv) **When opening stock is more than the closing stock:** In other words when production is less than the sales, profit shown by marginal costing will be more than that shown by absorption costing. This is because a part of fixed cost from the preceding period is added to the current year's cost of goods sold in the form of opening stock.

9. DIFFERENTIAL COST ANALYSIS [JUN'19]

Differential Cost is the change in the costs which results from the adoption of an alternative course of action. The alternative actions may arise due to change in sales volume, price, product mix (by increasing, reducing or stopping the production of certain items), or methods of production, sales, or sales promotion, or they may be due to '**make or buy**' or '**take or refuse**' decisions. When the change in costs occurs due to change in the activity from one level to another, differential cost is referred to as incremental cost or decremental cost, if a decrease in output is being considered, i.e. total increase in cost divided by the total increase in output. However, accountants generally do not distinguish between differential cost and incremental cost and the two terms are used to mean one and the same thing.

The essential features of differential costs are as follows:

1. The basis data used for differential cost analysis are costs, revenue and the investment factors which are relevant in the problem for which the analysis is undertaken.
2. Total differential costs rather than the costs per unit are considered.
3. Differential cost analysis is made outside the accounting records.
4. As the differences in the costs at two levels are considered, absolute costs at each level are not as relevant as the difference between the two. Thus, items of costs which do not change but are identical for the alternatives under consideration, are ignored.
5. The differentials are measured from a common base point or position.
6. The stage at which the difference between the revenue and the cost is the highest, measured from the common base point, determines the choice from amongst a number of alternative actions.
7. In computing differential costs, historical or standard costs may be used but they should be adjusted to the requirements of future conditions.
8. The elements and items of cost to be considered in differential cost analysis will depend upon the nature of the problem and the alternatives being considered.

10. DIFFERENTIAL COSTS ANALYSIS AND MARGINAL COSTING

Although the techniques of **differential costs analysis are similar to those of marginal costing, the two should not be confused**. The points of similarity and difference between differential costs analysis and marginal costing are summarized below:

Similarity:

- (a) Both the techniques of cost analysis and cost presentation.
- (b) Both are made use of by the management in decision making and in formulating policies.
- (c) The concepts of differential costs and marginal costs mainly arise out of the difference in the behaviour of fixed and variable costs.

- (d) Differential costs compare favourably with the economist's definition of marginal cost, viz. that marginal cost is the amount which at any given volume of output is changed if output is increased or decreased by one unit.

Difference:

- (a) Differential cost analysis can be made in the case of both absorption costing as well as marginal costing.
- (b) While marginal costing excludes the entire fixed costs, some of the fixed costs may be taken into account as being relevant for the purpose of differential cost analysis.
- (c) Marginal costs may be embodied in the accounting system whereas differential costs are worked out separately as analysis statements.
- (d) In marginal costing, margin of contribution and contribution ratio are the main yardsticks for performance evaluation and for decision making. In differential cost analysis, differential costs are compared with the incremental or decremental revenues, as the case may be.

11. PRACTICAL APPLICATION OF DIFFERENTIAL COSTS

They are useful in managerial decisions, which are enumerated below:

- (i) Determination of **most profitable levels** of production and price.
- (ii) **Acceptance of offer** at a lower price or offering a quotation at lower selling price in order to increase capacity.
- (iii) It is used to decide whether it will be more profitable to **sell a product as it is or to process it further** into a different product to be sold at an increased price.
- (iv) Determining the suitable price at which **raw material** may be **purchased**.
- (v) Decision of **adding a new product** or business segment.
- (vi) **Discontinuing** a product or business segment in order to avoid or reduce the present loss or increase profit.
- (vii) Changing the **product mix**.
- (viii) **Make or buy** decisions.
- (ix) Decision regarding **alternative capital investment** and plant **replacement**.
- (x) Decision regarding **change in method of production**.

12. TOOLS AND TECHNIQUES OF MARGINAL COSTING

1. Contribution:

In common parlance, contribution is the reward for the efforts of the entrepreneur or owner of a business concern. From this, one can get in his mind that contribution means profit. But it is not so. Technically or in Costing terminology, contribution means not only

profit but also fixed cost. That is why; it is defined as the amount recovered towards fixed cost and profit.

Contribution is helpful in determination of profitability of the products and/or priorities for profitabilities of the products. When there are two or more products, the product having more contribution is more profitable.

For example: The following are the three products with selling price and cost details:

	Amount (₹)		
Particulars	A	B	C
Selling price	100	150	200
Variable cost	50	70	100
Contribution	50	80	100

In the above example, one can say that the product 'C' is more profitable because, it has more contribution. This proposition of product having more contribution is more profitable is valid, as long as, there are no limitations on any factor of production. In this context, factors of production means, the factors that are responsible for producing the products such as material, labour, machine hours, demand for sales etc..

2. Limiting Factor (or) Key Factor:

In the above example, we find that product having more contribution is more profitable. However, when there is a limitation on any input factor, the profitability of the product cannot simply be determined by finding out the contribution of the unit, but it can be found out by ascertaining the contribution per unit of that factor of production which is limited in the given situation. Such factor of production which is limited in the question is called key factor or limiting factor.

Continuing the above example, it may be explained as follows:

The three products take same raw material. A takes 1 kg, B requires 2 kgs, C requires 5 kgs and the raw material is not abundant.

Then profitability of the above products is determined as follows:

$$\text{Profitability} = \left(\frac{\text{Contribution}}{\text{Key Factor}} \right)$$

A	B	C
50 / 1 = ₹ 50	80 / 2 = ₹ 40	100 / 5 = ₹ 20

Now, product A is more profitable because it has more contribution per kg of material.

The key factor can also be called as scarce factor or Governing factor or Limiting factor or Constraining factor etc., whatever may be the name, it indicates the limitation on the particular factor of production.

From the above, it is essentially understandable that contribution is helpful in determination of profitability of the products, priorities for profitability of the products and in particular, profitabilities when there are limitation on any factor.

2. Profit Volume Ratio (P/V Ratio) or Contribution Ratio:

First of all, a ratio is a statistical or mathematical tool with the help of which a relationship can be established between the variables of the same kind. Further, it may be expressed in different forms such as fractional form, quotient, percentage, decimal form, and proportional form.

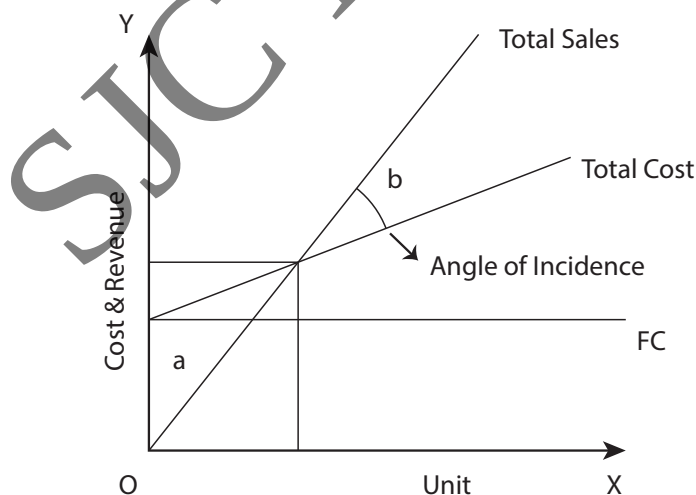
For example:

Gross profit ratio: It may be expressed as follows:

- Gross profit is $\frac{1}{4}$ th of sales
- Sales is 4 times that of gross profit
- Gross profit ratio is 25%
- Gross profit is 0.25 of sales and lastly
- Gross profit and sales are in the ratio of 1: 4

3. Break Even Point:

When someone asks a layman about his business he may reply that it is alright. But a technical man may reply that it is break even. So, **Break Even** means the volume of production or sales where there is no profit or loss. In other words, **Break Even Point** is the volume of production or sales where total costs are equal to revenue. It helps in finding out the relationship of costs and revenues to output. In understanding the breakeven point, cost, volume and profit are always used. The break even analysis is used to answer many questions of the management in day to day business. The formal break even chart is as follows:



a = Losses b = Profits

When no. of units are expressed on X-axis and costs and revenues are expressed on Y-axis, three lines are drawn i.e., fixed cost line, total cost line and total sales line. In the above graph we find there is an intersection point of the total sales line and total cost line and from that intersection point if a perpendicular is drawn to X-axis, we find break even units. Similarly, from the same intersection point a parallel line is drawn to X-axis so that it cuts

Y-axis, where we find Break Even point in terms of value. This is how, the formal pictorial representation of the Break Even chart.

At the intersection point of the total cost line and total sales line, an angle is formed called Angle of Incidence, which is explained as follows:

4. **Angle of Incidence:**

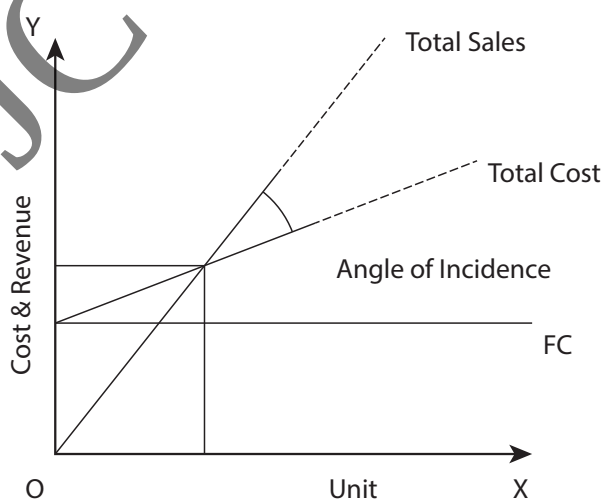
Angle of Incidence is an angle formed at the intersection point of total sales line and total cost line in a formal break even chart. If the angle is larger, the rate of growth of profit is higher and if the angle is lower, the rate of growth of profit is lower. So, growth of profit or profitability rate is depicted by Angle of Incidence.

5. **Break Even Analysis (or) Cost-Volume-Profit Analysis (CVP analysis):**

From the breakeven charts breakeven point and profits at a glance can be found out. Besides, management makes profit planning with the help of breakeven charts. It can clearly be understood by way of charts to know the changes in profit due to changes in costs and output. Such profit planning is made with the variables mainly cost, profit and volume, such an analysis is called **breakeven analysis**. Throughout the charts relationship is established among the cost, volume and profit, it is also called **Cost-Volume-Profit Analysis (CVP analysis)**. That is why it is popularly said by S.C.Kuchal in his book "**Financial Management - An Analytical and Conceptual Approach**", that Cost-volume-profit analysis, break even analysis and profit graphs are interchangeable words. The analysis is further explained as follows:

The change in profit can be studied through Break even charts in different situations in the following manner:

(i) **Increase in No. of Units**

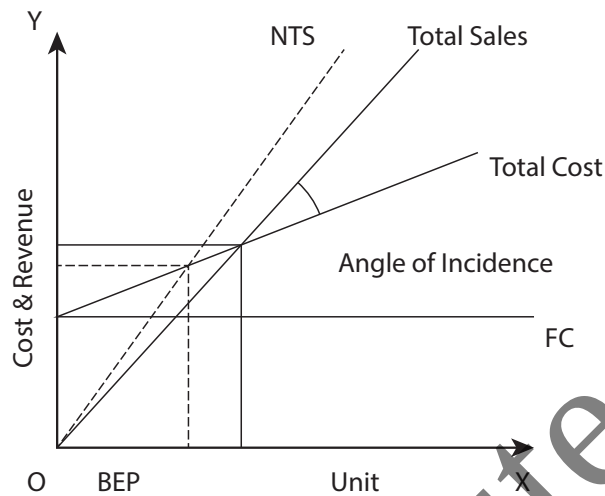


'.....' line indicates increase in total cost and total sales.

In the above chart, if we clearly observe we find that there is no change in BEP even if there is increase or decrease in No. of units.

(ii) **Increase in Sales due to increase in selling price.**

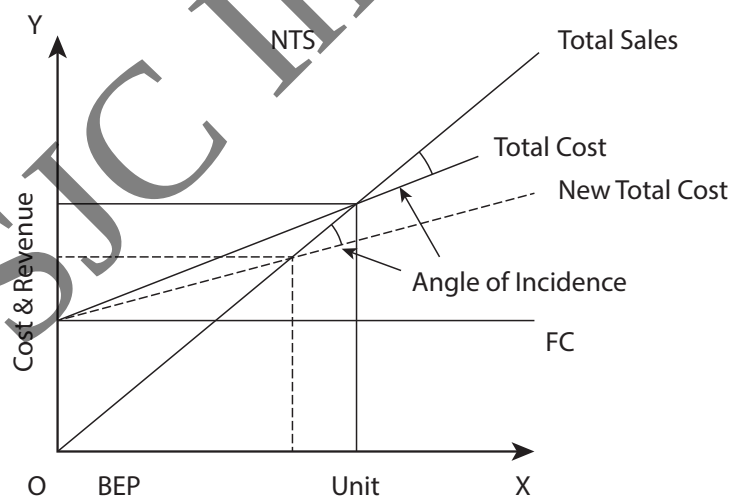
NTS = New Total Sales line



'.....' line indicates changes in break even point and changes in sales.

From the above chart, we observe that profit is increased by increasing the selling price and also, if there is change in selling price, BEP also changes. If selling price is increased then BEP decreases. If selling price is decreased then BEP increases. Thus, we say that there is an inverse relationship between selling price and BEP.

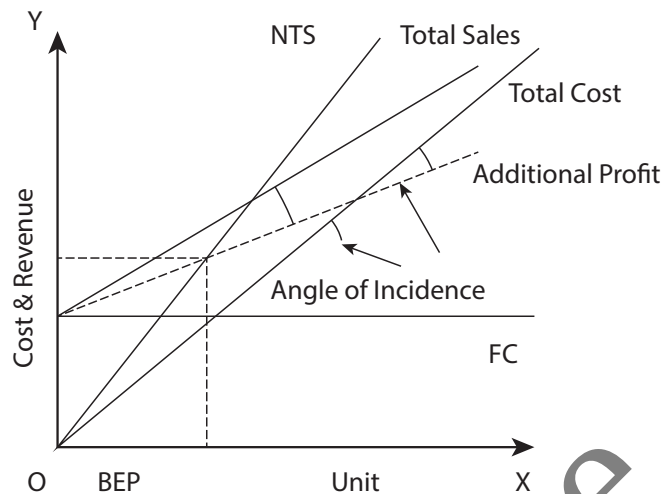
(iii) **Decrease in variable cost:**



'.....' line indicates decrease in total cost and decrease in B.E.P

From the above chart, we observe that when variable costs are decreased, no doubt, profit is increased. If there is change in variable cost then BEP also changes. If variable cost is decreased then BEP also decreases. If variable cost is increased then BEP also increases. Thus there is direct relationship between variable cost and BEP.

(iv) **Change in fixed cost:**

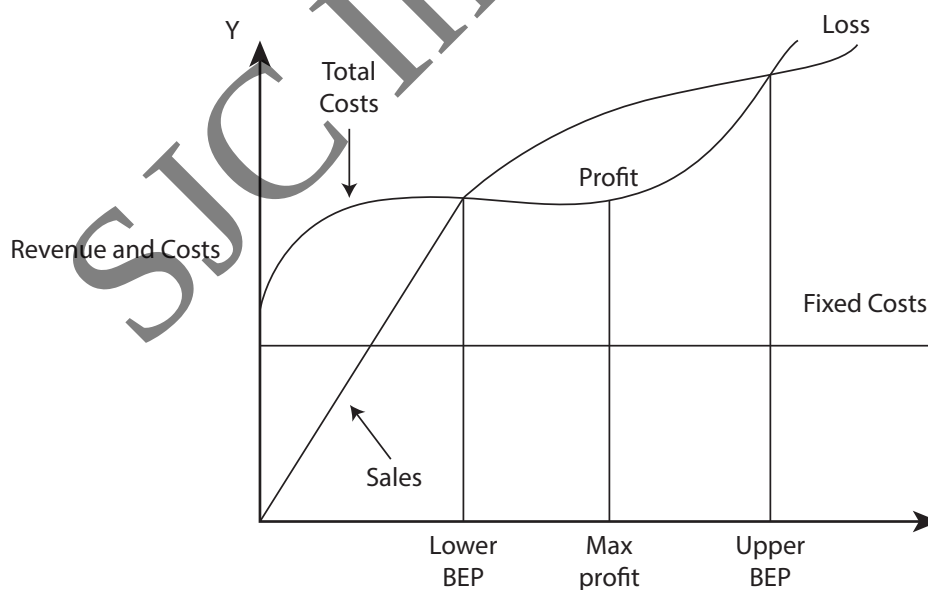


'.....' line indicates decrease in fixed cost and total cost and also decrease in BEP. NTC = New Total Cost Line

NFC = New Fixed Cost Line

From the above chart also we find that there is increase in profit due to decrease in fixed cost. If fixed cost is increased then BEP also increases. If fixed cost is decreased then BEP also decreases. Thus there is a direct relationship between fixed cost and BEP.

6. **Non linear Break Even Chart:**



In some cases on account of non-linear behaviour of cost and sales there may be two or more break even points. In such a case the optimum profit is earned where the difference between the sales and the total costs is the largest. It is obvious that the business should produce only upto this level. This is being illustrated in the above chart.

7. Cash Break-Even Point:

When break-even point is calculated only with those fixed costs which are payable in cash, such a break-even point is known as **cash break-even point**. This means that depreciation and other non-cash fixed costs are excluded from the fixed costs in computing cash break-even point. Its formula is-

Cash break even point = Cash fixed costs / Contribution per unit.

8. Profit Volume Chart:

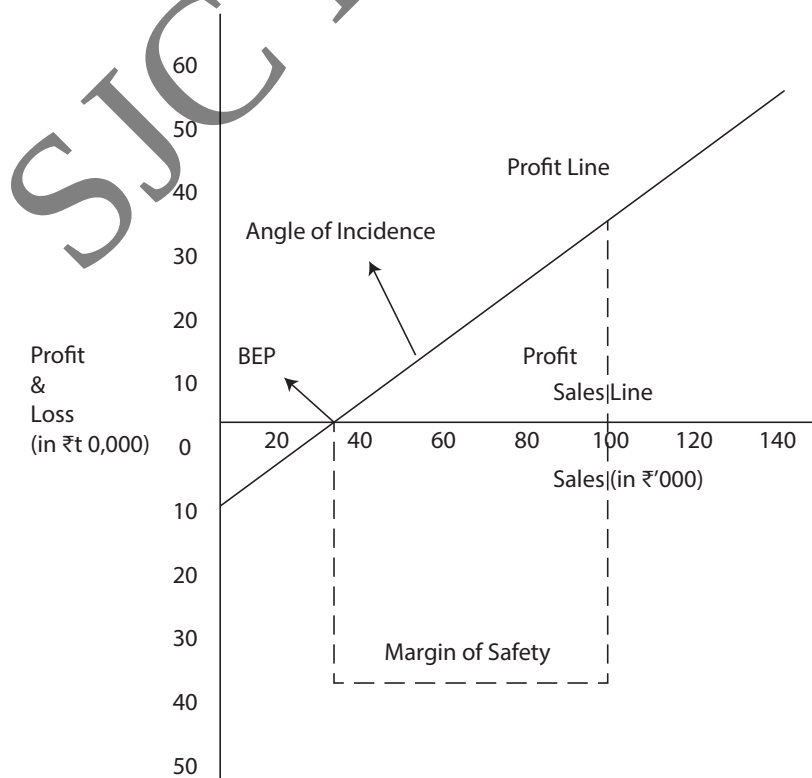
Profit-volume chart prominently exhibits the relationship between profit and sales volume. The normal break-even charts suffer from one limitation. Profit cannot be read directly from the chart. It is essential to deduct total cost from sale to know the profit figure. The profit graph overcomes the difficulty by plotting profit directly against an activity. These charts are easy to understand and their preparation involves drawing sales curve and profit curve. The point at which profit line cuts the sales line is called break-even point. Taking the methods and objects under consideration, the profit-volume chart can be further divided into following categories i.e.,

a. Simple Profit-Volume Chart:

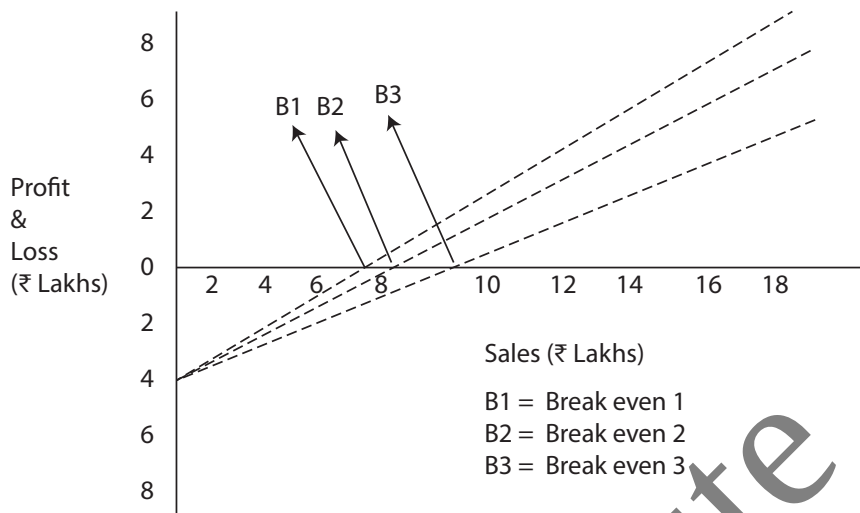
Its preparation involves the following steps:

- (i) Finding out profit at any two levels of activity.
- (ii) Drawing sales line.
- (iii) Drawing profit line.

Simple Profit-Volume chart is shown below:



- b. Profit volume chart showing different breakeven point at different price levels is shown below:



9. Uses and applications of Break even Analysis (Or) Profit Charts (Or) Cost Volume Profit Analysis:

The important uses to which cost-volume profit analysis or break-even analysis or profit charts may be put to use are:

- Forecasting costs and profits as a result of change in Volume determination of costs, revenue and variable cost per unit at various levels of output.
- Fixation of sales Volume level to earn or cover given revenue, return on capital employed, or rate of dividend.
- Determination of effect of change in Volume due to plant expansion or acceptance of order, with or without increase in costs or in other words, determination of the quantum of profit to be obtained with increased or decreased volume of sales.
- Determination of comparative profitability of each product line, project or profit plan.
- Suggestion for shift in sales mix.
- Determination of optimum sales volume.
- Evaluating the effect of reduction or increase in price, or price differentiation in different markets.
- Highlighting the impact of increase or decrease in fixed and variable costs on profit.
- Studying the effect of costs having a high proportion of fixed costs and low variable costs and vice-versa.
- Inter-firm comparison of profitability.
- Determination of sale price which would give a desired profit for break-even.
- Determination of the cash requirements as a desired volume of output, with the help of cash break-even charts.

- (m) Break-even analysis emphasizes the importance of capacity utilization for achieving economy.
- (n) During severe recession, the comparative effects of a shutdown or continued operation at a loss are indicated.
- (o) The effect on total cost of a change in the fixed overhead is more clearly demonstrated through break-even charts.

10. **Limitations of Break-even Analysis:**

- (a) That Costs are either fixed or variable and all costs are clearly segregated into their fixed and variable elements. This cannot possibly be done accurately and the difficulties and complications involved in such segregation make the break-even point inaccurate.
- (b) That the behaviour of both costs and revenue is not entirely related to changes in volume.
- (c) That costs and revenue patterns are linear over levels of output being considered. In practice, this is not always so and the linear relationship is true only within a short run relevant range.
- (d) That fixed costs remain constant and variable costs vary in proportion to the volume. Fixed costs are constant only within a limited range and are liable to change at varying levels of activity and also over a long period, particularly when additional plants and equipments are introduced.
- (e) That sales mix is constant or only one product is manufactured. A combined analysis taking all the products of the mix does not reflect the correct position regarding individual products.
- (f) That production and sales figures are identical or the change in opening and closing stocks of the finished product is not significant.
- (g) That the units of production on the various product range are identical. Otherwise, it is difficult to find a homogeneous factor to represent volume.
- (h) That the activities and productivity of the concern remain unchanged during the period of study.
- (i) As output is continuously varied within a limited range, the contribution margin remains relatively constant.

This is possible mainly where the output is more or less homogeneous as in the case of process industries.

11. **Margin of Safety:**

It is the sales point beyond the breakeven point. Margin of safety can be obtained by subtracting break even sales from Total sales. It is useful to determine financial soundness of business enterprise. If margin of safety is high, then the financial position of the enterprise is sound.

Uniform Costing in Profit Planning



Learning Objective

1. SCOPE OF UNIFORM COSTING
2. REQUISITES FOR INSTALLATION OF A UNIFORM COSTING SYSTEM
3. FIELDS COVERED BY UNIFORM COSTING
4. ADVANTAGES OF UNIFORM COSTING
5. LIMITATIONS OF UNIFORM COSTING
6. INTER FIRM COMPARISON
7. LIMITATIONS OF INTER-FIRM COMPARISON

1. INTRODUCTION

Uniform Costing is not a separate method or type of Costing. It is a technique of Costing and can be applied to any industry. Uniform Costing may be defined as the application and use of the same costing principles and procedures by different organisations under the same management or on a common understanding between members of an association. The main feature of uniform costing is that whatever be the method of costing used, it is applied uniformly in a number of concerns in the same industry, or even in different but similar industries. This enables cost and accounting data of the member undertakings to be compiled on a comparable basis so that useful and crucial decisions can be taken. The principles and methods adopted for the accumulation, analysis, apportionment and allocation of costs vary so widely from concern to concern that comparison of costs is rendered difficult and unrealistic. Uniform Costing attempts to establish uniform methods so that comparison of performances in the various undertakings can be made to the common advantage of all the constituent units.

2. SCOPE OF UNIFORM COSTING

Uniform Costing methods may be **advantageously** applied:

- (a) In a single enterprise having a number of branches or units, each of which may be a separate manufacturing unit.
- (b) In a number of concerns in the same industry bound together through a trade association or otherwise, and
- (c) In industries which are similar in nature such as gas and electricity, various types of transport, and cotton, jute and woolen textiles.

The need for application of Uniform Costing System exists in a business, irrespective of the circumstances and conditions prevailing therein. In concerns which are members of a trade association, the procedure for Uniform Costing may be devised and controlled by the association or by any other central body specially formed for the purpose.

3. REQUISITES FOR INSTALLATION OF A UNIFORM COSTING SYSTEM [MTP DEC'19]

The organisational set up for implementing the principles and methods of Uniform Costing may take different forms. It may range from a small association of a number of concerns who agree to have uniform information regarding a few specific cost accounting respects, to be a large organisation which has a fully developed scheme covering all the aspects of costing. **The success of a uniform costing system will depend upon the following:**

- (a) There should be a spirit of **mutual trust, co-operation** and a **policy of give and take** amongst the participating members.
- (b) There should be a **free exchange of ideas and methods**.

- (c) The **bigger units should be prepared to share** with the smaller ones, improvements, achievements of efficiency, benefits of research and know-how.
- (d) There should **not be any hiding** or withholding of information.
- (e) There should be **no rivalry** or sense of jealousy amongst the members.

In the application of Uniform Costing, the fundamental requirement is, therefore, **to locate such differences and to eliminate or overcome**, as far as practicable, the causes giving rise to such differences. **The basic reasons for the differences may be as follows:**

(a) **Size and organisational set up of the business:**

The number and size of the departments, sections and services also vary from one concern to another according to their size and organisation. The difficulty in operating Uniform Cost Systems for concerns which vary widely in regard to size and type of business may to some extent be overcome by arranging the various units in a number of size or type ranges, and applying different uniform systems for each such type.

(b) **Methods of production:**

The use of different types of machines, plant and equipments, degree of mechanization, difference in materials mix and sequence and nature of operations and processes are mainly responsible for the difference in costs.

(c) **Methods and principles of cost accounting applied:**

It is in this sphere that the largest degree of difference arises. Undertakings manufacturing identical or similar products and having the same system of cost accounting would generally employ different methods of treatment of expenditure on buying, storage and issue of materials, pricing of stores issues, payment to workers, basis of classification and absorption of overhead, calculation of depreciation, charging rent on freehold or leasehold assets etc.

4. FIELDS COVERED BY UNIFORM COSTING

There is no system of Uniform Costing which may be found to fit in all circumstances. The system to be installed should be tailored to meet the needs of each individual case. The essential points on which uniformity is normally required may be summarized as follows:

- (a) Whether **costs** are required for the **individual products** i.e for the **cost units** or for **cost centres**.
- (b) The **method of costing** to be applied.
- (c) The **technique** employed such as Standard Costing, Marginal Costing.
- (d) Items to be **excluded** from costs.
- (e) The basis of **departmentalization**.
- (f) The **basis of allocation** of costs to departments and/or service department costs to production departments.

- (g) The **methods of application administration, selling and distribution overhead** to cost of sales.
- (h) The method of **valuation of work-in-progress**.
- (i) Methods of **treating cost of spoilage, defective work, scrap and wastage**.
- (j) Methods of **accounting of overtime pay, bonus** and other miscellaneous **allowances** paid to workers.
- (k) Whether **purchase, material handling and upkeep expenses** are added to the cost of stores or are treated as overhead expenses.
- (l) The **system of materials control-pricing** of issues and valuation of stock.
- (m) The **system of classification** and **coding of accounts**.
- (n) The **method of recording accounting information**.

5. ADVANTAGES OF UNIFORM COSTING [RTP JUN'18]

Main **advantages** of a Uniform Costing System are summarised below:

- (i) It provides **comparative information to the members of the organisation** / association which may by them **to reduce or eliminate the evil effects of competition** and unnecessary expenses arising from competition.
- (ii) It enables the **industry to submit the statutory** bodies reliable and accurate data which might be required to regulate pricing policy or for other purposes.
- (iii) It enables the **member concerns to compare their own cost data** with that of the others detect the weakness and to take corrective steps for improvement in efficiency.
- (iv) The **benefits** of research and development can be **passed on the smaller members** of the association lead to economy of the industry as a whole.
- (v) It provides **all valuable features of sound cost accounting** such as valued and efficiency of the workers, machines, methods, etc., current reports of comparing major cost items with the predetermined standards, etc.
- (vi) It serves as a **prerequisite to Cost Audit** and inter firm comparison.
- (vii) Uniform Costing is a useful tool for **management control**. Performance of individual units can be measured against norms set for the industry as a whole.
- (viii) It avoids **cut-throat completion** by ensuring that competition among member units proceeds on healthy lines.
- (ix) The **process of pricing policy** becomes easier when Uniform Costing is adopted.
- (x) By showing the one best way of doing things, Uniform Costing **creates cost consciousness** and provides the best system of cost control and cost presentation in the entire industry.
- (xi) Uniform costing **simplifies the work of wage boards** set up to fix minimum wages and fair wages for an industry.

6. LIMITATIONS OF UNIFORM COSTING [MTP DEC'17; MTP DEC'21]

- (i) Uniform costing presumes the application of same principles and methods of Costing in each of the member firms. But **individual units generally differ** in respect of certain key factors and methods.
- (ii) For **smaller units the cost of installation** and operation of Uniform Costing System may be more than the benefits derived by them.
- (iii) Uniform costing may create conditions that are likely **to develop monopolistic tendencies** within the industry. Prices may be raised artificially and supplies curtailed.

If complete agreement between the members is not forthcoming, the statistics presented cannot be relied upon. This weakens the Uniform Costing System and reduces its usefulness.

7. INTER FIRM COMPARISON

Inter-firm comparison as the name denotes means the techniques of evaluating the performances, efficiencies, deficiencies, costs and profits of similar nature of firms engaged in the same industry or business. It consists of exchange of information, voluntarily of course, concerning production, sales cost with various types of break-up, prices, profits, etc., among the firms who are interested or willing to make the device a success. The basic purposes of such comparison are to find out the work points in an organisation and to improve the efficiency by taking appropriate measures to wipe out the weakness gradually over a period of time.

The benefits which are derived from Inter-firm Comparison are appended below :

- (a) Inter-firm Comparison makes the management of the organisation **aware of strengths and weakness** in relation to other organisations in same industry.
- (b) As only the **significant items are reported to the Management** time and efforts are not unnecessary wasted.
- (c) The management **is able to keep up to data information of the trends and ratios** and it becomes easier for them to take the necessary steps for improvement.
- (d) It develops **cost consciousness** among the members of the industry.
- (e) Information about the organisation is **made available freely** without the fear of disclosure of confidential data to outside market or public.
- (f) **Specialized knowledge and experience** of professionally run and successful organisations are made **available to smaller units** who can take the advantages it may be possible for them to have such an infrastructure.
- (g) **The industry as a whole benefits** from the process due to increased productivity, standardization of products, elimination of unfair comparison and the trade practices.
- (h) **Reliable and collective data** enhance the organising power in deal in with various authorities and Government bodies.

- (i) **Inter firm comparison** assists in a big way in identifying industry sickness and gives a timely warning so that effective remedial steps can be taken to save the organisation.

8. LIMITATIONS OF INTER-FIRM COMPARISON:

The practical difficulties that are likely to arise in the implementation of a scheme of inter-firm comparison are:

- (a) The **top management may not be convinced** of the utility of inter-firm comparison.
- (b) **Reluctance to disclose data** which a concern considers to be confidential.
- (c) A **sense of complacency** on the part of the management who may be satisfied with the present level of profits.
- (d) **Absence of a proper system of Cost Accounting** so that the costing figures supplied may not be relied upon for comparison purposes.
- (e) Non-availability of a **suitable base for comparison**.

These difficulties may be overcome to a large extent by taking the following steps:

- (a) 'Selling' the scheme through **education and propaganda**. Publication of articles in journals and periodicals, and lectures, seminars and personal discussions may prove useful.
- (b) **Installation of a system** which ensures complete **secrecy**.
- (c) Introduction of a **scientific cost system**.

Quality Cost Management



Total Quality Management (TQM)



Praise Analysis



Six Sigma



Pareto Analysis

Total Quality Management (TQM)



Learning Objective

1. THE UNDERLYING PRINCIPLES OF TQM
2. STEPS IN TOTAL QUALITY MANAGEMENT
3. 6 P'S
4. 4 P'S

SJC Institute

1. TOTAL QUALITY MANAGEMENT

Total Quality Management is a philosophy of continuously improving the quality of all the products and processes in response to continuous feedback for meeting the customers' requirements. It aims to do things right the first time, rather than need to fix problems after they emerge (A company should avoid defects rather than correct them). Its **basic objective is customer satisfaction**.

The elements of TQM are:

Total	Quality involves everyone and all activities in the company (Mobilizing the whole organization to achieve quality continuously and economically)
Quality	Understanding and meeting the customers' requirements. (Satisfying the customers first time every time)
Management	Quality can and must be managed (Avoid defects rather than correct them)

TQM is a vision based, customer focused, prevention oriented, continuously improvement strategy based on scientific approach adopted by cost conscious people committed to satisfy the customers first time every time. It aims at Managing an organization so that it excels in areas important to the customer.

2. THE UNDERLYING PRINCIPLES OF TQM [MTP DEC'18; MTP DEC'19; JUN'17]

The philosophy of TQM rest on the following principles which are enlisted below:

1. Clear exposition of the benefits of a project.
2. Total Employee involvement (TEI).
3. Process measurement.
4. Involvement of all customers and contributors.
5. Elimination of irrelevant data.
6. Understanding the needs of the whole process.
7. Use of graphical and pictorial techniques to achieve understanding.
8. Establishment of performance specifications and targets.
9. Use of errors to prompt continuous improvement.
10. Use of statistics to tell people how well they are doing

3. STEPS IN TOTAL QUALITY MANAGEMENT

Step 1: Identification of customers/customer groups:

Through a team approach (a **technique called Multi-Voting**), the Firm should identify major customer groups. This helps in generating priorities in the identification of customers and critical issues in the provision of decision-support information.

Step 2: Identifying customer expectations:

Once the major customer groups are identified, their expectations are listed. The question to be answered is - **What does the customer expect from the Firm?**

Step 3: Identifying customer decision-making requirements and product utilities:

By identifying the need to stay close to the customers and follow their suggestions, a decision-support system can be developed, incorporating both financial and non-financial and non-financial information, which seeks to satisfy user requirements. Hence, the Firm finds out the answer to - **What are the customer's decision-making requirements and product utilities?** The answer is sought by listing out managerial perceptions and not by actual interaction with the customers.

Step 4: Identifying perceived problems in decision-making process and product utilities:

Using participative processes such as brainstorming and multi-voting, the Firm seeks to list out its perception of problem areas and shortcomings in meeting customer requirements. This will list out areas of weakness where the greatest impact could be achieved through the implementation of improvements. The Firm identifies the answer to the question - **What problem areas do we perceive in the decision-making process?**

Step 5: Comparison with other Firms and benchmarking:

Detailed and systematic internal deliberations allow the Firm to develop a clear idea of their own strengths and weaknesses and of the areas of most significant deficiency. Benchmarking exercise allows the Firm to see how other Companies are coping with similar problems and opportunities.

Step 6: Customer Feedback:

Steps 1 to 5 provide a information base developed without reference to the customer. This is rectified at Steps 6 with a survey of representative customers, which embraces their views on perceived problem areas. Interaction with the customers and obtaining their views helps the Firm in correcting its own perceptions and refining its processes.

Steps 7 & 8: Identification of improvement opportunities and implementation of Quality Improvement Process:

The outcomes of the customer survey, benchmarking and internal analysis, provides the inputs for Steps 7 and 8, i.e. the identification of improvement opportunities and the implementation of a formal improvement process. This is done through a six-step process called PRAISE, for short.

4. 6C'S

The essential requirements for successful implementation are described as the **six C's of TQM**. These are:

The 6C's	
Commitment	If a TQM culture is to be developed, total commitment must come from top management. It is not sufficient to delegate ' quality ' issues to a single person. Quality expectations must be made clear by the top management, together with the support and training required for its achievement.
Culture	Training lies at the centre of effecting a change in culture and attitudes. Negative perceptions must be changed to encourage individual contributions and to make 'quality' a normal part of everyone's job.
Continuous improvement	TQM should be recognised as a ' continuous process '. It is not a 'one-time programme'. There will always be room for improvement, however small it may be.
Co-operation	TQM visualises Total Employee Involvement (TEI). Employee involvement and co-operation should be sought in the development of improvement strategies and associated performance measures.
Customer focus	The needs of external customers (in receipt of the final product or service) and also the internal customers (colleagues who receive and supply goods, services or information), should be the prime focus.
Control	Documentation, procedures and awareness of current best practice are essential if TQM implementations are to function appropriately. Unless control procedures are in place, improvements cannot be monitored and measured nor deficiencies corrected.

It is possible that the organisation is led to **Total Quality Paralysis**, instead of improvement, by improper implementation of TQM. To avoid such disruption and paralysis the following principles (called the four P's) of TQM should be followed:

5. 4P'S [JUN'18; DEC'17; MTP DEC'19]

The 4P's	
People	To avoid misdirection, TQM teams should consist of team spirited individuals who have a flair for accepting and meeting challenges. Individuals who are not ideally suited to the participatory process of TQM, should not be involved at all, e.g. lack of enthusiasm, non-attendance at TQM meetings, failure to complete delegated work, remaining a "Mute Spectator" at TQM meetings, etc.
Process	It is essential to approach problem-solving practically and to regard the formal process as a system designed to prevent participants from jumping to conclusions. As such, it will provide a means to facilitate the generation of alternatives while ensuring that important discussion stages are not omitted.

Problem	Problems need to be approached in a systematic manner, with teams tackling solvable problems with a direct economic impact, allowing for immediate feedback together with recognition of the contribution made by individual participants.
Preparation	Additional training on creative thinking and statistical processes are needed in order to give participants a greater appreciation of the diversity of the process. This training must quickly be extended beyond the immediate accounting circle to include employees at supervisory levels and also who are involved at the data input stagey

SJC Institute

Praise Analysis



Learning Objective

1. SIX STEP ACTIVITY SEQUENCE
2. DIFFICULTIES IN PRAISE ANALYSIS
3. IMPLEMENTATION OF PRAISE PROCESS

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1. SIX STEP ACTIVITY SEQUENCE

Identification of improvement opportunities and implementation of quality improvement process, of the TQM Process is through a six-step activity sequence, identified by the acronym 'PRAISE'.

Step	Activity	Elements
1	Problem Identification	• Areas of customer dissatisfaction. • Absence of competitive advantage.
2	Ranking	• Prioritise problems and opportunities by – 1. Perceived importance, and 2. Ease of measurement and solution.
3	Analysis	• Ask "Why?" to identify possible causes. Keep asking 'Why?' beyond to the move symptoms and to avoid jumping to premature conclusion;,, • Ask 'What?' to consider potential implications. • Ask 'How much?' to quantify cause and effect.
4	Innovation	• Use creative thinking to generate potential solutions. • Operationalise these solutions by identifying – 1. Barriers to implementation, 2. Available enablers, and 3. People whose co-operation must be sought.
5	Solution	• Implement the preferred solution. • Take appropriate action to bring about the required changes. • Reinforce with training and documentation back-up.
6	Evaluation	• Monitor the effectiveness of actions. • Establish and interpret performance indicators to track progress towards objectives • Identify the potential for further improvements and return to Step 1.

2. DIFFICULTIES IN PRAISE ANALYSIS

Step	Activity	Difficulties	Remedies
1	Problem Identification	- Effects of a problem are apparent, but the problems themselves are difficult to identify. - Problem may be identifiable, but it is difficult to identify a measurable improvement opportunity. - Some problems are too vague to define e.g. morale, communication, productivity etc.	- Participative approaches like brainstorming, multi- voting, panel discussion. - Quantification and precise definition of problems.
2	Ranking	- Difference in perception of individuals in ranking - Difference in preferences based on functions, e.g. production, finance, marketing etc. - Lack of consensus between individuals.	- Participative Approach. - Subordination of individual to group interest.
3	Analysis	Adoption of adhoc approaches and quick-fix solutions.	- Lateral Thinking. - Brainstorming.
4	Innovation	- Lack of creativity or expertise. - Inability to operationalise ideas, i.e. convert thoughts into action points.	Systematic evaluation of all aspects of each strategy.
5	Solution	Resistance from middle managers.	- Effective internal communication. - Training of personnel and managers. - Participative approach.
6	Evaluation	- Problems in implementation. - Lack of measurable data for comparison of expectations with actuals.	- Effective Control System to track actuals. - Feedback system.

PRAISE system are - (a) **Quality Control** - the search for continuous improvements in quality - and (b) **Total Employee Involvement** - the co-operation and commitment of employees. This **dual approach** provides a single focus - **the customer** - whose increased satisfaction remains the primary goal of the procedure.

3. IMPLEMENTATION OF PRAISE PROCESS

A three-point action plan for implementation of the process is -

1. **Small to Big Issues:** Big improvement opportunities are generally complex and require extensive interdepartmental co-operation. The choice of a relatively small problem in the first instance provides a greater chance of success. Therefore, the TQM team has to proceed from small to big issues gradually.
2. **Solvable problem:** The problem selected should not be trivial, but it should be one with a potential impact and a clear improvement opportunity. Measurable progress towards implementation should be accomplished within a reasonable time in order to maintain the motivation of participants and advertise the success of the improvement itself.
3. **Recognition of participants:** The successful projects and team members should receive appropriate recognition. Prominent individuals should be rewarded for their efforts through monetary / non-monetary prizes as a measure of personal recognition and as encouragement to others.

4. PRAISE ANALYSIS [MTP JUN'19]

PRAISE Analysis is a six step process of the Total Quality Management where identification of improvement opportunities and implementation of quality improvement process takes place. **Six Steps are :** Problem identification, Ranking, Analysis, Innovation, Solution and evaluation. These steps are represented by acronym PRAISE.

Process involves

- (i) Problem identification of customer dissatisfaction
- (ii) Ranking of the problems and opportunities as per priorities
- (iii) Analysis of possible causes of problem, potential implications and quantification of cause and effect
- (iv) Innovation by creative thinking to generate potential solutions
- (v) Solution implementation by making required changes in the systems and reinforcing the same with training and documentation backup
- (vi) Evaluation by monitoring the effectiveness of the actions and identify the potential for further improvements and return to step 1.

Six Sigma



Learning Objective

1. DMAIC
2. DMIADV
3. KEY ROLES REQUIRED FOR SUCCESSFUL IMPLEMENTATION OF SIX SIGMA
4. SIX SIGMA PROCESS IN QUALITY CONTROL PROCESS

SJC Institute

1. SIX SIGMA

Six Sigma is a set of practices developed by Motorola to systematically improve process by eliminating defects. A defect is defined as non conformity of a product or service to its specifications. The term six Sigma refers to the ability of highly capable processes to produce output with specifications. In particular, processes that operate with six sigma quality produce at defect levels below 3.4 defects per million opportunities (DPMO). Six Sigma's implicit goal is to improve all process to that level of quality or better

2. DMAIC [MTP DEC'17; MTP JUN'18]

Basic methodology consists of the following five (5) steps:

- **Define** the process improvement goals that are consistent with customer demands and enterprise strategy.
- **Measure** the current process and collect relevant data for future comparison.
- **Analyze** to verify relationship and causality of factors. Determine what the relationship is, and attempt to ensure that all factors have been considered.
- **Improve** or optimize the process based upon the analysis using techniques like Design of Experiments.
- **Control** to ensure that any variances are corrected before they result in defects. Set up pilot runs to establish process capability, transition to production and thereafter continuously measure the process and institute control mechanisms.

3. DMIADV [MTP DEC'17; MTP JUN'18]

Basic methodology consists of the following five steps:

- **Define** the goals of the design activity that are consistent with customer demands and enterprise strategy.
- **Measure** and identify CTQs (critical to qualities), product capabilities, production process capability, and risk assessments.
- **Analyze** to develop and design alternatives, create high-level design and evaluate design capability to select the best design.
- **Design** details, optimize the design, and plan for design verification. This phase may require simulations.
- **Verify** the design, set up pilot runs, implement production process and handover to process owners.

Some people have used **DMAICR (Realize)**. Others content that focusing on the financial gains realized through Six Sigma is counter-productive and that said financial gains are simply by products of a good process improvement.

4. KEY ROLES REQUIRED FOR SUCCESSFUL IMPLEMENTATION OF SIX SIGMA

Six Sigma identifies several key roles for its successful implementation:

1. **Executive Leadership** includes CEO and other key top management team members. They are responsible for setting up a vision for Six Sigma implementation. They also empower the other role holders with the freedom and resources to explore new ideas for breakthrough improvements.
2. **Champions** are responsible for the Six Sigma implementation across the organization in an integrated manner. The Executive Leadership draws them from the upper management. Champions also act as mentors to Black Belts. At GE this level of certification is now called **"Quality Leader"**.
3. **Master Black Belts**, identified by champions, act as in-house expert coaches for the organization on Six Sigma. They devote 100% of their time to Six Sigma. They assist champions and guide Black Belts and Green Belts. Apart from the usual rigour of statistics, their time is spent on ensuring integrated deployment of Six Sigma across various functions and departments.
4. **Experts** this level of skill is used primarily within Aerospace and Defense Business Sectors. Experts work across company boundaries, improving services, processes, and products for their suppliers, their entire campuses, and for their customers. **Raytheon** Incorporated was one of the first companies to introduce Experts to their organizations. At Raytheon, Experts work not only across multiple sites, but across business divisions, incorporating lessons learned throughout the company.
5. **Black Belts** operate under Master Black Belts to apply Six Sigma methodology to specific projects. They devote 100% of their time to Six Sigma. They primarily focus on Six Sigma project execution, whereas Champions and Master Black Belts focus on identifying projects/functions for Six Sigma.
6. **Green Belts** are the employees who take up Six Sigma implementation along with their other job responsibilities. They operate under the guidance of Black Belts and support them in achieving the overall results.
7. **Yellow Belts** are employees who have been trained in Six Sigma techniques as part of a corporate-wide initiative, but have not completed a Six Sigma project and are not expected to actively engage in quality improvement activities.

5. SIX SIGMA PROCESS IN QUALITY CONTROL PROCESS. [MTP DEC'19; MTP JUN'17; MTP DEC'17; MTP JUN'18; MTP DEC'18; MTP JUN'20; DEC'19; RTP JUN'18]

Six Sigma is a set of practices originally developed by Motorola to systematically improve processes by eliminating defects. A defect is defined as non-conformity of a product or service to its specifications.

While the particulars of the methodology were originally formulated by **Bill Smith at Motorola** in 1986, Six Sigma was heavily inspired by six preceding decades of quality improvement methodologies such as quality control, TQM, and Zero Defects. Like its predecessors, Six Sigma asserts the following:

- (a) Continuous efforts to reduce variation in process outputs is key to business success
- (b) Manufacturing and business processes can be measured, analyzed, improved and controlled
- (c) Succeeding at achieving sustained quality improvement requires commitment from the entire organization, particularly from top-level management.

The term “**Six Sigma**” refers to the ability of highly capable processes to produce output within specification. In particular, processes that operate with six sigma quality produce at defect levels below 3.4 defects per (one) million opportunities (DPMO). **Six Sigma's** implicit goal is to improve all processes to that level of quality or better.

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Pareto Analysis



Learning Objective

1. USEFULNESS OF PARETO ANALYSIS
2. APPLICATION OF PARETO ANALYSIS

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1. INTRODUCTION

Pareto Analysis is a rule that recommends **focus on the most important aspects of the decision making** in order to simplify the process of decision making. It is based on the 80 : 20 rule that was a phenomenon first observed by **Vilfredo Pareto**, a nineteenth century Italian economist. He noticed that 80% of the wealth of Milan was owned by 20% of its citizens. This phenomenon, or some kind of approximation of it say, (70: 30 etc.) can be observed in many different business situations. The management can use it in a number of different circumstances to direct management attention to the key control mechanism or planning aspects. It helps to clearly establish top priorities and to identify both profitable and unprofitable targets.

2. USEFULNESS OF PARETO ANALYSIS

It provides the mechanism to control and direct effort by fact, not by emotions. It helps to clearly establish top priorities and to identify both profitable and unprofitable targets.

Pareto analysis is useful to:

- (i) Prioritize problems, goals, and objectives to Identify root causes.
- (ii) Select and define key quality improvement programs.
- (iii) Select key customer relations and service programs.
- (iv) Select key employee relations improvement programs.
- (v) Select and define key performance improvement programs.
- (vi) Maximize research and product development time.
- (vii) Verify operating procedures and manufacturing processes.
- (viii) Product or services sales and distribution.
- (ix) Allocate physical, financial and human resources.

3. APPLICATION OF PARETO ANALYSIS

Pareto analysis may be applicable in the presentation of Performance Indicators data through selection of representative process characteristics that truly determine or directly or indirectly influence or conform the desired quality or performance result or outcome. The Pareto Analysis is generally applicable to the following business situations:

(i) **Pricing of a product:**

- In the case of a firm dealing with multi products, it would not be possible for it to analyse **cost-profit-price-volume relationships** for all of them. In practice, in case of such firm approximately 20% of products may account for about 80% of total sales revenue. Pareto Analysis is **used for analysing the firm estimated sales revenues from various products** and it might indicate that approximately 80% of its total sales revenue is earned from about 20% of its products.

- Such analysis **helps the top management to delegate the pricing decision** for approximately 80% of its products to the lower levels of management, thus freeing themselves to concentrate on the pricing decisions for products approximately 20% which are essential for the company's survival.
- Thus, a **firm can adopt more sophisticated pricing methods for small proportion of products** that jointly accounts for approximately **80%** of total sales revenue. For the remaining **80%** of the products which account for **20%** of total sales revenue the firm may use cost based pricing method.

(ii) **Customer Profitability analysis:**

- Instead of analysing products, customers can be analysed for their relative profitability to the organisation.
- Again it is often found that approximately **20%** of **customers** generate **80%** of the **profit**. There will always be some customers who are less profitable than others, just as some products are less profitable than others.
- Such an **analysis** is useful tool for evaluation of the portfolio of customer profile and decision making such as whether to continue serving a same customer group, what is the extent of promotion expenses to be incurred.

(iii) **ABC analysis- Stock Control:** Another application of Pareto analysis is in **stock control** where it may be found that only a few of the goods in stock make up most of the value. In practice approximately **20% of the total quantity of stock may account for about 80% of its value**. The outcome of such analysis is that by concentrating on small proportion of **stock items that jointly accounts for 80% of the total value, a firm may well be able to control most of monetary investment in stocks**.

(iv) **Application in Activity Based Costing:** In Activity Based Costing it is often said that **20% of an organisation cost drivers are responsible for 80% of the total cost**. By analysing, monitoring and controlling those cost drivers that cause most cost, a better control and understanding of overheads will be obtained.

(v) **Quality Control:**

- Pareto analysis seeks to discover from an analysis of defect report or customer complaints which **"vital few"** causes are responsible for most of the reported problems.
- Often, **80%** of reported problems can usually be traced to **20%** of the various underlying causes. By concentrating once efforts on rectifying the vital **20%**, one can have the greatest immediate impact on product quality.
- The Pareto Analysis indicates how frequently each type of failure (**defect**) occurs. The purpose of the analysis is to direct management attention to the area where the best returns can be achieved by solving most of quality problems, perhaps just with a single action.

Example:

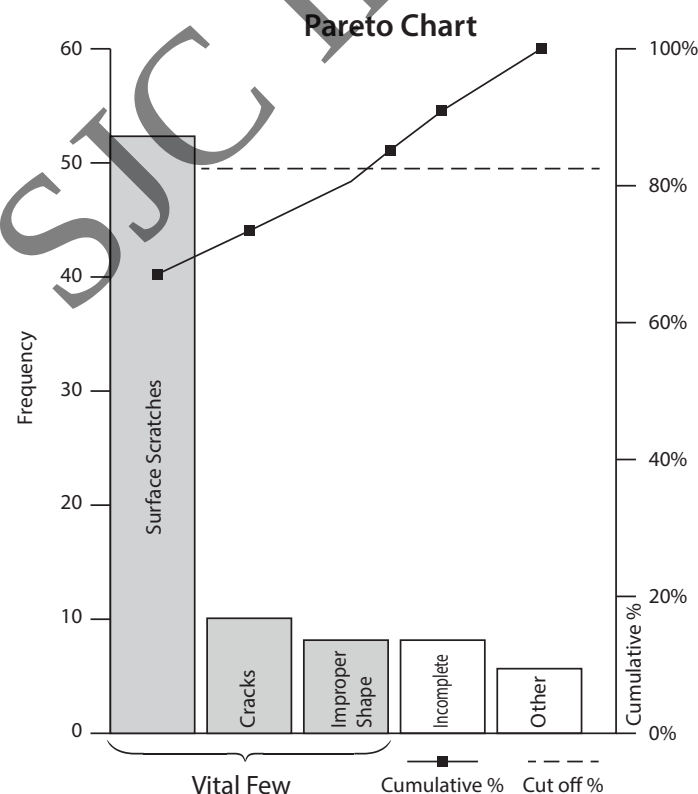
A Toy company performs a Pareto analysis, given a set of 'defect types' and frequencies of their occurrence. The sample data consists of information about 84 defective items. The items have been classified by their 'defect types' as follows:

Defect Types	No. of items
Cracks (due to mishandling of raw material)	10
Improper shapes	8
Incomplete	8
Surface scratches	53
Other (due to bad quality raw material)	5

Frequency table indicating the frequency of occurrence of defects in decreasing order of their occurrence will be as follows:

Defect type	No. of items	(%)	Cumulative %
Surface scratches	53	63.0952	63.0952
Cracks	10	11.9048	75.0000
Improper shape	8	9.5238	84.5238
Incomplete	8	9.5238	94.0476
Other	5	5.9524	100.00

The Pareto chart is then constructed for defect type.



The **purpose** of Pareto analysis in this example, **is to direct attention to the area where best returns can be achieved by solving most of the quality problems, perhaps just with a single action.** In this case, use of good quality raw material say plastic may solve 63% of problem and if raw material is handled properly at least 75% the problems may be overcome.

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Learning Objective

1. TYPES OF BENCH MARKING
2. BENCH TRENDING AND DIFFERENCE WITH BENCH MARKING
3. STAGES IN THE PROCESS OF BENCH MARKING
4. PRE-REQUISITES OF BENCH MARKING
5. DIFFICULTIES IN IMPLEMENTATION OF BENCH MARKING

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1. DEFINITIONS

Benching Marking: Traditionally control involves comparison of the actual results with an established standard or target. The practice of setting targets using external information is known as 'Bench marking'.

Bench marking is the establishment - through data gathering of targets and comparatives, with which performance is sought to be assessed.

After examining the firm's present position, benchmarking may provide a basis for establishing better standards of performance. It focuses on improvement in key areas and sets targets which are challenging but evidently achievable. Bench marking implies that there is one best way of doing business and orients the firm accordingly. It is a catching-up exercise and depends on the accurate information about the comparative company – be it inside the group or an outside firm.

Benchmark is the continuous process of enlisting the best practices in the world for the process, goals and objectives leading to world-class levels of achievement.

2. TYPES OF BENCHMARKING

The different types of Benchmarking are:

- (i) Product Benchmarking (Reverse Engineering)
 - (ii) Competitive Benchmarking
 - (iii) Process Benchmarking
 - (iv) Internal Benchmarking
 - (v) Strategic Benchmarking
 - (vi) Global Benchmarking
- (i) **Product Benchmarking (Reverse Engineering):** is an age old practice of product oriented reverse engineering. Every organization buys its rival's products and tears down to find out how the features and performances etc., compare with its products. This could be the starting point for improvement.
 - (ii) **Competitive Benchmarking:** This has moved beyond product-oriented comparisons to include comparisons of process with those of competitors. In this type, the process studied may include marketing, finance, HR, R&D etc.,
 - (iii) **Process Benchmarking:** is the activity of measuring discrete performance and functionality against organization through performance in excellent analogous business process e.g. for supply chain management – the best practice would be that of Mumbai Dubbawallas.
 - (iv) **Internal Benchmarking:** is an application of process benchmarking, within an organization by comparing the performance of similar business units or business process.

- (v) **Strategic Benchmarking:** differs from operational benchmarking in its scope. It helps to develop a vision of the changed organizations. It will develop core competencies that will help sustained competitive advantage.
- (vi) **Global Benchmarking:** is an extension of Strategic Benchmarking to include benchmarking partners on a global scale. E.g. Ford Co. of USA benchmarked its A/c payable functions with that of Mazada in Japan and found to its astonishment that the entire function was managed by 5 persons as against 500 in Ford.

3. BENCH TRENDING AND DIFFERENCE WITH BENCH MARKING [MTP JUN'17; MTP JUN'18; MTP DEC'18; MTP JUN'20]

Bench Trending: Continuous monitoring of specific process performance with a selected group of benchmarking is a systematic and continuous measurement process of comparing through measuring an organization business processes against business leaders (role models) anywhere in the world, to gain information that will help organization take action to improve its performance. The continuous process of enlisting the best practices in the world for the processes, goals and objectives leading to world class levels of achievement.

Benchmarking is the process of comparing the cost, time or quality of what one organization does against what another organization does. The result is often a business case for making changes in order to make improvements.

Benchmarking is a powerful management tool because it overcomes “**paradigm blindness**”. **Paradigm Blindness can be summed up as the mode of thinking, “the way we do it is the best because this is the way we’ve always done it”.** Bench Marking opens organizations to new methods, ideas and tools to improve their effectiveness. It helps crack through resistance to change by demonstrating other methods of solving problems than the one currently employed and demonstrating that they work, because they are being used by others.

- (a) Identify your problem areas.
- (b) Identify other industries that have similar processes.
- (c) Identify organizations that are leaders in these areas.
- (d) Survey companies for measures and practices
- (e) Visit the “**best practice**” companies to identify leading edge practices.
- (f) Implement new and improved business practices.

4. STAGES IN THE PROCESS OF BENCH MARKING

The process of benchmarking involves the following stages:

Stage	Description
1	Planning -
	(a) Determination of Benchmarking goal statement,
	(b) Identification of best performance
	(c) Establishment of the benchmarking or process improvement team, and
	(d) Defining the relevant benchmarking measures
2	Collection of Data and Information
3	Analysis of the findings based on the data collected in Stage 2
4	Formulation and implementation of recommendations
5	Constant monitoring and reviewing

Various stages in the process of Bench Marking detailed understanding

Stage 1: Planning

- (a) **Determination of benchmarking goal statement:** This requires identification of areas to be benchmarked, which uses the following criteria –

Benchmark for Customer Satisfaction	Benchmark for improving Bottom line (Profit)
• Consistency of product or service	• Waste and reject levels
• Process cycle time	• Inventory levels
• Delivery performance	• Work-in-progress
• Responsiveness to customer requirements	• Cost of Sales
• Adaptability to special needs	• Sales per employee

- (c) **Identification of best performance:** The next step is seeking the “best”. To arrive at the best is both expensive and time consuming, so it is better to identify a Company which has recorded performance success in a similar area.
- (d) **Establishment of the benchmarking or process improvement team:** This should include persons who are most knowledgeable about the internal operations and will be directly affected by changes due to benchmarking.
- (e) **Defining the relevant benchmarking measures:** Relevant measures will not be restricted to include the measures used by the Firm today, but they will be refined into measures that comprehend the true performance differences. Developing good measurement is key or critical to successful benchmarking.

Stage 2: Collection of data and information: This involves the following steps –

- (a) **Compile information and data** on performance. They may include mapping processes.
- (b) **Select** and contact partners.

- (c) **Develop a mutual understanding** about the procedures to be followed and, if necessary, prepare a Benchmarking Protocol with partners.
- (d) Prepare questions and agree terminology and performance measures to be used.
- (e) Distribute a schedule of questions to each partner.
- (f) **Undertake information** and data collection by chosen method for example, interviews, site-visits, telephone tax and e-mail.
- (g) **Collect the findings** to enable analysis.

Stage 3: Analysis of findings:

- (a) **Review the findings** and produce tables, charts and graphs to support the analysis
- (b) **Identify gaps** in performance between out Firm and better performers.
- (c) **Seek explanations** for the gaps in performance. The performance gaps can be positive, negative or zero.
- (d) **Ensure that comparisons** are meaningful and credible
- (e) **Communicate** the findings to those who are affected.
- (f) **Identify realistic opportunities** for improvements. The negative performance gap indicates an undesirable competitive position and provides a basis for performance improvement. If there is no gap it may indicate a neutral position relative to the performance being benchmarked. The zero position should be analysed for identifying means to transform its performance to a level of superiority or positive gap.

Stage 4: Recommendations:

Making recommendations	Implementing recommendations
Deciding the feasibility of making the improvements in the light of conditions that apply within own Firm	Implement the action plans
Agreement on the improvements that are likely to be feasible	Monitor performance
Producing a report on the Benchmarking in which the recommendations are included	Reward and communicate success.
Obtaining the support of owners/management for making the changes needed.	Keep owners/management informed of progress
Developing action plan(s) for implementation.	

Stage 5: Monitoring and reviewing: This involves –

- (a) **Evaluating the benchmarking** process undertaken and the results of the improvements against objectives and success criteria plus overall efficiency and effectiveness.

- (b) **Documenting** the lessons learnt and make them available to others.
- (c) **Periodically** re-considering the benchmarks for continuous improvement.

5. PRE-REQUISITES OF BENCH MARKING

1. **Commitment:** Senior Managers should support benchmarking fully and must be omitted to continuous improvements.
2. **Clarity of Objectives:** The objectives should be clearly defined at the preliminary stage. Benchmarking teams have a clear picture of their Firm's performance before approaching others for comparisons.
3. **Appropriate Scope:** The scope of the work should be appropriate in the light of the objectives, resources, time available and the experience level of those involved.
4. **Resources:** Sufficient resources must be available to complete projects within the required time scale.
5. **Skills:** Benchmarking teams should have appropriate skills and competencies.
6. **Communication:** Stakeholders, and also staff and their representatives, are to be kept informed of the reasons for benchmarking.

6. DEFINE DIFFICULTIES IN IMPLEMENTATION OF BENCH MARKING

1. **Time consuming:** Benchmarking is time consuming and at times difficult. It has significant requirement of staff time and Company resources. Companies may waste time in benchmarking non-critical functions.
2. **Lack of management Support:** Benchmarking implementation requires the direct involvement of all managers. The drive to be best in the industry or world cannot be delegated.
3. **Resistance from employees:** It is likely that their maybe resistance from employees.
4. **Paper Goals:** Companies can become pre-occupied with the measures. The goal becomes not to improve process, but to match the best practices at any cost.
5. **Copy-paste attitude:** The key element in benchmarking is the adaptation of a best practice to tailor it to a company's needs and culture. Without that step, a company merely adopts another company's process. This approach condemns benchmarking to fail leading to a failure of bench marking goals.

Yield Management



Learning Objective

1. KEY STRATEGIES
2. EFFECTIVE PRICING
3. THREE IMPORTANT APPROACHES
4. PRODUCT DIFFERENTIATION
5. COST LEADERSHIP
6. YIELD MANAGEMENT

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1. CONCEPT

Concept Pricing can make or break a business. Setting prices for the products or services does not simply come down to a simple calculation. Prices can be practical tools for making ends meet. To figure out the best way to set prices, it is worthwhile to examine the idea of what the business wants that its pricing strategy should achieve.

Product prices determine the revenue stream of a business. Prices must be sufficient to cover the costs and profit. Before lowering prices, it is preferable to lower costs to maintain a stable profit margin and a stable cash flow into the business. Any pricing strategy must be chosen to ensure a maximum profit. Knowing the market and customer base are key elements to choosing the right pricing strategy.

2. KEY STRATEGIES

The key pricing strategies may, broadly, be listed as:

- i. Profit Orientation
- ii. Competition Based
- iii. Demand Based
- iv. Cost Plus
- v. Mark-up

3. EFFECTIVE PRICING

Effective Pricing is the one that satisfies all the stakeholders, viz. the producer, distributor and the consumer. It fits into the criteria of profit orientation, competition, demand base as also the cost plus and markup. In a way it may be called long run calibrator of price equilibrium over a business cycle. It is the price set by the producer and accepted by all. It is what all stakeholders can bear.

Steps involved in determining an effective price may be listed as:

- i. Analysis of Financial Statements
- ii. Analysis of Cost Behaviour
- iii. Analysis of the Profit Gap
- iv. Evolving Cost Reduction Strategies
- v. Determination of Feasible Prices for different Capacity level
- vi. Determination of Effective Price
- vii. Establishing Cost Controls
- viii. Review, Revise and Reset

4. THREE IMPORTANT APPROACHES

Three important approaches that warrant discussion in relation to effective pricing are:

- (i) Product Differentiation
- (ii) Cost Leadership
- (iii) Yield Management

5. PRODUCT DIFFERENTIATION

Product Differentiation is the process of distinguishing a product or service from others, to make it more attractive to a particular target market. This involves differentiating it from competitors' products as well as the firm's own products¹.

The strategy of Product Differentiation is adopted to build up specific competitive advantages over competitors by tapping the unique resource endowments exclusive to an entity. The major sources of product differentiation may be traced to differences in quality, differences in functional features, and differences in availability (e.g., timing and location).

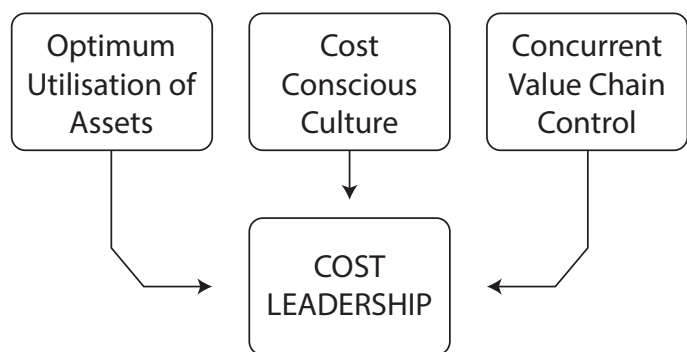
The objective of differentiation is to develop a position that potential customers see it as unique. As a result, the unique features of the product create a perception of esteem value for the product, which goes beyond pricing considerations. Well established differentiation makes customers in a given segment develop a lower sensitivity to the non-price features of the product.

Many a time product differentiation is driven by the factors of esteem value. The implication of differentiation often enables the possibility of charging a price premium.

One of the innovative research papers observes that the growing market of physically challenged persons can be a source of competitive advantage for the airlines if they differentiate their products and services by fulfilling the needs of the physically challenged. That is where the innovative value of Product Differentiation could lie.

6. COST LEADERSHIP

Cost Leadership is a generic strategy adopted to gain competitive advantage. The Strategy aims at the firm winning market share by appealing to cost conscious or price-sensitive customers. This is achieved by having the lowest prices in the target market segment, or at least the lowest price to value ratio (price compared to what customers



Dimensions of Cost Leadership

¹ The concept was proposed by Edward Chamberlin in 1933 in his 'Theory of Monopolistic Competition'.

receive). To succeed at offering the lowest price while still achieving profitability and a high return on investment, the firm must be able to operate at a lower cost than its rivals.

Cost leadership strategy drives the management to constantly work on reducing costs at every level and to remain competitive as also profitable. The three-fold dimensions, in this context, consist of:

- (a) Optimum Utilisation of Assets
- (b) Cost Conscious Culture
- (c) Concurrent Value Chain Control

7. YIELD MANAGEMENT

Concept:

Yield Management is a set of revenue maximization strategies and tactics meant to improve the business profitability. It is a technique that determines the best pricing policy for optimising profits. It adopts the principle of pricing the products and services at what the market can bear. In the process, it facilitates optimum utilisation of the resources.

Yield management was devised as the scientific way of dynamically managing prices, inventories, and capacities of perishable services. It is a scientific technique that combines Operations Research, Statistics and Customer Relationship Management (CRM); and categorizes customers into price bands. It is the process of understanding, anticipating and influencing consumer behaviour in order to maximize revenue and profits from fixed and perishable resources such as airline seats, hotel rooms, hospital beds, etc. The underlying challenge is to sell the right resources to the right customer at the right time for the right price. The process may lead to price discrimination, wherein an enterprise charges different prices to different customers for identical goods or services.

Yield management is a large revenue generator for several major industries. The general principles of revenue management are widely applicable all across, even though each particular application needs to carefully address the requirements of a specific industry. It is the art and science of price-driven and capacity-based profit maximization. It is a proven technique that helps service industries to maximize revenue.

Yield Management involves several aspects of management control, including price management, revenue streams management, and distribution channel management, just to name a few. It is a multidisciplinary strategy that blends the elements of marketing, operations, and financial management into a highly successful integrated approach. A revenue manager ought to work in cohesion with the other departments while designing and implementing revenue management strategies.

Yield Management Strategy is being used by many a sector such as Aviation, Hospitality, Health Care, Power Distribution, Telecommunications, etc. The demonstrative examples include lower tariffs for advance

Fixed
Capacity

Perishable
Products

Low Marginal
Costs

Price Elastic
Demand

Segmented
Market

Advance
Booking

reservation by airlines; weekend discounts by hotels; time sensitive tariffs by power generation & distribution companies; the differential pricings adopted by Telecom Services, Broadcasting Media, Railways, and so on.

Features

The industries that are amenable to the strategy of Yield Management do exhibit specific features such as Fixed Capacity, Perishable Products, Low Marginal Costs, Price Elastic Demand, Segmented Market, and Advance Booking of the products and services that are being offered.

Fixed Capacity: The first of the precincts, governing yield management, is that the quantum of the products or services available for sale is governed by the principle of Fixed Capacity. In relation to the airline industry seats are the products; in case of hotel industry hotel rooms are the products; and the capacity of hospitals is the number beds they have for patients. The capacity of the seats in a flight remains fixed as also the rooms in a hotel or beds in a hospital.

In line with the upward and downward swings in demand, it is not feasible to add extra seats for any particular trip of a flight, reduce the rooms in a hotel, or do away with some beds in a hospital - on a day-to-day basis. The capacity of the enterprise, thus, remains constant for the time being.

Perishable Products: The second of the precincts relates to the perishable character of the products and services. The life span of the products and services is very limited in character; the products and services tend to perish after a time limit; and they cease to be of no value thereafter. An airline seat that remains vacant for the trip or a hotel room that remains unoccupied for a night or a hospital bed that does not have a patient for the day - adds no value at all for the period. These products and services cannot be stored in a warehouse and their value cannot be preserved. If not sold in time, the value is lost forever.

Low Marginal Costs: Yield management is of especially high relevance in cases where the fixed costs are relatively high compared to the variable costs. Such of these industries are heavily capital intensive with high fixed costs and low marginal costs. Once the capacity is created by incurring the fixed (capacity) costs, the cost of serving an additional customer is quite meagre. Rationally, an additional unit of the product can be offered at a lower margin if the demand can be increased. The high fixed cost and low marginal cost nature of the business enables the application of price differentiation as a tool to generate additional revenues. The fundamental principle is that the sales can be pushed up till the point that marginal revenue continues to be greater than marginal costs.

Price Elastic Demand: The Demand for the products and services where Yield Management can be applied is characterized by price elastic demand. Lower prices would draw more customers whereas higher prices would wean away the unwilling customers. There could also be seasonal fluctuations in the demand. In that, in peak season, the supplier can increase the revenues by raising the prices, while during lean season the sale volumes can be maintained by lowering prices. A higher profit through higher prices is the guiding parameter for peak season; and better capacity utilisation through lower prices happens to be the key strategy to tide over the pitfalls of slack season. Past data will offer the manager a way to forecast as to when and how these periods of high and low demand may occur.

Segmented Market: The market for the products and services is segmented wherein different

classes of customers are willing to pay different prices for using the same amount of the products or services. If all customers would pay the same price for using the same quantum of resources, the challenge would perhaps be limited to selling as quickly as possible and minimize the holding costs. Airlines and Hotels typically segment their customer base into a set of categories based on the price each category is willing to pay. Typical categories include the business traveller and the vacation traveller, because demand patterns for each of these categories may vary significantly, the service providers find it difficult to satisfy all of the demand simultaneously.

A good example is the comparison between the time-conscious business executive and the price sensitive vacation customer. The former is willing to pay a higher price in exchange for flexibility of being able to book at the last minute while the latter is willing to give up some flexibility for the sake of a more inexpensive pricing. Yield Management tries to maximize revenues by managing the trade-off between a low occupancy and higher rate scenario of business customers versus a high occupancy and lower rate of vacation customers. Such a strategy allows airlines and hotels to fill the seats and rooms that would otherwise have been empty.

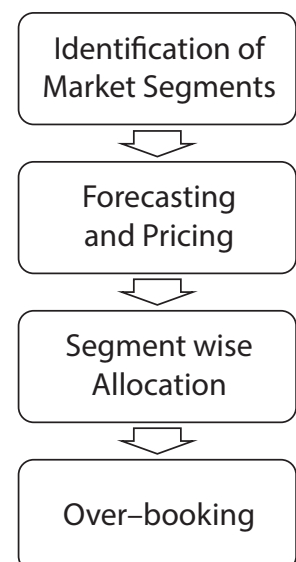
Advance Booking: More often than not, requests for bookings of the products and services start early. It could be an airline seat, hotel room, or a maternity bed; enough scope exists for the customer to foresee a time line and make the reservations well in advance. Therefore, the suppliers have enough leeway to adjust prices based on the variation between advance bookings and expected demand. If all products are sold at the same time, the supplier does not have the flexibility to adjust prices upward if demand picks up later. The trade-off occurs when a supplier is faced with the option of accepting an early reservation from a customer who wants a low price, or waiting to see if a higher paying customer will eventually show up.

The Process

The system of Yield Management is based on optimization methodologies developed from advanced statistical and analytical models. In order to arrive at a solution, the processors need to evaluate several decisions, which require a significant investment of skills, hardware and time.

Many Yield Management practitioners prefer to breakdown the actual business scenario into four sub-problems, viz. Identification of Market Segments, Forecasting and Pricing, Segment-wise Allocation of Inventory, and Overbooking. An individual solution to some or all of these sub-problems is arrived thereafter. This would significantly reduce the number of potential non-optimal decisions thereby providing fewer choices, leading to quicker results.

Identification of Market Segments: The first and foremost step in Yield Management system is the identification of the various market segments for the products and services, followed by implementation of a differential pricing scheme. The objective is expansion of the market through customer temptation. In relation to aviation, it may be observed that customers in the business class segment are



**Process of
Yield Management**

less sensitive to higher prices as opposed to those in the vacation segment. Yield Management system helps the service providers to create additional price-points by building physical and logical fences around the different market segments.

Forecasting and Pricing: The next step in Yield Management process is forecasting demand and pricing of the different market segments. Pricing and demand are inter-related and need to be coordinated. Considering the example of the hotel industry, demand for a room is cyclical in nature depending upon the day of a week or months of a year and follows a trend of demand growth due to economic

upswing. These forecasts are seldom precise but provide the decision-maker with an approximate set of inputs that are used in the planning process. Yield Management models help pinpoint demand by minimizing uncertainty and producing the best possible forecast.

Segment-wise Allocation: The next important step in a Yield Management process is the allocation of the products and services to different market segments. The ratio of discounted versus full priced products is not fixed during the reservation period; rather, it is tweaked appropriately as the date of providing the service approaches. The opportunity cost of selling a discounted product instead of a full priced one has to be measured in order to make the best decision. Thus, when a customer approaches the hotel for a discounted price, the manager needs to evaluate this scenario with the expected revenue from another customer who might come at a later date, willing to pay a higher price for the same room. The manager would accept the request only if the discounted price now is more than the expected price at which the room might be booked by the second customer. The key word here is 'expected'. Yield Management systems use mathematical algorithms to arrive at this decision using techniques such as Littlewoods and Expectation Maximization, referred to as the EM algorithm.

Over-booking: Overbooking is the practice of intentionally selling more products than are available in order to offset the effect of cancellations and no-shows. Studies estimate that although a hotel is fully booked, a small percentage of the rooms may remain vacant on any given date. Poor overbooking decisions can prove to be very expensive for the hotel. In the short run, it is only a loss of room revenue, but over the long-term, casualties may include decreased customer loyalty, loss of hotel reputation, etc. American Airlines developed an optimization model that maximizes net revenues associated with overbooking decisions for the airline industry. The driving force behind the model is the evaluation of the trade-off between additional revenue accrued by selling an already-reserved seat versus the downside from doing so. It has been found that net revenue increases with overbooking until the point where the downside from overbooking a seat exceeds customer revenues. Beyond that point, the negative impact of overbooking increases rapidly because fewer and fewer customers appreciate being turned away.

Complexities

Yield Management system does enable increased revenues; at the same time, it can be quite complicated to design; and requires high levels of expertise for implementation. Some of the concerns being

Performance
Measurement

Differential
Pricing

Employee
Motivations

Customer
Response

Complexities of Yield Management

faced by the industry in the implementation of a robust and accurate Yield Management system may be stated as difficulties in performance measurement, impact on customer loyalty, impact on employee motivations and certainty in customer response.

Performance Measurement: Measuring performance of any Yield Management system is a major issue. Occupancy rates and yield are measures that are affected by external competition. An ideal measurement can be done using an opportunity model that indicates where the company stands in comparison to its maximum.

Customer Loyalty: Differential pricing is here to stay and customers seem resigned to the fact that the service provider charges different prices for the same service. However, some customers do not like this practice and penalize the service provider by not becoming a patron. Therefore, in a fiercely competitive environment where quality of service is the key to success, Yield Management may not work. In evaluating the efficiency of a Yield Management system, the trade-off between generating short-term profits and creating long-term customer loyalty and mindshare needs to be studied carefully.

When Yield Management was introduced in the early 1990s, primarily in the airline industry, many suggested that despite the obvious immediate increase in revenues, it might harm customer satisfaction and loyalty, interfere with relationship marketing, and drive customers from firms that used Yield Management to firms that do not use Yield Management. To some extent, frequent flier programs were developed as a response to regain customer loyalty and reward frequent & high yield passengers. Today, Yield Management is nearly universal in many industries, apart from airlines.

Employee Motivations: From an operational point of view, Yield Management can impact the motivational level of the employees. In many cases, Yield Management takes much of the guess work out of employees, thereby reducing their decision-making responsibilities. Sometimes, employees taking reservations are paid a percentage of the sales they make, motivating them to make group bookings, which in turn may be contradictory with the objectives of a Yield Management system.

Customer Response: Despite optimising revenue in theory, introduction of yield management can sometimes fail to achieve this in practice because of corporate image problems. In 2002, Deutsche Bahn, the German national railway company, experimented with yield management for frequent loyalty card passengers. The fixed pricing model that had existed for decades was replaced with a more demand-responsive pricing model, but this reform proved highly unpopular with passengers, leading to widespread protests and a decline in passenger numbers. Yield Management would be successful only if customer response can be gauged properly.

Capacity Optimisation

Adoption of Yield Management implies predicting potential capacity, and developing a pricing strategy that will enable optimum capacity utilisation and maximum revenue. Yield Management attempts to derive the operational solutions by means of methods similar to aggregate and hierarchical production planning techniques often employed in the manufacturing industry.

Firms that engage in yield management usually use software-based systems to do so. The Internet has greatly facilitated this process. Enterprises that use yield management periodically review

transactions for goods or services already supplied and for goods or services to be supplied in the future. The models attempt to forecast total demand for all products or services they provide, by market segment and price point. Since total demand normally exceeds what the particular firm can produce in that period, the models attempt to optimize the firm's outputs to maximize revenue.

The optimization attempts to answer the question: Given our operating constraints, what is the best mix of products and or services for us to produce and sell in the period, and at what prices, to generate the highest expected revenue? Optimization can help the firm adjust prices and to allocate capacity among market segments to maximize expected revenues.

Taking the example of airlines, the passenger capacity is fixed for every scheduled flight; and when the aircraft departs, the unsold seats can be said to have perished. Selling of these vacant seats even at lower prices would lead to augmentation of the trip revenues and better utilization of passenger capacity; and that is where revenue management comes into play.

Airlines keep monitoring the trend of reservations and respond by offering the probable vacant seats at discounted prices to leisure travellers. Statistical analysis of past data helps in forecasting demand and establishing the appropriate price bands. The same modus operandi can be extended to Hotels, Car Rentals, Road Transport, Rail Transport, Hospitals, Stadiums, Cinema Halls, Apartment Housing, and so on.

The application of Yield Management enhances the capacity utilisation whereby fixed costs per unit are brought down to the feasible minimum. Thus, fulfilling the objectives of Marginal Costing. Obviously, Yield Management is an extension of the techniques of Cost Volume Profit Analysis of Marginal Costing.

Yield Competencies

According to Professor Peter Bell, Richard C. Ivey School of Business, "Revenue management concepts will be applied to almost everything that will be sold and will prove to be such a powerful competitive weapon that major firms will be living, and in many cases dying, according to revenue management algorithms."

As part of ongoing changes in the industry, companies throughout the world spectrum are placing a strong emphasis on implementing major operational changes. Beyond recognizing that, meaningful cost reductions must be achieved without compromising safety, capacity and service levels, they are also looking at reducing costs by increasing flexibility and improving asset utilization through Yield Management strategy. In doing so, they continue to reassess their true core competencies that lead to optimize business efficiencies and increase profitability.

Yield Competency matrix may be drawn by finding answers to the following six simple questions that govern the specific features of Yield Management.

1. Is the unit of product or service governed by the principle of Fixed Capacity?
2. Is the unit of product or service Perishable?
3. Does the product or service warrant Low Marginal Costs?
4. Is the unit of product or service prone to Price Elastic Demand?

5. Is the unit of product or service poised for a Segmented Market?
6. Can the unit of product or service be offered by means of Advance Booking?

If the answers are “Yes”, it is time to tread towards the path of Yield Management and catch the train of Revenue Maximisation.

As the Yield Management, being software driven application, its scope can be unending. The path is open for a plethora of Indian SMEs encompassing Transportation, Hospitality, Logistics, Education, Power Distribution, etc. A seat in a bus, a table in a restaurant, some space in a warehouse, a seat in a college, power for farmers, flat in a rental apartment - everything is amenable to YM! It is for the Cost Managers to search out, evolve and implement the strategy of Yield Management across ‘Make in India’.

If customer is the King, Yield Management is the Strategy!

Case Study 1: Wal-Mart pulls off its growth through Price Perfection

Many retailers are trying to replicate the success of Wal-Mart Stores Inc., the world’s largest retailer that was founded over 60 years ago, on “Everyday Low” prices. The central goal of Wal-Mart is to keep retail prices low and the company has been very successful at this. Experts estimate that Wal-Mart saves shoppers at least 15 percent on a typical cart of groceries.

Wal-Mart has grown, over the years, into more than just the world’s largest retailer. It is an economic force and a cultural phenomenon. It all started with a simple philosophy from founder Sam Walton: “Offer shoppers lower prices than they get anywhere else”. That basic strategy has shaped Wal-Mart’s culture and driven the company’s growth.

Sam Walton opened his first ‘five-and-dime’ in 1950 and his vision was to keep prices as low as possible. Even if his margins weren’t as fat as competitors, he figured he could make up for that in volume. Wal-Mart has been able to keep its prices low through cutting-edge technology, a push to make suppliers sell merchandise at cheaper and cheaper prices, and a frugal corporate culture.

Wal-Mart pushed the retail industry to establish the universal bar code, which forced manufacturers to adopt common labelling. Over a decade back, Wal-Mart became the first major retailer to demand manufacturers use Radio Frequency Identification Technology (RFID).

The stories of how Wal-Mart pushes manufacturers into selling the same product at lower and lower prices are legendary. One example is Lakewood Engineering & Manufacturing Co. in Chicago, a fan manufacturer. In the early 1990s, a 20-inch box fan costs \$20. Wal-Mart pushed the manufacturer to lower the price, and Lakewood responded by automating the production process. Lakewood also badgered its own suppliers to knock down the prices of parts. Then, in 2000, Lakewood opened a factory in China, where workers could be as cheap as 25 cents an hour. By 2003, the price on the fan in a Wal-Mart store had dropped to about \$10.

In a 2003 Los Angeles Times article, part of a Pulitzer Prize-winning series about Wal-Mart, tells of a Wal-Mart buyer named Celia Clancy, who was in charge of clothing and demanded that each supplier either lower the price or increase the quality every year on every item. This philosophy is known as “plus one.” In “The Wal-Mart Effect,” author Charles Fishman discusses how the price of a four-pack of GE light bulbs decreased from \$2.19 to 88 cents during a five-year period.

Wal-Mart's impact extends beyond just small suppliers. It also affects how even major, established companies like Coca-Cola and PepsiCo do business. At Wal-Mart's request, Coke and its largest bottler Coca-Cola Enterprises announced that they are changing the way they deliver Powerade in the United States, altering a basic distribution

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NOTES

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Part IV Practice Questions

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Q1

Ex. Book No.

Pg. No.

CMA Mat

The anticipated sales of Electronic Corporation Ltd. is ₹ 4,00,000 and unit selling price is ₹ 20 each. The per unit cost of direct material is ₹ 9, labour is ₹ 3 and other variable expenses are ₹ 3 per unit. The company is earning a net profit of 5% and to improve the profitability, the following proposals were discussed at the Executive Committee Meeting:

- The present administrative setup is on the regional basis and it was felt that centralization will reduce the fixed cost by ₹ 12,000.
- The Production Manager has agreed that he will try to work on a cost reduction programme which will reduce the cost by ₹ 1 per unit but there will be little impact on the quality which will be negligible to the customer.
- The Sales Manager opposed the two proposals and suggests that it may be possible to increase the number of units sold by 20%, provided the selling price is reduced by 5%
- Alternatively, as per Sales Manager, if the selling price is increased by 10%, the sales number of units will be reduced by 5%.

As the Cost and Management Accountant of the company, **evaluate** the aforesaid four proposals and also **put forward** your suggestions to improve the situation.

Reference Evaluation of Proposals	What's New

Answer

- Computation of Number of units to be Sold, Breakeven & envisaged Reduction in FC

Item	Splash	Flash	Total
No. of units to be sold	3,00,000	1,00,000	4,00,000
Break even units	1,80,000	60,000	
Cost Reduction envisaged in FC (₹)	4,320	750	5,070

- Percentage of Profit on Capital Employed = 23.195 i.e.. > 23%

Recommendation: The Sales Manager's proposal can be adopted.

3. Evaluation of Alternatives

Alternative	(i)	(ii)	(iii)	(iv)
Proposal	Central administration (Reduction in F.C. by ₹ 12,000)	Variable cost Reduction by ₹ 1 per unit	20% increase in Sales units with 5% reduction in selling price	10% increase in Selling price and 5% reduction in sales units
Ranking	3	2	Reject	1
Suggestion				Accept

Recommendation: As Cost and Management Accountant of the company, it will be recommended a combination of proposals (i) and (iii) which will, together, generate an additional profit of ₹ 45,000 (12,000 + 33,000).



CMA Mat

The income statement of Ashok Gears Ltd. is summarized as below:

Net Revenue	₹ 80,00,000
Less: Expenses (including ₹40,00,000 of Fixed Cost)	₹ 88,00,000
Net Loss	₹ 8,00,000

The manager believes that an increase of ₹20,00,000 as fixed expenditure in advertising outlays will increase the sales substantially. His plan was approved by the Board.

You are required to **calculate**:

- At **what** sales volume will the Company have break even?
- What** sales volume will result in a Net Profit of ₹ 4,00,000?

Reference	What's New
Sales Volume to Break Even	

Answer

(i) Computation of Break-Even Sales

Net Revenue	= ₹ 80,00,000	
Variable Expenses	= ₹ (88,00,000 - 40,00,000)	= ₹ 48,00,000
Contribution	= (80,00,000 - 48,00,000)	= ₹ 32,00,000
Contribution Margin Ratio (PV Ratio)	= 32,00,000 ÷ 80,00,000	= 40%

Revised Fixed Cost	= (Existing 40,00,000 + 20,00,000 of Advertising) = ₹ 60,00,000	
Break Even Sales	= (Fixed Cost ÷ PV Ratio)	= (60,00,000 ÷ 40%)
		= ₹ 1,50,00,000

(ii) Computation of sales level to earn a Net Profit of ₹ 4,00,000

Targeted Contribution	= (Fixed Cost + Desired Profit)	= (60,00,000 + 4,00,000)
		= ₹ 64,00,000
Required Sales	= (Targeted Contribution ÷ PV Ratio)	= (64,00,000 ÷ 40%)
		= ₹ 1,60,00,000

(Explanatory Comment: The problem brings forth the primary application of marginal costing in manufacturing sector for the purposes of calculating the BEP and profit planning.)

Q3

Ex. Book No.

Pg. No.

CMA Mat

S K started a catering service to supply food to patients admitted in hospitals and those who were recovering from the pandemic. The cost of food and the disposable packing during a month of 30 days is ₹ 100 per meal. She sells each meal at ₹ 160. She has an arrangement with a delivery agency that quickly reaches for pick up and delivers to the customers. The agency charges her ₹ 120 per delivery, on condition that not more than 10 meals be transported by one person on one trip, i.e., ₹ 120 is charged irrespective of the number of meals subject to a maximum of 10 meals. She incurs a fixed expense of ₹ 1,44,000 per month.

- At **what** volume of sales will she break-even at the earliest? How many deliveries will be required?
- If she is able to sell 5000 meals in a month, **what** will be her maximum profits?
- What** will be her worst income at this level? Assume she will not entertain any delivery for less than 3 meals per trip.

Reference

BE Sales and Profit Analysis - Slab Cost

What's **New**

Answer

(a) **Break Even at the earliest and Number of Deliveries**

The earliest break-even will occur when the transport cost per unit is the minimum, i.e., when all her sales are delivered in batches of 10. Then:

Cost of Food & Disposable Packing – ₹ 100 Delivery Cost = $120 \div 10 = ₹ 12$ per meal

Contribution per meal = $(160 - 100 - 12) = ₹ 48$.

$$\begin{aligned}\text{Break Even Sales} &= \text{Fixed Cost} \div \text{Contribution per Unit} \\ &= (1,44,000 \div 48) = 3000 \text{ meals per month}\end{aligned}$$

$$\text{Number of deliveries required} = (3000 \div 10) = 300 \text{ per month}$$

(b) **Profit at maximum sales of 5,000 per month**

$$\text{Contribution at a sale of 5,000 meals} = 5000 \times 48 = ₹ 2,40,000$$

$$\text{Fixed Cost} = ₹ 1,44,000$$

$$\text{Profit} = (2,40,000 - 1,44,000) = ₹ 96,000$$

(c) **Worst income at the level of 5,000 meals**

$$\text{Contribution per Meal before delivery Cost} = (160 - 100) = ₹ 60$$

$$\text{Delivery cost at 3 meals per delivery} = (120 \div 3) = 40$$

$$\text{Contribution per meal} = (60 - 40) = ₹ 20$$

$$\text{Contribution at a sale of 5,000 meals} = 5000 \times 20 = ₹ 1,00,000$$

$$\text{Profit} = (1,00,000 - 1,44,000) = ₹ 44,000$$

In a worst situation the loss is ₹ 44,000

Q4

Ex. Book No.

Pg. No.

CMA Mat

T Ltd. produces and sells a product. The company expects the following revenues and costs in 2022:

Revenues (400 sets sold @ ₹600 per product)	₹ 2,40,000
Variable costs	₹ 1,60,000
Fixed costs	₹ 50,000

What amount of sales must T Ltd. has to earn a target net income of ₹63,000 if they have a tax rate of 30%?

Reference	What's New
Amount of sales to earn target income	

Answer

$$\text{Sales} = ₹ 2,40,000$$

$$\text{Variable Costs} = ₹ 1,60,000$$

$$\text{Current Contribution} = (2,40,000 - 1,60,000) = ₹ 80,000$$

$$\text{Contribution of Sales Ratio} = (80,000 \div 2,40,000) = 1/3$$

Target Net Income (Net Profit) = ₹63,000

Tax Rate = 30%

Profit Before Tax = {(Net Profit ÷ (1-Tax Rate))}
 = {63,000 ÷ (1-30%)} = ₹90,000

Fixed Costs + ₹50,000

Target Contribution = (PBT + FC) = 90,000 + 50,000
 = ₹1,40,000

Target Sales = (Target Contribution ÷ C/S Ratio)
 = (₹1,40,000 ÷ 1/3) = ₹4,20,000

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Chapter 2

Transfer Pricing

Q1

Ex. Book No.

Pg. No.

CMA Mat

AB Limited has two divisions Alpha & Beta. Alpha produces components, two units of which are required for one unit of final product produced by Beta. Alpha has a capacity to produce 20,000 units and entire quantity is supplied to Beta @ ₹ 200/unit. Variable cost component at Alpha is ₹ 190 & fixed cost ₹ 20 per unit. For final product of Beta, per unit variable cost excluding component is ₹ 700, fixed cost ₹ 200 and selling price is ₹ 1500. Alpha has placed a proposal for increasing the transfer price to ₹ 220 i.e.. the market price. Facility at Alpha can be rented out @ ₹ 3.00 Lakhs p.a. Manager at Alpha wants to opt for this alternative. Beta can buy this component from outside market @ ₹ 210. If capacity of Alpha is augmented to 40,000 units with an additional investment of ₹ 15 lakhs, it can sell 20,000 units to external market and balance to Beta @ ₹ 210 per unit. Fixed cost for Alpha will be up by ₹ 1.00 lakh.

Evaluate and **give** your opinion on:

- Facility of Alpha is rented out and Beta buys from market @ ₹210 per unit
- Alpha sells to outside market @ ₹220 and Beta buys @ 210 per unit from market
- Capacity enhancement at a cost of capital of 12% p.a.

Reference

Evaluation of Alternatives for Profit Planning

What's New

Answer

(i) **Present position on transfer of Alpha @ ₹200 to Beta**

Particulars	Division Alpha	Division Beta
Units sold	20,000	10,000
Selling price per unit (₹)	200	1,500
Variable cost per unit (₹)	190	700 + (2 × 200) = 1,100
Contribution per unit (₹)	10	400
Fixed cost per unit (₹)	20	200
Profit per unit (₹)	(-) 10	200
Total Profit / Loss (₹)	(-)2,00,000	20,00,000

Overall profit for the company	= (-2,00,000 + 20,00,000)	= ₹ 18,00,000
Total Fixed Cost for Alpha	= (20,000 × 20)	= ₹ 4,00,000
Total Fixed Cost for Beta	= (10,000 × 200)	= ₹ 20,00,000

- (ii) **Alternative (a), i.e.. Facility of Alpha is rented out and Beta buys from market @ ₹ 210 per unit**

Particulars	Division Alpha	Division Beta
Units sold	0	10,000
Selling price per unit (₹)		1,500
Variable cost per unit (₹)		700 + (2 × 210) = 1,120
Contribution per unit (₹)		380
Total Contribution (₹)		38,00,000
Fixed Cost (₹)		20,00,000
Rental Income (₹)	3,00,000	
Total Profit (₹)	3,00,000	18,00,000

Overall profit for the company = (3,00,000 + 18,00,000) = ₹ 21,00,000

- (iii) **Alternative (b), i.e.. Alpha sells to outside market @ ₹220 and Beta buys @ 210 per unit from market**

Particulars	Division Alpha	Division Beta
Units sold	20000	10000
Selling price per unit (₹)	220	1500
Variable cost per unit (₹)	190	700 + (2×210) = 1120
Contribution per unit (₹)	30	380
Total Contribution (₹)	6,00,000	38,00,000
Fixed Cost (₹)	4,00,000	20,00,000
Total Profit (₹)	2,00,000	18,00,000

Overall profit for the company = (2,00,000 + 18,00,000) = ₹ 20,00,000

- (iv) **Alternative (c), i.e., Capacity enhancement of Alpha at a cost of capital of 12% p.a.**

Particulars	Division Alpha (Sale)	Division Alpha (Transfer)	Division Beta
Units sold	20000	20000	10000

Marginal Costing

Q1

Ex. Book No.

Pg. No.

CMA Mat

ABC Computer Ltd. is planning to introduce a new computer "Speedo". The maximum production capacity will be 40,000 units per annum. The company plans to produce full capacity in the first year. The cost per computer is as follows:

	(₹)
Direct material	6,000
Direct labour	5,000
Variable factory overheads	3,000

Fixed factory overheads is ₹1600 Lakhs and selling and distribution overheads will be ₹800 lakhs per annum. Fixed factory overheads are estimated on the basis of full capacity. The marketing department has come out with the following price and demand forecast for the first year.

Selling Price Range	Sales Volume
14001 – 24000	36000
24001 – 30000	32000
30001 – 36000	18000
36001 – 42000	10000

ABC Computers has decided to price the computers at full cost plus 20% for the first year

- Work out** the price/unit at which ABC wishes to sell the computer for the first year and arrive at the demand.
- Can** you work out a better proposal?
- Determine** the profit and the value of stock for year 1 for figures obtained under (a) above following Marginal cost approach (for the demand arrived at based on company's pricing policy for 1st year)
- Determine** the profit and the value of stock for year 1 for figures obtained under (a) above following Absorption cost approach (for the demand determined for better proposal for 1st year)



Reference

Evaluation of Alternative Choices and Profit under Absorption and Marginal Costing

What's **New**

Answer

- (a) **Computation of Price/unit at which ABC wishes to sell the computer for the first year**

Particulars	Per unit (₹)	Total for 40000 units (₹ In Lakhs)
Variable Costs		
Direct Materials	6000	2400
Direct Labour	5000	2000
Variable factory overheads	3000	1200
Total Variable Costs	14000	5600
Fixed Costs		
Factory Overheads	4000	1600
Selling and Distribution overheads	2000	800
Total Fixed Costs	6000	2400
Total Costs	20000	8000
Add Markup @ 20%	4000	1600
Selling Price	24000	9600

At a selling price of ₹ 24,000 per unit, the maximum demand, as given in the price – sales matrices, is 36,000. Therefore, the price per unit at which ABC wishes to sell the computer for the first year is ₹ 24,000 units and the demand is 36,000 units.

- (b) **Workings for a better the Proposal**

Volume	Max Selling Price per unit (₹)	Contribution per unit (₹) = (Price – Total Variable Costs of ₹ 14000)	Contribution for the entire Volume of Sales (₹ Lakhs)
36000	24,000	10,000	3,600
32000	30,000	16,000	5,120
18000	36,000	22,000	3,960
10000	42,000	28,000	2,800

The contribution is maximum (₹ 5,120 lakhs) at a sale volume of 32,000 units which, evidently, is a better proposal

- (c) **Determination of the profit and the value of stock for year 1 for figures obtained under (a) above following Marginal Cost Approach: (Demand as per First Year)**

Number of units produced	=	40,000
Number of Units to be sold	=	36,000
Selling Price	=	₹ 24,000
Contribution per Unit	=	₹ 10,000 (24,000 – 14000)

Contribution for the entire volume of Sales = $(36,000 \times 10,000) = ₹ 3600$ lakhs

Fixed Costs = (Factory Overheads + Selling and Distribution Overheads)

= $1600 + 800 = ₹ 2400$ lakhs

Profit on Sales = (Contribution – Fixed Costs)	=	$(3,600 - 2,400) = ₹ 1,200$ lakhs
Closing Stock = (Production – Sales)	=	$(40000 - 36000) = 4000$ units
Value of Closing Stock @ Marginal Cost	=	$(4,000 \times 14,000) = ₹ 560$ lakhs
Profit on Production	=	$(1,200 + 560) = ₹ 1,760$ lakhs

(d) **Determination of the profit and the value of stock for year 1 for figures obtained under (a) above following Absorption Cost Approach:**

Number of units produced = 40,000

Number of Units to be sold = 32,000

Selling Price = ₹ 30,000

Contribution per Unit = ₹ 16,000

Contribution for the entire volume of Sales = $(32,000 \times 16,000) = ₹ 5,120$ lakhs

Fixed Costs Absorbed on No. of Units sold = (Factory Overheads @ ₹ 4,000 per unit + Selling and Distribution

Overheads @ ₹ 2,000 per unit) = $1,280 + 640 = ₹ 1,920$ lakhs

Profit on Sales	= (Contribution – Fixed Costs)	= $(5,120 - 1,920)$	= ₹ 3,200 lakhs
Closing Stock	= (Production – Sales)	= $(40,000 - 32,000)$	= 8000 units
Rate per Valuation	= (Variable Cost + Factory Overheads)	= $(14,000 + 4,000)$	= ₹ 18,000

Value of Closing Stock @ Absorbed Cost = $(8,000 \times 18,000) = ₹ 1,440$ lakhs

Profit on Production = $(3,200 + 1,440) = ₹ 6,560$ lakhs

Note: The aspect over absorption of overheads is not considered while computing the profit.

(Commentary: The problem reveals multiple conceptual dimensions during the course of evaluation of alternatives.)

Decision Making

Q1

Ex. Book No.

Pg. No.

CMA Mat

Chakra Ltd. manufactures Mixer Grinders. The manufacture involves an assembly of various parts which are processed in the machine shop and purchased components. The on/off switch is presently being purchased from a vendor at ₹4.50 each, annual requirement being 20,000 pieces. The production manager has put up a proposal two months back to make the switch in the machine shop. He had suggested that the company would make profit and save taxes on bought out switch. The costing department was asked to make an estimate of making the item which showed that the cost of making was ₹4.73. The purchase department continues buying the item on the basis of the cost estimate given to them. Recently, the Vendor has sent a letter requesting the purchase department to grant increase in price of 10% minimum per switch as the input costs had gone up. The costing department was once again requested to estimate cost of making the switch. The costing department re-estimated the costs using current prices and observed that the cost of making has gone up to ₹5.33. Purchase department again decided to continue buying as it was cheaper to buy than make.

The cost estimate prepared by the costing department was as under:

	Annual costs	
	Previous (₹)	Current (₹)
Direct Materials	40,000	48,000
Direct Labour	20,000	22,000
Overheads	30,000	31,500
Total cost at current price	90,000	1,01,500
Add: expected increase 5%	4,500	5,075
Expected manufacturing cost	94,500	1,06,575
Cost per price	4.73	5.33

Twenty-five per cent of the overheads are fixed.

Required:

Do you agree with the decision of buying considering the relevant costs? If the cost of making or buying is more or less, **what** factors other than cost will influence making decision?

	Reference	What's New
	Make or Buy	

Answer

Twenty-five per cent of the overheads are fixed and hence not relevant for decision making.

Fixed Overheads:

Previous = 25% of 30,000 = ₹ 7,500

Current = 25% of 31,500 = ₹ 7,875

Variable Overheads:

Previous = 75% of 30,000 = ₹ 22,500

Current = 75% of 31,500 = ₹ 23,625

Statement of variable costs of making on/off switch before and after price increase

Particulars	Previous (₹)	Current (₹)
Materials	40,000	48,000
Labour	20,000	22,000
Overhead (Variable)	22,500	23,625
Total Variable Cost	82,500	93,625
Number of Units	20,000	20,000
Making Cost per Switch	4.125 (82,500/20,000)	4.68 (93,625/20,000)
Cost of Buying	4.50	4.95 (4.50 × 110/100)

It is not advisable that purchase department continues to buy the switch because variable cost of making is less than the buying cost.

Relevant Costing

Q1

Ex. Book No.

Pg. No.

CMA Mat

The accountant of XYZ Ltd. has prepared the following estimate on the basis of which he has advised that a contract should not be accepted at the price offered. The estimate (₹) was as follows:

Material X in stock at original cost	1,50,000
Material Y on order at contract price	1,80,000
Material Z to be ordered at current price	3,00,000
Skilled Labour	5,40,000
Unskilled Labour	3,00,000
Supervisory Cost	1,00,000
General Overheads	1,80,000
Total Cost	17,50,000
Price offered	14,00,000
Net Loss (Price offered – Total Cost)	3,50,000

The following details are available about the cost components listed above.

- Material X is an obsolete material. It can be used on another product W, the material for which is available at ₹1,35,000 (Material X requires some adaptation to be used which costs ₹15,000). It may take some time before W's order is confirmed. Until then storage will cost ₹12,000.
- Material Y is ordered for some other product which is no longer required. It now has a residual value of ₹1,55,000.
- Skilled labour can work on other contracts which are presently operated by semi-skilled labour at a cost of ₹4,00,000
- Unskilled labour are specifically employed for this contract
- Supervisory staff will remain whether or not the contract is accepted. Only two of them can replace other positions where the salary is ₹50,000.
- Overheads are charged at 33 1/3% of skilled labour. Only ₹1,25,000 would be avoidable

You are **required** to answer the following questions using relevant cost approach:

- Relevant costs of material X, Y and Z
- Relevant cost of labour-skilled and unskilled
- Relevant cost of Supervisory cost and General overheads
- If the contract is accepted, what would be the resulting financial impact on XYZ's profit

Reference	What's New
Relevant Cost of Material, Labour and Overhead	

Answer

(a) Relevant costs of material X, Y and Z

Material X (Obsolete)

Material X in stock at original cost	= ₹ 1,50,000
Reuse Value	= ₹ 1,35,000
Adaptation Cost	= ₹ 15,000
Storage Cost	= ₹ 12,000
Relevant Cost of Material X	= (1,35,000 – 15,000 – 12,000) = ₹ 1,08,000

Material Y (No longer required)

Material Y on order at contract price	= ₹ 1,50,000
Residual Value	= ₹ 1,55,000
Relevant Cost of Material Y	= ₹ 1,55,000

Material Z (To be ordered)

Material Z to be ordered at current price	= ₹ 3,00,000
Relevant Cost of Material Z	= ₹ 3,00,000

Material X is an obsolete material but can be used as substitute of some other material available at ₹1,35,000 after incurring an adaptation cost of ₹15,000 and Storage cost of ₹12,000. While using Material 'X' for current work, these costs can be saved, so relevant cost = 1,35,000 - 15,000 - 12,000 = 1,08,000

(b) Relevant cost of labour-skilled and unskilled

Skilled Labour (Can replace unskilled labour)

Cost of skilled labour	= ₹ 5,40,000
Replacement Cost (in place of unskilled labour)	= ₹ 4,00,000
Relevant Cost of Skilled Labour	= ₹ 4,00,000

Unskilled Labour (Specifically Employed)

Cost of unskilled labour	= ₹ 3,00,000
Relevant Cost of Unskilled Labour	= ₹ 3,00,000

[Lower of the Above]

(c) Relevant cost of Supervisory cost and General overheads

Supervisory cost = ₹ 1,00,000

Replacement Value for others = ₹ 50,000

Relevant Supervisory Cost = ₹ 50,000

Avoidable General Overheads = ₹ 1,25,000

Relevant Costs of General Overheads = ₹ 1,25,000

(d) Computation of Financial Impact

Serial	Element	Amount (₹)
A	Price Offered	14,00,000
B	Relevant Costs	
	1. Material X	1,08,000
	2. Material Y	1,55,000
	3. Material Z	3,00,000
	4. Skilled Labour	4,00,000
	5. Unskilled Labour	3,00,000
	6. Supervisory Cost	50,000
	7. General Overheads	1,25,000
	8. Total (1...7)	14,38,000
C	Financial Impact (A-B)	(38,000)

Observation: The loss is much less than what the accountant has worked out. However, if the contract is accepted, XYZ's profit will be reduced by ₹ 38,000/-.

Chapter 6 Target Costing

Q1

Ex. Book No.

Pg. No.

CMA Mat

A Company requires ₹ 85,00,000 in sales to meet its target net profit. Its contribution margin is 30% and the fixed costs are ₹15,00,000. **What** is the target net profit?

Reference	What's New
Target Net Profit	

Answer

Sales	= ₹85,00,000
Contribution	= 30% of Sales = $(85,00,000 \times 30\%) = ₹25,50,000$
Fixed Costs	= ₹15,00,000
Target Net Profit	= (Contribution – Fixed Costs)
	= $(25,50,000 - 15,00,000)$
	= ₹10,50,000

Q2

Ex. Book No.

Pg. No.

CMA Mat

Marketing department of an organisation estimates that 40,000 new mixers could be sold annually at a price of ₹6,000 each. To design, develop and produce these new mixers an investment of ₹40,00,00,000 would be required. The company desires a 15% return on investment (ROI).

What should be the target cost to manufacture, sell, distribute and service one mixer?

Reference	What's New
Target Cost	

Answer

Projected sales	= (40,000 mixers × ₹6,000 per mixer) = ₹ 24,00,00,000
Desired profit	= (15% of 40,00,00,000) = ₹6,00,00,000
Target Cost for 40,000 mixers	= Projected Sales – Desired Profit
	= 24,00,00,000 – 6,00,00,000
	= ₹18,00,00,000
Target cost per mixer	= (18,00,00,000 ÷ 40,000 mixers) = ₹4,500

Q3

Ex. Book No.

Pg. No.

CMA Mat

'B' manufacturing Company sells its product at ₹1,000 per unit. Due to competition, its competitors are likely to reduce the price by 15%. B wants to respond aggressively by cutting down its price by 20% and expects that the present volume of 1,50,000 units p.a. will increase to 2,00,000 p.a. B wants to earn a 10% target profit on sales.

What should be the Target cost per unit of the product?

Reference	What's New
Target Cost per unit	

Answer

Serial	Particulars	Workings	(₹)
1	Selling Price at Present	Given	1,000.00
2	Proposed Reduction	20% of 1000	200.00
3	Target selling price	(1 – 2)	800.00
4	Target profit margin	10% of 800	80.00
5	Target costs per unit	(3 – 4)	720.00

Q4

Ex. Book No.

Pg. No.

CMA Mat

CELO Company has the capacity of production of 80,000 units and presently sells 20,000 units at ₹100 each. The demand is sensitive to selling price and it has been observed that for every reduction of ₹10 in Selling Price, the demand is doubled.

Required:

- What** should be the Target Cost at full capacity, if Profit Margin on Sale is 25%?
- What** should be the Cost Reduction Scheme at full capacity if at the present level 40% of the cost is variable and Total Fixed Cost is ₹36 lakhs?
- If Rate of Return desired is 16%, **what** will be the maximum investment at full capacity?

Reference	What's New
Target Cost, Cost Reduction Scheme, Rate of Return	

Answer

a. Target Cost at Full Capacity

Projected Demand

Selling Price (₹ Per Unit)	Demand (Units)	Capacity Utilisation
100	20,000	25%
90	$(20,000 \times 2) = 40,000$	50%
80	$(40,000 \times 2) = 80,000$	100%

Selling Price at Full Capacity = ₹80.00

Target Profit = 25% on Sales = ₹20.00

Target Cost at Full Capacity = $(80 - 20) = ₹ 60.00$ per unit

b. Cost Reduction Scheme

(i) Computation of Variable Cost per unit

At the Present Capacity of 20,000 units

Selling Price = ₹100.00 per unit

Profit Margin = 25% on Sales = ₹25.00

Total Cost = $(100 - 25) = ₹ 75.00$

Variable Cost = 40% of total cost = 40% of 75 = ₹30.00

(ii) Existing Projections of Total Cost at full capacity

Total Variable Cost = $(₹30 \times 80000) = ₹24.00$ lakhs

Total Fixed Cost = ₹36.00 Lakhs

Total Cost = $(24.00 + 36.00) = 60.00$ lakhs

(iii) Target Cost = $(60 \times 80000) = 48.00$ lakhs

(iv) Cost Reduction Scheme

Cost Reduction Needed = (Existing Cost – Target Cost) = (60.00 – 48.00) = ₹ 12.00 lakhs

c. **Maximum Investment at full capacity**

(i) Target Profit at full Capacity

Sales = 80.00 × 80,000 units = ₹64.00 lakhs

Target Cost = ₹48.00 lakhs

Target Profit = (64.00 – 48.00) = ₹16.00 lakhs

(ii) Rate of Return on Investment = 16%

(iii) Minimum Investment

Investment Needed = (Target Profit ÷ Target Return on Investment)
 = (16.00 ÷ 16%) = ₹ 100.00 lakhs



Ex. Book No.

Pg. No.

CMA Mat

The operation costs of a product produced by ABC Ltd are ₹53. Presently, the company produces only 600 units p.a. to sell at ₹55 per unit due to hard competition in the market. But with existing facilities, production can be increased to 1,000 units if additional production can be sold in the market. The company accordingly introduced target costing on market research, new design for the product and changes in the process so that costs are brought down substantially and market share can be increased. The estimates for the next year are:

Target selling price	₹50 per unit
Target profit margin	10% on sales
Target volume	900 units

Required:

- (a) **Calculate** target costs per unit and target costs for the expected volume; and
 (b) **Compare** existing profit with target profit.



Reference

Targeted Cost

What's New

Answer

(a) **Statement of Target Costs**

Particulars	Per Unit (₹)	For 900 units (₹)
Target selling price	50	45,000
Less: Target profit margin (10% of sales)	5	4,500
Target costs	45	40,500

(b) **Comparative Profit Statement**

Particulars	Existing Position		Proposed Position	
	Per unit (₹)	600 units (₹)	Per unit (₹)	900 units (₹)
Sales	55	33,000	50	45,000
Less: Costs	53	31,800	45	40,500
Profit	2	1,200	5	4,500

Q6

Ex. Book No.

Pg. No.

CMA Mat

S Ltd. has sales of 2,00,000 units at a price of ₹100.00 per unit and profit of ₹70.00 Lakhs in the current year. Due to stiff competition, next year the Company has to reduce its price of product @ 3% to achieve same target volume of sales. The cost structure and profit for the current year is given as below:

Particulars	(₹ Lakhs)
Direct Material	50.00
Direct Wages	40.00
Variable Factory Overheads	15.00
Fixed Overheads including Sales & Admin Expenses	25.00
Total Cost	130.00

To achieve the Target Cost to maintain the same profit, the Company is evaluating the proposal to reduce Labour Cost and Fixed Factory Overheads. A Vendor supplying the Machine suitable for the Company's operations has offered an advanced technology Semi-Automatic Machine of ₹10 Lakhs as replacement of Old Machine worth ₹3 Lakhs. The Vendor is agreeable to take back the Old Machine at ₹1 Lakh only. The Company's policy is to charge depreciation at 15% on WDV. The Maintenance Charge of the Existing Machine is ₹1 Lakh per annum whereas there will be warranty of services free of cost for the New Machine first two years. There are 7 Supervisors whose Salary is ₹1.50 Lakhs per annum. The New Machine having Conveyor Belt is expected to help in cost cutting measures in the following ways -

- (1) Improve Productivity of workers by 10%
- (2) Cut-down Material Wastage by 5%
- (3) Elimination of services of Supervisors because of automatic facilities of the machine
- (4) Saving in Packaging Cost by ₹1 Lakhs.

Assuming Cost of Capital to be 15%, **calculate** how many Supervisors should be removed from the production activities to achieve the Target Cost.

Reference	What's New
Removal of Supervisor to achieve target Cost	

Answer

A. Targeted Cost Reduction

Targeted price Reduction = 3% of 200 lakhs = ₹6 lakhs

Targeted Cost Reduction = ₹6 lakhs

B. Net Savings on account of New Machine

1. Savings on account of the New Machine

- Reduction in wages due to Improve Productivity of workers by 10%
 $= \{40 \text{ lakhs} - [(40 \text{ lakhs} \div 110) \times 100] = (40.00 - 36.36) = ₹3.64 \text{ lakhs}$
- Cut-down Material Wastage by 5% = 5% of 50 lakhs = ₹2.50 lakhs
- Saving in Packaging Cost = ₹1.00 lakhs
- Saving in Maintenance Cost = ₹1.00 lakhs
- Total Savings = $3.64 + 2.50 + 1.00 + 1.00 = ₹8.14 \text{ lakhs}$

2. Additional Costs on account of the New Machine

- Loss in Disposal of Old Machine = $(₹3 \text{ lakhs} - ₹1 \text{ lakhs}) = ₹2.00 \text{ lakhs}$
- Difference in Depreciation = $(₹10 \text{ lakhs} - ₹3 \text{ lakhs}) \times 15\% = ₹1.05 \text{ lakhs}$
- Cost of Capital Investment = $(₹10 \text{ lakhs} \times 15\%) = ₹1.50 \text{ lakhs}$
- Total Additional Costs = $(2.00 + 1.05 + 1.50) = ₹4.55 \text{ lakhs}$

C. Supervisors to be Removed

Short Fall = $(A-B) = (6.00 - 3.59) = ₹2.41 \text{ lakhs}$

Number of Supervisors to be removed

$= (2.41 \text{ lakhs} \div 1.50 \text{ lakhs per supervisors}) = 1.61$

i.e., say 2 Supervisors

Life Cycle Costing

Q1

Ex. Book No.

Pg. No.

CMA Mat

SRM Ltd. has developed a new product 'Kent' which is about to be launched into the market and anticipates to sell 80,000 of these units at a sale price of ₹ 300 over the product's life cycle of four years. Data pertaining to product 'Kent' are as follows:

Costs of Design and Development of Moulding Dies and Other tools	₹ 10,25,000
Manufacturing costs	₹ 125 per unit
Selling costs	₹ 12,500 per year + ₹ 100 per unit
Administration costs	₹ 50,000 per year
Warranty expenses	5 replacement parts per 25 units @ ₹ 10 per part, 1 visit per 500 units (cost ₹ 500 per visit)

Required:

- Compute** the product Kent's Life Cycle Cost.
- Suppose SRM Ltd. can increase sales volume by 25% through 15% decrease in selling price, **should** SRM Ltd. choose the lower price?

Reference	What's New
Life Cycle Cost and evaluation of proposals	

Answer

- Statement showing Kent's Life Cycle Cost (80,000 units)**

Particulars	Amount (₹)
Costs of Design and Development of Moulding Dies and Other tools	10,25,000
Manufacturing costs (125 × 80,000 units)	1,00,00,000
Selling costs (₹ 100 × 80,000 units + ₹ 12,500 × 4 years)	80,50,000
Administration costs (₹ 50,000 × 4 years)	2,00,000
Warranty expenses	
Replacement costs: {(80,000 units ÷ 25 units) × 5 parts × ₹ 10}	1,60,000
Visit costs: {(80,000 units ÷ 500 units × 1 visit × ₹ 500)}	80,000
Total Cost	1,95,15,000

(ii) **Statement showing Kent's Life Cycle Cost (1,00,000 units)**

Particulars	Amount (₹)
Costs of Design and Development of Moulding Dies and Other tools	10,25,000
Manufacturing costs ($125 \times 1,00,000$ units)	1,25,00,000
Selling costs ($₹ 100 \times 1,00,000$ units + $₹ 12,500 \times 4$ years)	1,00,50,000
Administration costs ($₹ 50,000 \times 4$ years)	2,00,000
Warranty expenses	
Replacement costs: $\{(1,00,000 \text{ units} \div 25 \text{ units}) \times 5 \text{ parts} \times ₹ 10\}$	2,00,000
Visit costs: $\{(1,00,000 \text{ units} \div 500 \text{ units}) \times 1 \text{ visit} \times ₹ 500\}$	1,00,000
Total Cost	2,40,75,000

Statement showing Kent's Life Time Profit

Particulars	₹ at the level of 80,000 units	₹ at the level of 1,00,000 units
Sales	$(80,000 \times 300) = 2,40,00,000$	$(1,00,000 \times 255) = 2,55,00,000$
Total cost	1,95,15,000	2,40,75,000
Profit	44,85,000	14,25,000

Observation: Reducing the, price by 15% will decrease profit by ₹ 30,60,000/-. Therefore, SRM Ltd. should not cut the price.

Activity Based Cost Management

Q1

Ex. Book No.

Pg. No.

CMA Mat

A company manufactures 500 units of product AX. The following details are available:

Material cost to manufacture	₹ 1,50,000
Labour cost	₹ 2,65,000
Material Reordering Cost	₹ 4,500
Material Handling Cost	₹ 2,500
Material orders	35
Material movements	20

What is the Total Material cost under Activity based costing?



Reference

Activity Based Cost

What's **New**

Answer

Total Material Cost under Activity Based Costing

Serial	Particulars	(₹)
1	Material cost to manufacture	1, 50,000
2	Material Reordering Cost	4,500
3	Material Handling Cost	2,500
4	Total Material Cost-	1, 57,000

Explanatory Comment: Material Reordering Cost and Material Handling Cost are directly traced to the Total Material Cost under the system of ABC.

Q2

Ex. Book No.

Pg. No.

CMA Mat

Production overheads of XYZ Manufacturers Pvt. Ltd. for 500 units of product X are

Machine oriented activity cost: ₹ 1,35,400

Material ordering overheads: ₹ 69,570

Machine hours are 1.50 hrs per unit and No. of material orders are 6 per unit.

Raw material cost ₹ 300 per unit and labour cost ₹ 150 per unit.

What is the Total cost of X per Unit?

Reference	What's New
Activity Based Cost	

Answer

(i) **Machine Oriented Cost per Unit**

Machine oriented activity cost for 500 units = ₹ 1,35,400

Machine hours for 500 units = $1.5 \times 500 = 750$

Machine Oriented Cost per hour = $(135400 \div 750) = ₹ 180.53$

Machine Oriented Cost per Unit = $(180.53 \times 1.5) = ₹ 270.80$

(ii) **Material Ordering Cost per Unit**

Material Ordering Cost = ₹ 69,570

Material Orders per unit = 6

Material Orders for 500 units = $6 \times 500 = 3000$

Material Ordering Cost per Order = $(69570 \div 3000) = ₹ 23.19$

Material Ordering Cost per Unit = $(23.19 \times 6) = ₹ 139.14$

(iii) **Total Cost of X per Unit**

Serial	Particulars	(₹)
1	Raw Material cost	300.00
2	Labour cost	150.00
3	Machine Oriented Cost	270.80
4	Material Ordering Cost	139.14
5	Total Cost	859.94

Explanatory Comment: Costs of activities, viz. Material Orientation and Material Ordering have been computed per unit of Cost Driver.

Q3

Ex. Book No.

Pg. No.

CMA Mat

AML Ltd is engaged in the production of three types of ice-cream products viz. Coco, Strawberry & Vanilla. The Company presently sells 50,000 units of Coco @ ₹ 25 per unit, Strawberry 20,000 units @ ₹ 20 per unit and Vanilla 60,000 units @ ₹ 15 per unit. The demand is sensitive to selling price and it has been observed that every reduction of ₹ 1 per unit in selling price increases the demand for each product by 10% to the previous level. The company has the production capacity of 60,500 units of Coco, 24,200 units of Strawberry, and 72600 units of Vanilla. The company marks up 25% of the cost of product.

The management decides to apply ABC analysis. For this purpose, it identifies four activities as store support costs. The cost driver rates are as follows.

Activity	Cost Driver Rate
Ordering	₹ 800 per purchase order
Delivery	₹ 700 per delivery
Shelf Stocking	₹ 199 per hour
Customer Support and Assistance	₹ 1.10 per unit sold

The other relevant information for the products is as follows

	Coco	Strawberry	Vanilla
Direct Material p.u. (₹)	8	6	5
Direct Wages p.u. (₹)	5	4	3
No. of purchase orders	35	30	15
No. of Deliveries	112	66	48
Shelf stocking hours	130	150	160

Under the traditional costing system, store support costs are charged @ 30% of prime cost.

Required:

- Calculate** the unit cost and total cost of each product at the maximum level using traditional costing.
- Calculate** the unit cost and total cost of each product at the maximum level using activity-based Costing.

Reference	What's New
Traditional costing vs ABC	

Answer
(i) Computations under Traditional Costing

Element	Coco	Strawberry	Vanilla
Unit Cost (₹ per Unit)			
Direct Material	8	6	5
Direct Labour	5	4	3
Prime Cost	13	10	8
Store Support Costs @ 30% on PC	3.90	3.00	2.40
Total	16.90	13.00	10.40
Number of Units	60,500	24,200	72,600
Total Cost (₹)			
Direct Material	4,84,000	1,45,200	3,63,000
Direct Labour	3,02,500	96,800	2,17,800
Prime Cost	7,86,500	2,42,000	5,80,800
Store Support Costs	2,35,950	72,600	1,74,240
Total	10,22,450	3,14,600	7,55,040

(ii) Computations under Activity Based Costing
a. Computation of Store Support Costs per Unit

Activity	Coco	Strawberry	Vanilla
Ordering Cost			
Number of Purchase Orders	35	30	15
Cost @ ₹ 800 per order	28,000	24,000	12,000
Delivery Cost			
Number of Deliveries	112	66	48
Cost @ ₹ 700 per delivery	78,400	46,200	33,600
Shelf Stocking Cost			
Shelf stocking hours	130	150	160
Cost @ ₹ 199 per hour	25,870	29,850	31,840
Customer Support			
Cost Number of Units Sold	60,500	24,200	72,600
Cost @ ₹ 1.10 per unit	66,550	26,620	79,860
Total Cost (₹)	1,98,820	1,26,670	1,57,300
Number of Units	60,500	24,200	72,600
Store Support Costs per Unit (₹)	3.286	5.234	2.167

b. **Statement of Costs**

Element	Coco	Strawberry	Vanilla
Unit Cost (₹ per Unit)			
Direct Material	8	6	5
Direct Labour	5	4	3
Prime Cost	13	10	8
Store Support Costs	3.286	5.234	2.167
Total (Prime Cost + Store Support Cost)	16.286	15.234	10.167
Number of Units	60,500	24,200	72,600
Total Cost (₹)			
Direct Material	4,84,000	1,45,200	3,63,000
Direct Labour	3,02,500	96,800	2,17,800
Prime Cost	7,86,500	2,42,000	5,80,800
Store Support Cost	1,98,803	1,26,663	1,57,324
Total	9,85,303	3,68,663	7,38,124

Explanatory Comments: The following statement draws comparison of unit costs and total costs under the traditional and ABC systems

Statement of Comparison

Particulars	Coco	Strawberry	Vanilla
Unit Costs			
Under Traditional System	16.90	13.00	10.40
Under ABC System	16.286	15.234	10.167
Total Costs			
Under Traditional System	10,22,450	3,14,600	7,55,040
Under ABC System	9,85,303	3,68,663	7,38,124

The differences between the cost computations under the two systems are obvious and self-explanatory. Prime cost remaining the same, the differences arose on account of bettering the accuracy in the distribution of store support costs under ABC.

Standard Costing

Q1

Ex. Book No.

Pg. No.

CMA Mat

Find Sales Price Planning Variance and Sales Price Operating Variance and **Comment**.

Product	Budget Sale Price (₹)	Revised Sales Price (₹)	Actual Sales Price (₹)	Budget Sale Units	Actual Sale Units
P1	20	15	18	2000	1900
P2	25	30	25	2000	2300
P3	25	27	28	2000	2000

Reference

What's New

Planning and Operating Variance - Sales

Answer

Sales Price Planning Variance = Actual Sale Units (Revised Sale Price – Budget Sale Price)

Product	Actual Sale Units	Revised Sales Price - Budget Sale Price	Sales Price Planning Variance	Nature
P1	1900	(15 - 20) = -5	1900 × -5 = -9500	Adverse
P2	2300	(30 - 25) = 5	2300 × 5 = 11500	Favourable
P3	2000	(27 - 25) = 2	2000 × 2 = 4000	Favourable
Total			6000	Favourable

Sales Price Operating Variance = Actual Sale Units (Actual Sale Price – Revised Sale Price)

Product	Actual Sale Units	Actual Sales Price - Revised Sale Price	Sales Price Planning Variance	Nature
P1	1900	(18 - 15) = 3	1900 × 3 = 5700	Favourable
P2	2300	(25 - 30) = -5	2300 × -5 = -11500	Adverse
P3	2000	(28 - 27) = 1	2000 × 1 = 2000	Favourable
Total			-3800	Adverse

Comments :

There are a number of factors causing a change in the product costs to change. These factors can be planned or unplanned events. A change in the cost of any product will compel the management to change the selling price. The budgeted or standard selling price will need to be revised; the difference in the selling price for actual number of units sold will then give the variance in the sale price planning.

Causes for sales price planning variance include:

- Change in the raw materials prices, compelling management to revise sale prices significantly
- Inefficient operations or unskilled labour, causes the product prices to rise or conversely efficient production to control the costs
- Market competition, with fewer competitors likely to achieve favourable sale price variance and intense competition to result in adverse

Competitive and attractive sales price for a product can be the difference between a successful and failed product launch. Also, for market leaders, to maintain the market share it is important to keep the variances in check. A well-planned sale price can help a company gain competitive advantage in the market.

Achieving the revised sales price is the responsibility of the operational managers, however, some controllable or unforeseen factors can still cause the difference in the actual sale prices. The deviation in the revised sales price and

the actual sales price is operational sales price variance. Some external factors, such as new entrants in the market can compel the management to sell the products at lower than revised prices. Some operational inefficiency such as unavailability of important raw material components can cause an increase the production costs.

Here are some of the factors that contribute to budget revisions and sales price operating variances:

- The threat of new competitors compelling management to lower the sales prices
- Unavailability of input components or increase in the raw material prices
- Inefficiency in operations leading to wastes and idle labour hours
- Seasonal product demand or lack of competition may tempt the management to increase sales prices
- Operational efficiencies and economies of scale can also lead to favourable sales price variances

Revising the sales price budgets is inevitable for any management; however, closing the revised and actual sale price gaps should be the real focus. A careful interpretation of the drivers behind the variances can help achieve the desired goals. Sales price largely depend on the input components, securing long term supplier contracts, bulk buying discounts, and regular supplies can help reduce costs. Efficient operations and labour can also contribute towards lower product costs that can help achieving favourable sales price variance.

Q2

Ex. Book No.

Pg. No.

CMA Mat

Green Chemicals produces agriculture fertilizers with following information provided. The management analysed past data and set the budgeted rates as following:

Standard hours per unit of product	1.1
Standard direct labour rate per hour	₹ 18.50
Standard usage of material per unit	1.2 kg per unit
Standard price of material per unit	₹ 70

During production times, the management revised the budgets with updated information as:

Revised price of material per unit	₹ 71
Revised labour rate per hour	₹ 20 per hour
Revised hours per unit of product	1.05
Revised usage of material per unit	1.175 kg per unit

After the production period the company recorded the following actual results:

Actual Production	15,400
Raw Material usage	16,555 KGs
Actual cost of raw material	₹ 11,91,960 or ₹ 72 per KG
Actual labour costs	16,632 hours and ₹ 3,24,324 or ₹ 19.50 per hour.

Calculate the Raw Material and Direct labour Planning & Operational Variances.

Reference Material and Labour Variance	What's New Planning & Operational Variances

Answer**(i) Calculation of Raw Material Price Variances**

Raw Material Price Variance

$$= \text{Actual Quantity (Standard Price - Actual Price)}$$

$$= 16555 (70 - 72) = ₹ 33,110 \text{ Adverse}$$

Raw Material Price Planning variance

$$= \text{Actual Quantity (Standard Price - Revised Price)}$$

$$= 16555 (70 - 71) = ₹ 16,555 \text{ Adverse}$$

Raw Material Price Operational variance

$$= \text{Actual Quantity (Revised Price – Actual Price)}$$

$$= 16555 (71 - 70) = ₹ 16,555 \text{ Adverse}$$

Check

Raw Material Price Variance

$$= \text{Sum of Planning Variance and Operational Variance}$$

$$= (₹ 16,555 \text{ Adverse} + ₹ 16,555 \text{ Adverse})$$

$$= ₹ 33,110 \text{ Adverse}$$

(ii) Calculation of Raw Material Usage Variances

Raw Material Usage Variance

$$= \text{Standard Price (Standard Quantity – Actual Quantity)}$$

$$= 70 [(15400 \times 1.2) - 16555]$$

$$= 70 (18480 - 16555) = ₹ 1,34,750 \text{ Favourable}$$

Raw Material Usage Planning Variance

$$= \text{Standard Price (Standard Quantity – Revised Quantity)}$$

$$= 70 [(15400 \times 1.2) - [(15400 \times 1.175)]]$$

$$= 70 (18480 - 18095) = ₹ 26,950 \text{ Favourable}$$

Raw Material Usage Operational Variance

$$= \text{Standard Price (Revised Quantity – Actual Quantity)}$$

$$= 70 [(15400 \times 1.175) - 16555]$$

$$= 70 (18095 - 16555) = ₹ 1,07,800 \text{ Favourable}$$

Check

Raw Material Usage Variance

$$= \text{Sum of Planning Variance and Operational Variance}$$

$$= (₹ 26,950 \text{ Favourable} + ₹ 1,07,800 \text{ Favourable})$$

$$= ₹ 1,34,750 \text{ Favourable}$$

Raw Material Variance = Standard Cost – Actual Cost

$$= (15400 \text{ units} \times 1.2 \text{ kg} \times ₹ 70) - (11,91,960)$$

$$= (12,93,600 - 11,91,960) = ₹ 1,01,640 \text{ Favourable}$$

Or

Raw Material Variance

$$= \text{Sum of Price Variance and Usage Variance}$$

$$= (₹ 33,110 \text{ Adverse} + ₹ 1,34,750 \text{ Favourable})$$

$$= ₹ 1,01,640 \text{ Favourable}$$

(iii) Calculation of Labour Rate Variances

Direct Labour Rate Variance

$$= \text{Actual Hours (Standard Rate - Actual Rate)}$$

$$= 16632 (18.50 - 19.50) = ₹ 16,632 \text{ Adverse}$$

Direct Labour Rate Planning variance

$$= \text{Actual Hours (Standard Rate - Revised Rate)}$$

$$= 16632 (18.50 - 20.00) = ₹ 24,948 \text{ Adverse}$$

Direct Labour Rate operational variance

$$= \text{Actual Hours (Revised Rate - Actual Rate)}$$

$$= 16632 (20.00 - 19.50) = ₹ 8,316 \text{ Favourable}$$

Check

Direct Labour Rate Variance

$$= \text{Sum of Planning Variance and Operational Variance}$$

$$= (₹ 24,948 \text{ Adverse} + ₹ 8,316 \text{ Favourable}) = ₹ 16,632 \text{ Adverse}$$

(iv) Calculation of Labour Efficiency Variances

Direct Labour Efficiency Variance

$$= \text{Standard Rate (Standard Hours - Actual Hours)}$$

$$= 18.50 [(15400 \times 1.1) - 16632]$$

$$= 18.50 (16940 - 16632) = ₹ 5698 \text{ Favourable}$$

Direct labour Efficiency Planning Variance

$$= \text{Standard Rate (Standard Hours - Revised Hours)}$$

$$= 18.50 [(15400 \times 1.1) - [(15400 \times .05)]]$$

$$= 18.50 (16940 - 16170) = ₹ 14,245 \text{ Favourable}$$

Direct Labour Efficiency Operational Variance

$$= \text{Standard Rate (Revised Hours - Actual Hours)}$$

$$= 18.50 [(15400 \times 1.05) - 16632]$$

$$= 18.50 (16170 - 16632) = ₹ 8547 \text{ Adverse}$$

Check

Direct Labour Efficiency Variance

$$= \text{Sum of Planning Variance and Operational Variance}$$

$$= (₹ 14,245 \text{ Favourable} + ₹ 8,547 \text{ Adverse})$$

$$= ₹ 5698 \text{ Favourable}$$

Direct Labour Variance

$$= \text{Standard Cost} - \text{Actual Cost}$$

$$= (15400 \text{ units} \times 1.1 \text{ hours} \times ₹ 18.50) - (16632 \text{ hours} \times 19.50)$$

$$= (3,13,390 - 3,24,324) = ₹ 10,934 \text{ Adverse}$$

Or

Direct Labour Variance

$$= \text{Sum of Rate Variance and Efficiency Variance}$$

$$= (₹ 16,632 \text{ Adverse} + ₹ 5698 \text{ Favourable}) = ₹ 10,934 \text{ Adverse}$$

(v) Summary

The summary of variances may be presented in a tabular form as follows:

Serial	Description	Planning	Operational	Total
1	Raw Material			
a	Raw Material Price Variance	16,555 (A)	16,555 (A)	33,110 (A)
b	Raw Material Usage Variance	26,950 (F)	1,07,800 (F)	1,34,750 (F)
c	Sub Total	10,395 (F)	91,245 (F)	1,01,640 (F)
2	Direct Labour			
a	Direct Labour Rate Variance	24,948 (A)	8,316 (F)	16,632 (A)
b	Direct Labour Efficiency Variance	14,245 (F)	8547 (A)	5,698 (F)
c	Sub Total	10,703 (A)	231 (A)	10,934 (A)
3	Total	308 (A)	91,014 (F)	90,706 (F)

(vi) Observations

- The planning variance consists of ₹ 10,395 (Favourable) with respect to Raw Material and ₹ 10,703 (Adverse) in relation to Direct Labour, both together aggregating to ₹ 308 (Adverse). Even though the variance looks smaller at the aggregate level, it is substantial at the element level and hence needs further probing as to the causes and effects.
- The operational variance consists of ₹ 91,245 (Favourable) with respect to Raw Material and ₹ 231 (Adverse) in relation to Direct Labour, both together aggregating to ₹ 91,014 (Favourable). Favourable Direct Labour Operational Variance of ₹ 8,316 is neutralized by Adverse Direct Labour Efficiency Operational Variance of ₹ 8,547. Therefore, all the elements of variance (material as also labour) warrant further probing as to the causes and effects.


Q3

Ex. Book No.

Pg. No.

CMA Mat

The following is a flexible budget of FB Co. Ltd. For a production department.

	Level of Activity		
Direct Labour Hours	4000	5000	6000
Number of Units	8000	10000	12000
Fixed Overhead (₹)	5000	5000	5000
Variable Overhead (₹)	800	1000	1200
Total Overheads (₹)	5800	6000	6200

Normal Level of activity was 5000 direct labour hours. Actual Results were:

Direct Labour hours – 4800

Variable Overhead – ₹ 900

Output in Units – 10400

Fixed Overhead – ₹ 5100

Compute Fixed overhead cost, volume and expenditure variances, variable overhead cost, efficiency and expenditure variances, efficiency, capacity and activity ratios.

 **Reference**

Fixed overhead Variances and Variable Overhead Variance & It's ratios

What's **New**

Answer

Step 1: Initial Workings

Normal level of activity has been expressed in terms of direct labour hours. Accordingly:

Standard Labour Hours per unit of Output = $(10,000 \div 5,000) = 2$ hours per unit

Standard Labour Hours for Actual Output = $(10,400 \div 2) = 5,200$ hours

Standard Rate of Recovery for FOH = $(5,000 \div 5,000) = ₹1$ per labour hour

Standard Rate of Recovery for VOH = $(1,000 \div 5,000) = ₹0.20$ per labour hour

Step 2: FOH Variances

Description	Formula	Workings	Variance
FOH Cost Variance	SRSH – AOH	$(1 \times 5200) - 5100 = 100$ (F)	₹ 100 (F)
FOH Volume Variance	SR (SH – BH)	$1 \times (5200 - 5000) = 200$ (F)	₹ 200 (F)
FOH Expenditure Variance	SRBH – AOH	$(1 \times 5000) - 5100 = 100$ (A)	₹ 100 (A)

Step 3: VOH Variances

Description	Formula	Workings	Variance
VOH Cost Variance	SRSH – AOH	$(0.2 \times 5200) - 900 = 140 \text{ (F)}$	₹ 140 (F)
VOH Volume Variance	SR (SH – BH)	$0.2 \times (5200 - 4800) = 80 \text{ (F)}$	₹ 80 (F)
VOH Expenditure Variance	SRAH – AOH	$(0.2 \times 4800) - 900 = 60 \text{ (F)}$	₹ 60 (F)

Step 4: Ratios

Description	Formula	Workings	Ratio
Efficiency Ratio	SH ÷ AH	$5200 \div 4800$	108.33%
Capacity Ratio	AH ÷ BH	$4800 \div 5000$	96 %
Activity Ratio	SH ÷ BH	$5200 \div 5000$	104%

SJC Institute

Linear Programming

Q1

Ex. Book No.

Pg. No.

CMA Mat

A shopkeeper deals in two items – Wall hangings and Artificial plants. He has ₹ 50000 to invest and a space to store 100 pieces at the most. Costs of Wall hangings and Artificial plants are respectively ₹ 450 and ₹ 200 each. He can sell a Wall hanging at a profit of ₹ 80 and an Artificial plant at a profit of ₹ 37. Assuming that he can sell all the items that he buys, **formulate a Linear Programming problem** in order to maximize his profit.

Reference

Maximisation of profit

What's New

Answer

Let x_1 be the number of Wall hangings and x_2 be the number of Artificial plants that the dealer buys and sells. So the Decision Variables in this case are x_1 and x_2 respectively.

Total Profit of the shopkeeper can be expressed as $Z = 80x_1 + 37x_2$ and this is the Objective Function.

Similarly Total Cost of buying the items can be given as $(450x_1 + 200x_2)$ which should not exceed the investment capacity of the shopkeeper i.e ₹ 50000. It can be expressed as $450x_1 + 200x_2 \leq 50000$

Again there is a space limitation and the shopkeeper can store at the most 100 pieces. We can express this as $x_1 + x_2 \leq 100$

Moreover x_1 and x_2 , being physical quantities, should be non-negative. So $x_1 \geq 0$ and $x_2 \geq 0$

Thus the **mathematical formulation of the given LPP is –**

Maximize $Z = 80x_1 + 37x_2$

Subject to the constraints

$450x_1 + 200x_2 \leq 50000$

$x_1 + x_2 \leq 100$

$x_1 \geq 0$ and $x_2 \geq 0$



Ex. Book No.

Pg. No.

CMA Mat

A dealer of cement has two warehouses M and N with stocks of 30000 and 20000 bags of cement respectively. Three customers A, B and C have placed order on the dealer for 15000, 20000 and 15000 bags respectively. Costs of transportation per 1000 bags of cement from different warehouses to different customers are given below.

Transportation Cost (₹ ₹00) per 1000 bags			
To	A	B	C
From			
M	40	20	20
N	20	60	40

The dealer wants to find how to fulfill the orders so that the transportation cost is minimum. **Formulate** the problem.

Reference

Formulation of LPP

What's New

Answer

As transportation costs are given per 1000 bags, we assume 1 unit = 1000 bags

Let Warehouse M supplies x_1 units to A and x_2 units to B. As the stock of M is 30000 bags or 30 units, so C gets $(30 - x_1 - x_2)$ units from M.

Total requirement of A is 15000 bags or 15 units. Of this x_1 is supplied from M. Thus remaining $(15 - x_1)$ units is to be supplied from N.

Similarly B gets $(20 - x_2)$ units from N and C gets $[15 - (30 - x_1 - x_2)] = x_1 + x_2 - 15$ units from N.

Using the supplied values of Transportation Cost per unit we express Total Transportation Cost as -

$$Z = 4000x_1 + 2000x_2 + 2000(30 - x_1 - x_2) + 2000(15 - x_1) + 6000(20 - x_2) + 4000(x_1 + x_2 - 15)$$

$$\text{Or, } Z = 4000x_1 - 2000x_2 + 150000$$

As the problem deals with units of cement bags, each of the units mentioned above should be non-negative. Hence the constraints are -

$$x_1 \geq 0, x_2 \geq 0, 30 - x_1 - x_2 \geq 0 \text{ Or, } x_1 + x_2 \leq 30, 15 - x_1 \geq 0 \text{ Or, } x_1 \leq 15, 20 - x_2 \geq 0 \text{ Or, } x_2 \leq 20$$

$$x_1 + x_2 - 15 \geq 0 \text{ Or, } x_1 + x_2 \geq 15$$

Thus the mathematical formulation of the given LPP is -

$$\text{Minimize } Z = 4000x_1 - 2000x_2 + 150000$$

Subject to the constraints

$$x_1 + x_2 \leq 30$$

$$x_1 + x_2 \geq 15$$

$$x_1 \leq 15$$

$$x_2 \leq 20$$

$$x_1 \geq 0, x_2 \geq 0$$

Q3

Ex. Book No.

Pg. No.

CMA Mat

One kind of cake requires 200 grams of flour and 25 grams of fat and another kind of cake requires 100 grams of flour and 50 grams of fat. Find the maximum number of cakes that can be made from 5 kgs. of flour and 1 kg. of fat assuming there is no shortage of other ingredients required for making cakes.

Formulate LPP based on the information given and solve **graphically**.

Reference

Formulation of LPP and Solution

What's New

Graphically

Answer

Let x and y be the number of cakes of the first and second kinds respectively. Thus total number of cakes can be expressed as $Z = x + y$

Limitations on the availability of flour is expressed as $200x + 100y \leq 5000$ Or, $2x + y \leq 50$

Also limitations on the availability of fat is expressed as $25x + 50y \leq 1000$ Or, $x + 2y \leq 40$

As both x and y are physical quantities, they have to be non-negative i.e $x \geq 0$ and $y \geq 0$

So the mathematical formulation of the given LPP is –

Maximize $Z = x + y$

Subject to the constraints

$$2x + y \leq 50, x + 2y \leq 40, x \geq 0 \text{ and } y \geq 0$$

For solving the problem graphically first of all we need to get the Feasible Region where all the above and the space constraints are satisfied simultaneously. To get that we have to plot the constraint lines as explained below.

Converting the first constraint into an equation we get $2x + y = 50$ Or, $2x/50 + y/50 = 1$

$$\text{Or, } x/25 + y/50 = 1$$

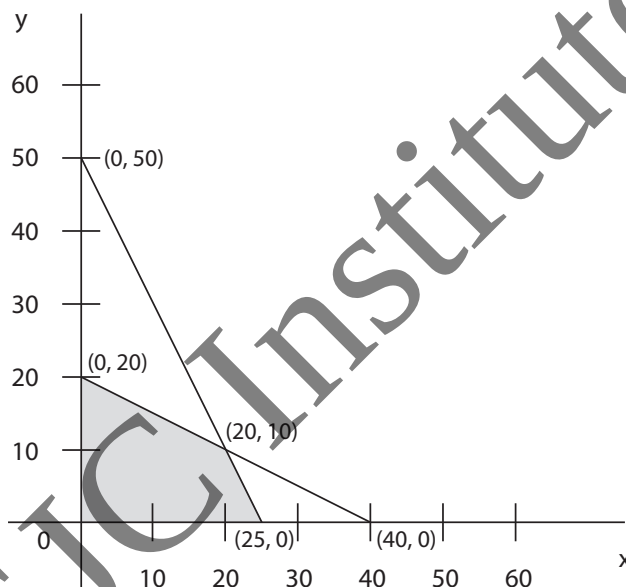
This is a straight line passing through the points (25,0) and (0,50). These points are plotted on a graph and subsequently joined to get the constraint line. The space below this line represents the constraint $2x + y \leq 50$

Similarly the second constraint is converted into the equation $x + 2y = 40$ Or, $x/40 + y/20 = 1$

This is also a straight line passing through the points (40,0) and (0,20). Points are plotted on the same graph and subsequently joined to get the line for the second constraint. The space below this line represents the constraint $x + 2y \leq 40$.

The lines for the other two constraints $x \geq 0$ and $y \geq 0$ i.e $x = 0$ and $y = 0$ are the two axes and the space above x axis denotes the region of $y \geq 0$ as well as the space to the right of y axis denotes the region of $x \geq 0$

The region common to all the above mentioned spaces denotes the Feasible Region. It is shown as the shaded area in the graph below.



Coordinates of the corner points of the Feasible Region are marked in the diagram above.

[It can be mentioned that the points (25,0) and (0,20) are already obtained and plotted to draw the constraint lines. But the coordinates of the point (20,10) have to be obtained either directly from the graph paper or by solving the simultaneous equations of the constraint lines i.e $2x + y = 50$ and $x + 2y = 40$. Solution of simultaneous equations is a preferred option because for the problems where the coordinates are fraction, the graph will not give a perfect value.]

Now the value of the Objective Function is computed at each of the Corner Points and shown in the table below.

Coordinates of the Corner Point	Value of the Objective Function ($Z = x + y$)
(25,0)	$25 + 0 = 25$
(20,10)	$20 + 10 = 30$
(0,20)	$0 + 20 = 20$

It is clear from the above values that Z is maximum at the point (20,10)

Thus the solution of the LPP is given as $Z_{\max} = 30$ and the corresponding values of the Decision Variables are x_1

$= 20$ and $x_2 = 10$. In other words, the maximum number of cakes to be made is 30 and this is achieved by making 20 nos. of the first type and 10 nos. of the second type.

Q4

Ex. Book No.

Pg. No.

CMA Mat

Mr. Lal is on a low cholesterol diet. During lunch at the office canteen he always chooses between two particular types of meal – Type A and Type B. The table below lists the amount of protein, carbohydrates and vitamins each meal provides along with the amount of cholesterol (which he is trying to minimize). He needs at least 200 grams of protein, 960 grams of carbohydrates and 40 grams of vitamins for lunch each month. Over this time period, how many days should he have Type A meal and how many days the Type B meal so that he gets adequate amount of protein, carbohydrates and vitamins and at the same time minimizes his cholesterol intake? Use Graphical Method.

	Type A meal	Type B meal
Protein (Grams)	8	16
Carbohydrates (Grams)	60	40
Vitamins (Grams)	2	2
Cholesterol (Miligrams)	60	50

Reference

Formulation and Solution of LPP

What's **New**

Graphically

Answer

Let, x = No. of days Mr. Lal will take Type A meal & y = No. of days Mr. Lal will take Type B meal

Since the goal is to minimize Mr. Lal's cholesterol intake, the Objective Function should represent the total cholesterol provided by both the meals.

So the Objective Function is $Z = 60x + 50y$

The constraints are given as follows –

$8x + 16y \geq 200$ (Constraint associated with the total protein provided by the two types of meals)

Or, $x + 2y \geq 25$

$60x + 40y \geq 960$ (Constraint associated with the total carbohydrates provided by the two types of meals)

Or. $3x + 2y \geq 48$

$2x + 2y \geq 40$ (Constraint associated with the total vitamins provided by the two types of meals)

Or, $x + y \geq 20$

Also x and y being number of days cannot be negative i.e $x \geq 0$ and $y \geq 0$

So the formulated LPP can be stated as –

Minimize $Z = 60x + 50y$

Subject to the constraints

$x + 2y \geq 25$

$3x + 2y \geq 48$

$x + y \geq 20$

$x \geq 0$ and $y \geq 0$

To find the feasible region, first of all the straight lines corresponding to the above constraints are drawn using the method followed in the previous illustration.

$x + 2y = 25$ Or, $x/25 + y/12.5 = 1$ is the first constraint line and it passes through (25,0) and (0,12.5)

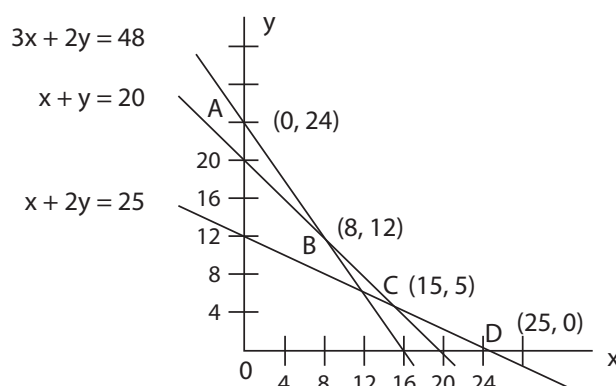
$3x + 2y = 48$ Or, $x/16 + y/24 = 1$ is the second constraint line which passes through (16,0) and (0,24)

$x + y = 20$ Or, $x/20 + y/20 = 1$ is the third constraint line and it passes through (20,0) and (0,20)

$x = 0$ is the axis of y and $y = 0$ is the axis of x

Now the constraint inequalities are graphed and the common region of the same is shaded as shown in the diagram below. It can be mentioned that the region of feasibility in this case is unbounded on the upper side. But that is not a matter of concern because the problem deals with minimization of the Objective Function which is confined to the corner points of the lower boundary of the envelope. As per the diagram, such corner points are A, B, C and D. Of these points coordinates of A and D are directly available from the graph because they lie on the axes. Coordinates of C and D can also be obtained from the graph. But it is suggested to get those by solving the simultaneous equations.

Coordinates of B is obtained by solving the equations $3x + 2y = 48$ and $x + y = 20$ and those of C is obtained by solving $x + y = 20$ and $x + 2y = 25$



Now the value of the Objective Function is computed at each of the corner points and shown in the table below.

Coordinates of the Corner Point	Value of the Objective Function ($Z = 60x + 50y$)
A (0,24)	$60.0 + 50.24 = 1200$
B (8,12)	$60.8 + 50.12 = 1080$
C (15,5)	$60.15 + 50.5 = 1150$
D (25,0)	$60.25 + 50.0 = 1500$

It is clear from the table above that the value of the Objective Function is minimum at B (8, 12).

Thus the solution of the LPP is given as $Z_{\min} = 1080$ miligrams and the corresponding values of the decision variables are $x = 8$ and $y = 12$

Hence Mr. Lal should take Type A meal for 8 days and Type B for 12 days to intake least cholesterol.

Q5

Ex. Book No.

Pg. No.

CMA Mat

Solve graphically the following LPP –

Maximize $M = 50x_1 + 60x_2$

subject to the constraints $2x_1 + x_2 \leq 300$

$3x_1 + 4x_2 \leq 509$

$4x_1 + 7x_2 \leq 812$

$x_1 \geq 0, x_2 \geq 0$



Reference

Formulation and Solution of LPP

What's **New**

Graphically

Answer

Decision variables are x_1 and x_2 & the Objective function is $M = 50x_1 + 60x_2$

1st Constraint is $2x_1 + x_2 \leq 300$. Corresponding line is $L1 : 2x_1 + x_2 = 300$ Or, $x_1/150 + x_2/300 = 1$

2nd Constraint is $3x_1 + 4x_2 \leq 509$. Corresponding line is $L2 : 3x_1 + 4x_2 = 509$ Or, $x_1/169.67 + x_2/127.25 = 1$

3rd Constraint is $4x_1 + 7x_2 \leq 812$. Corresponding line is $L3 : 4x_1 + 7x_2 = 812$ Or, $x_1/203 + x_2/116 = 1$

4th Constraint is $x_1 \geq 0$. Corresponding line is $x_1 = 0$ & this the horizontal axis

5th Constraint is $x_2 \geq 0$. Corresponding line is $x_2 = 0$ & this is the vertical axis.

These constraints are graphed by following the method explained in the above illustrations. Subsequently the feasible region is obtained and shaded as shown in the figure below. This is

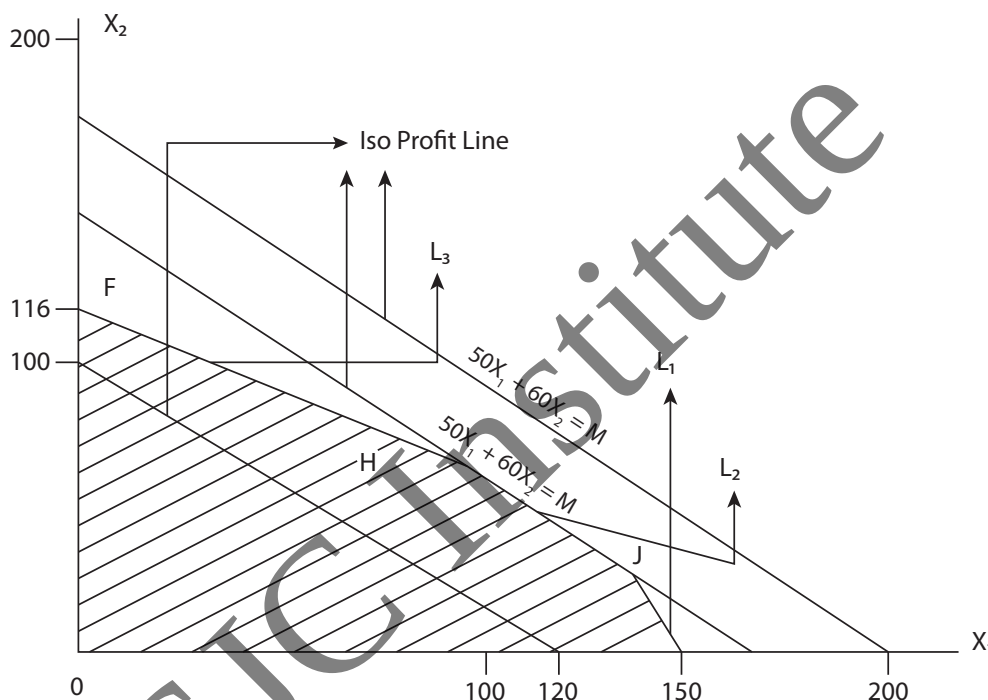
a convex polygon having vertices at O, F, H, J and C. Coordinates of O, F & C are easily obtained from the graph and they are respectively (0,0), (0,116) & (150,0) Coordinates of H and J are obtained by solving the simultaneous equations. H is the point of intersection of L_2 and L_3 & J is the point of intersection of L_1 and L_2

For finding coordinates of J we have $2x_1 + x_2 = 300$ — (1) and $3x_1 + 4x_2 = 509$ — (2)

Substituting $x_2 = 300 - 2x_1$ from (1) into (2) we have $3x_1 + 4(300 - 2x_1) = 509$ Or, $5x_1 = 691$ Or, $x_1 = 138.2$

Substituting $x_1 = 138.2$ in (1) we have $2 \times 138.2 + x_2 = 300$ Or, $x_2 = 23.6$. So J = (138.2, 23.6)

Similarly coordinates of H can be calculated.



The objective function $M = 50x_1 + 60x_2$ represents a family of straight lines with slope = - 5/6. Some of these lines will intersect with the feasible region and contain many feasible solutions while the others will not intersect and contain no feasible solution. Our aim is to find out that line of the family which intersects with the feasible region and reach farthest out from the origin (as this is a problem of maximization. For minimization it is just the reverse) Farthest is the line from the origin, greater is the value of M.

From the diagram it is clear that J is one such point farthest from the origin and lying on $50x_1 + 60x_2 = M$. Also coordinates of J are (138.2, 23.6).

At this point the value of the Objective Function is $50 \times 138.2 + 60 \times 23.6 = 8326$ and this is the required maximum value.

So the solution to the LPP is $M_{\max} = 8326$ and $x_1 = 138.2$ & $x_2 = 23.6$

[Note – In the above illustration the solution is unique because there is only one point J on the feasible region through which the Iso-profit line is passing. But there can be situations where the Iso-profit (or Iso-cost, as the case may be) line or the Objective Function line has the same

slope as one of the constraint lines. In that case the Iso- profit (or the Iso-cost) line will coincide with one of the outer boundary lines of the Feasible Region and infinite number of optimal solutions will result. This is a special case of LPP called **Multiple Optima** situation.]

Q6

Ex. Book No.

Pg. No.

CMA Mat

Sri Lanka, the third largest tea producing country has a production share of 9% of the international market and one of the world's leading exporters with a share of 19% of the global demand. Thus tea industry is crucial to enhance their economic competitiveness in the world market. The nature of the highly competitive global market has made scientific and reasonable production management increasingly important for tea companies to differentiate themselves from competitors. In order to enhance their competitive position, Sri Lankan tea manufacturers are giving serious thought to use optimization techniques like Linear Programming to find their best product mix to achieve maximization of profit. Dulwan Tea Company, established in 1974 is one of the leading tea exporters of the country. They use their own leaves which grow in their tea plantations. More than 2500 varieties of flavored and non-flavored tea products are produced and globally exported by the company. This brand is available in more than 90 countries in the world including UK, Poland, Canada, South Africa, Australia and New Zealand. Therefore how to optimize the production process yielding maximum profit is a critical and challenging task in front of the decision makers of Dulwan. After lot of deliberations among themselves, the management of Dulwan has decided to hire a Cost and Management consultant.

Accordingly they hired Mr. Kuppuswamy, a resident of Jafna, Sri Lanka and a well-known consultant of the island. In his first visit to the company the management explained to him the requirements and Mr. Kuppuswamy technically phrased the objective of the work as follows.

- To formulate a mathematical model that would suggest a viable product mix to ensure maximum profit of the company as well as evaluating performance of the proposed product mix.
- To highlight the peculiarities of using linear programming technique at a single operating procedure and prove that despite the obstacles, the application of the technique in determining the product mix enables Dulwan Tea Company to be more profitable than the otherwise.

Thereafter a team is formed from the existing employees of the company and under the guidance of Mr. Kuppuswamy they started working to formulate the problem as a Linear Programming model. Since the company is dealing with huge varieties of tea product, everybody could realize that solving such LPP manually is impossible. So it is decided to purchase a suitable software for the purpose and Mr. Kuppuswamy is requested to get at least three quotes from renowned global software companies. When the process is on, all of a sudden new opportunities open and the company decided to bid for supplying few of its very premium quality tea to the European market. But the management was not very sure as to which quality of tea they should try to sell so that the objective of profit maximization is fulfilled. Once again Mr. Kuppuswamy was approached and this time he decided to find the best product mix by solving the problem manually (as variety of very premium quality tea was not much and also the decision regarding which software to purchase not finalized).

During solution of the problem manually, at one stage the following Simplex Table is obtained

C_B	Product Mix	Quantity	x_1	x_2	x_3	s_1	s_2	s_3	A_1
2	x_1	4	1	2	1/2	0	0	1/4	0
0	s_2	12	0	0	-1	0	1	-1/2	0
0	s_1	12	0	6	0	1	0	1	-1
	C_j		2	4	1	0	0	0	-M
	Z_j	8	2	4	1	0	0	1/2	0
	$C_j - Z_j$		0	0	0	0	0	-1/2	-M

Answer the following questions, with proper explanation, related to the Simplex Table above.

- How many** varieties of very Premium quality tea are considered in the problem?
- Is** the solution given in the Table above Optimal?
- What** is the Objective Function?
- Is** there any alternate solution to the problem?
- Is** the solution feasible?
- What** is the optimum product mix and the maximum profit.
- If any alternate solution **is** possible then find it.

Reference	What's New
Simplex Method	

Answer

- From the table it is clear that there are three decision variables x_1 , x_2 and x_3 . So 3 varieties of very Premium quality tea are considered.
- The given table has either zero or negative entries in the Index Row or $(C_j - Z_j)$ Row. Also this is a problem of maximization. So the criteria of optimality is satisfied here. Hence the solution is optimal.
- C_j row represents the contribution per unit of different variables. Thus from the table, respective contributions of the Decision variables x_1 , x_2 and x_3 are 2, 4 and 1. So the Objective Function is $Z = 2x_1 + 4x_2 + x_3$.
- In the Optimal Table only decision variable x_1 is present in the Basis. Thus the other two decision variables x_2 and x_3 are Non-basic variables and corresponding to both the entries in the Index Row or $(C_j - Z_j)$ Row are zero. This indicates the presence of multiple optimum solutions. Hence there exists alternate solution to the problem.

- (v) As no Artificial Variable is present as Basic Variable in the final table, infeasibility of the solution is not applicable. Hence the solution is feasible.
- (vi) Optimum product mix is as follows –
- Quantity of Type 1 quality premium tea to be produced = $x_1 = 4$ quantity units and no units for the other two varieties of tea and. Maximum profit = 8 money units
- (vii) From the answer given in (v) above, we can say that two alternate solutions are possible – either that will contain x_2 or x_3 . So a new solution is found as below by arbitrarily choosing the column of x_2 as the Key Column.

Table -1 showing Optimal Solution

C_B	Product Mix	Quantity	x_1	x_2	x_3	s_1	s_2	s_3	A_1	Ratio	
2	x_1	4	1	2*	1/2	0	0	1/4	0	4/2 = 2	Key Row
0	s_2	12	0	0	-1	0	1	-1/2	0	12/0 = -	
0	s_1	12	0	6	0	1	0	1	-1	12/6 = 2	
	C_j		2	4	1	0	0	0	-M		
	Z_j	8	2	4	1	0	0	1/2	0		
	$C_j - Z_j$		0	0	0	0	0	-1/2	-M		
				Key Column							

Minimum Ratio column has two tied ratios corresponding to two basic variables x_1 and s_1 . We arbitrarily choose the row corresponding to x_1 as the Key Row. So the key element is 1 and the cell for that is shaded as shown. Hence in the next table x_2 enters and x_1 departs.

Further calculations are done by following the methodology explained in Illustration 6 before and shown in the next table below.

Table 2 showing Alternative Solution

C_B	Product Mix	Quantity	x_1	x_2	x_3	s_1	s_2	s_3	A_1	Ratio
4	x_2	2	1/2	1	1/4	0	0	1/8	0	
0	s_2	12	0	0	-1	0	1	-1/2	0	
0	s_1	0	-3	0	-3/2	1	0	1/4	-1	
	C_j		2	4	1	0	0	0	-M	
	Z_j	8	2	4	1	0	0	1/2	0	
	$C_j - Z_j$		0	0	0	0	0	-1/2	-M	

As all the entries in Index Row or $(C_j - Z_j)$ Row are zero or negative, the solution is optimal. Here also the maximum value of Objective Function is 8 money units and the optimum product mix is -

Quantity of Type 2 quality premium tea to be produced = $x_2 = 2$ quantity units and no units for the other two varieties.

Q7

CMA Mat

Obtain the Dual of the following Primal Problem.

Minimize $Z = 6000x_1 + 4000x_2$

subject to the constraints

$4x_1 + x_2 \geq 12$

$9x_1 + x_2 \geq 20$

$7x_1 + 3x_2 \geq 18,$

$10x_1 + 40x_2 \geq 40$ and $x_1 \text{ \& } x_2 \geq 0$

Reference

Formation of Dual

What's **New**

Minimisation

Answer

To facilitate formation of Dual of the given problem, the following table is constructed.

		PRIMAL (Minimization problem)			
DUAL (Maximization problem)	Decision Variables	x_1	x_2	Relation	RHS of Constraint
	y_1	4	1	$\geq$	12
	y_2	9	1	$\geq$	20
	y_3	7	3	$\geq$	18
	y_4	10	40	$\geq$	40
	Relation	$\leq$	$\leq$	-	-
	RHS of Constraint	6000	4000	-	-

Dual problem is given as –

Maximize $Z^* = 12y_1 + 20y_2 + 18y_3 + 40y_4$

Subject to the constraints

$4y_1 + 9y_2 + 7y_3 + 10y_4 \leq 6000$

$y_1 + y_2 + 3y_3 + 40y_4 \leq 4000$

$y_1, y_2, y_3 \text{ and } y_4 \geq 0$

[N.B - Following points should be noted from the above solution –

1. Primal is a problem of MINIMIZATION having Objective Function Z , while Dual is a problem of MAXIMIZATION having Objective Function Z^* .

2. The number of Decision Variables in Primal is two (i.e x_1 and x_2). Hence the number of Constraints in Dual is also two.
3. The number of Constraints in Primal is four. So the number of Decision Variables in Dual is also four (y_1, y_2, y_3 and y_4)
4. In Primal the coefficients of the Decision Variables in the Objective Function are 6000 and 4000 respectively. In Dual the Right Hand Side (RHS) constants of the Constraints are formed by these two.
5. Coefficients of the Decision Variables in the Constraints of the Primal are represented column-wise in the above Table which becomes the Coefficients of the Decision Variables in the Constraints of the Dual and represented row-wise in the solution.
6. Primal has greater than or equal to ($\geq$) type inequality Constraints while Dual has less than or equal to ($\leq$) type inequality Constraints.
7. Number of Non negativity Constraints are same as the Number of Decision Variables in both Primal and Dual.]

Q8

CMA Mat

Write the Dual of the following –

Maximize $Z = 5x_1 + 10x_2$ subject to $2x_1 - 3x_2 \leq 7$, $x_1 + 2x_2 = 4$ and $x_1, x_2 \geq 0$

Reference Formulation of Dual	What's New Maximisation

Answer

Given Primal is a problem of Maximization. So it's Dual can be written only if all the Constraints are less than or equal to type. But one of the Constraints is equal to type. Thus the same needs to be converted to less than type which is done as shown below.

$x_1 + 2x_2 = 4$ can be replaced by two inequalities $x_1 + 2x_2 \leq 4$ and $x_1 + 2x_2 \geq 4$

Again the inequality $x_1 + 2x_2 \geq 4$ can be re-written as $-(x_1 + 2x_2) \leq -4$ or, $-x_1 - 2x_2 \leq -4$

So the given Primal is re-written as

Maximize $Z = 5x_1 + 10x_2$

Subject to

$2x_1 - 3x_2 \leq 7$, $x_1 + 2x_2 \leq 4$, $-x_1 - 2x_2 \leq -4$ and $x_1, x_2 \geq 0$

DUAL (Minimization problem)	Decision Variables	PRIMAL (Maximization problem)			
		x_1	x_2	Relation	RHS of Constraint
	y_1	2	-3	$\leq$	7
	y_2	1	2	$\leq$	4
	y_3	-1	-2	$\leq$	-4
	Relation	$\geq$	$\geq$	-	-
	RHS of Constraint	5	10	-	-

Dual problem is as follows –

$$\text{Minimize } Z^* = 7y_1 + 4y_2 - 4y_3$$

Subject to

$$2y_1 + y_2 - y_3 \geq 5$$

$$-3y_1 + 2y_2 - 2y_3 \geq 10$$

$$y_1, y_2, y_3 \geq 0$$

Considering $y_2 - y_3 = y_4$ the Dual problem can be written as follows

$$\text{Minimize } Z^* = 7y_1 + 4y_4$$

Subject to

$$2y_1 + y_4 \geq 5$$

$$-3y_1 + 2y_4 \geq 10$$

$$y_1 \geq 0 \text{ and } y_4 \text{ is unrestricted in sign}$$

Q9

CMA Mat

Find the Dual program of the following LPP –

Maximize $Z = 3x_1 + 5x_2 + 7x_3$ subject to $x_1 + x_2 + 3x_3 \leq 10$, $4x_1 - x_2 + 2x_3 \geq 15$ and $x_1, x_2 \geq 0$ & x_3 is unrestricted in sign.

	Reference	What's New
	Formulation of Dual	Unrestricted Sign

Answer

As x_3 is unrestricted in sign, to maintain the basic assumption of non-negativity of the Decision Variables in LPP, it is assumed that x_3 is the difference of two non-negative variables x_4 and x_5 i.e $x_3 = x_4 - x_5$

So the given problem is re-written as

$$\text{Maximize } Z = 3x_1 + 5x_2 + 7x_4 - 7x_5$$

Subject to

$$x_1 + x_2 + 3x_4 - 3x_5 \leq 10$$

$$4x_1 - x_2 + 2x_4 - 2x_5 \geq 15$$

$$x_1, x_2, x_4, x_5 \geq 0$$

But the given problem relates to the case of Maximization. So for getting it's Dual, all the constraints should be of less than or equal to form. Here the 2nd constraint is of more than equal to form. It requires to be converted to less than equal to form. That is done as below.

$$4x_1 - x_2 + 2x_4 - 2x_5 \geq 15 \text{ Or, } -4x_1 + x_2 - 2x_4 + 2x_5 \leq -15$$

Thus the Primal Problem takes the form as below.

$$\text{Maximize } Z = 3x_1 + 5x_2 + 7x_4 - 7x_5$$

Subject to the constraints

$$x_1 + x_2 + 3x_4 - 3x_5 \leq 10$$

$$-4x_1 + x_2 - 2x_4 + 2x_5 \leq -15$$

$$x_1, x_2, x_4, x_5 \geq 0$$

The data are summarized in the Table below to facilitate formation of Dual.

		PRIMAL (Maximization problem)					
DUAL (Minimization problem)	Decision Variables	x_1	x_2	x_4	x_5	Relation	RHS of Constraint
	y_1	1	-1	3	-3	$\leq$	10
	y_2	-4	1	-2	2	$\leq$	-15
	Relation	$\geq$	$\geq$	$\geq$	$\geq$	-	-
	RHS of Constraint	3	5	7	-7	-	-

So the Dual Problem is –

$$\text{Minimize } Z^* = 10y_1 - 15y_2$$

Subject to the constraints

$$y_1 - 4y_2 \geq 3$$

$$y_1 + y_2 \geq 5$$

$$3y_1 - 2y_2 \geq 7$$

$$-3y_1 + 2y_2 \geq -7 \text{ Or, } 3y_1 - 2y_2 \leq 7$$

$$y_1, y_2 \geq 0$$

The 3rd and 4th Constraints can be combined to give an equality constraint $3y_1 - 2y_2 = 7$

Thus the required Dual is

$$\text{Minimize } Z^* = 10y_1 - 15y_2$$

Subject to the constraints

$$y_1 - 4y_2 \geq 3$$

$$y_1 + y_2 \geq 5$$

$$3y_1 - 2y_2 = 7$$

$$y_1, y_2 \geq 0$$

Q10

CMA Mat

A retired person has plans to invest in shares. He has been suggested by one of his friends who plays in the share market to invest in two shares A and B which gives dividends @ 12% and 4% p.a respectively. For an investment of ₹1, the growth in the market value of the shares A and B are respectively 10 paise and 40 paise in one year. The retired person wants to invest such that the dividend income is at least ₹600 p.a and the growth of initial investment in one year is at least ₹1000.

- (i) **Formulate** it as a Linear Programming Problem.
- (ii) **Write** its Dual.
- (iii) **Solve** the Dual using Simplex Method. Interpret the solution.



Reference

Formulation of LPP, Dual, Simplex Method

What's **New**

Answer

- (i) Let x_1 and x_2 be the number of units of the shares A and B to be purchased by the retired person. The LP can be formulated as –

$$\text{Minimize } Z = x_1 + x_2$$

Subject to the Constraints

$$0.12x_1 + 0.04x_2 \geq 600 \text{ (Constraint on the income from Dividend)}$$

$$0.10x_1 + 0.40x_2 \geq 1000 \text{ (Constraint on the income from Growth)}$$

$$x_1, x_2 \geq 0 \text{ (Non-negativity Constraint)}$$

This is the formulated Primal Problem

(ii) To **obtain the Dual**, the data are summarized in the table below.

DUAL (Maximization problem)	PRIMAL (Minimization problem)				
	Decision Variables	x_1	x_2	Relation	RHS of Constraint
	y_1	0.12	0.04	$\geq$	600
	y_2	0.10	0.40	$\geq$	1000
	Relation	$\leq$	$\leq$	-	-
	RHS of Constraint	1	1	-	-

The Dual is given as –

To Maximize $Z^* = 600y_1 + 1000y_2$

Subject to the Constraints

$$0.12y_1 + 0.10y_2 \leq 1$$

$$0.04y_1 + 0.40y_2 \leq 1$$

$$y_1, y_2 \geq 0$$

(iii) To solve the **Dual using Simplex Method**, we introduce non-negative Slack Variables (S_1 & S_2) to rewrite the Constraints as follows –

$$0.12y_1 + 0.10y_2 + S_1 = 1$$

$$0.04y_1 + 0.40y_2 + S_2 = 1$$

So the Dual LPP in standard form is written as

$$\text{Minimize } Z^* = 600y_1 + 1000y_2 + 0.S_1 + 0.S_2$$

Subject to the Constraints

$$0.12y_1 + 0.10y_2 + S_1 = 1$$

$$0.04y_1 + 0.40y_2 + S_2 = 1$$

$$y_1, y_2, S_1, S_2 \geq 0$$

Considering $y_1 = y_2 = 0$, the Initial Feasible Solution is $S_1 = 1$ & $S_2 = 1$. This is shown in the following Table.

Table – 1 Showing Initial Feasible Solution

C_j			600	1000	0	0	Min. Ratio	
C_B	Basic Variables	Solution Values (X_B)	y_1	y_2	S_1	S_2		
0	S_1	1	0.12	0.10	1	0	$1/0.10 = 10$	
0	S_2	1	0.04	0.40*	0	1	$1/0.04 = 2.5$	Key Row
$(Z_B)^* = 0$		$(Z_j)^*$	0	0	0	0	y_2 enters & S_2	
	-	$C_j - (Z_j)^*$	600	1000	0	0	departs	
			Key Col.					

Table – 2 Showing Improved Solution

C _j			600	1000	0	0	Min. Ratio	
C _B	Basic Variables	Solution Values (X _B)	y ₁	y ₂	S ₁	S ₂		
0	S ₁	0.75	0.11*	0	1	- 0.25	0.75/0.11 = 6.82	Key Row
1000	y ₂	2.5	0.10	1	0	2.5	2.5/0.10 = 25	
(Z _B)* = 2500	-	(Z _j)*	100	1000	0	2500	y ₁ enters & S ₁ departs	
		C _j – (Z _j)*	500	0	0	- 2500		
			Key Col.					

Table – 3 Showing Optimal Solution

C_j			600	1000	0	0	Min. Ratio
C_B	Basic Variables	Solution Values (X_B)	y_1	y_2	S_1	S_2	
600	y_1	6.82	1	0	9.1	-2.27	
1000	y_2	1.82	0	1	-0.91	2.73	
$(Z_B)^* = 5912$	-	$(Z_j)^*$	600	1000	4550	1368	
		$C_j - (Z_j)^*$	0	0	-4550	-1368	

As all the values in the Index Row are zero or negative, optimality condition has been attained. The optimum solution is –

Maximum value of $Z^* = 5912$ and it corresponds to $y_1 = 6.82$ and $y_2 = 1.82$

Here the values of y_1 and y_2 indicates the marginal worth of one unit of Shares A and B respectively. Also the shadow prices (that is the optimum values of the dual variables) are the coefficients of the surplus variables in the Objective Function of the Primal problem.

Also the values, with sign changed, in the Index Row of the optimum Dual solution corresponding to the non- basic variables (that is S_1 and S_2) indicate the optimum values of the decision variables (x_1 and x_2) of the Primal problem. Thus the optimum number of Shares A to be purchased (x_1) = 4550 and that of Share B to be purchased (x_2) = 1368

Also optimum values of Objective Function in both Primal and Dual should be same i.e $(Z)_{\text{Minimum}} = (Z^*)_{\text{Maximum}} = 5912$

[Above interpretation of optimum Dual Solution can be cross checked by solving the Primal using Simplex Method or otherwise]

Q11

CMA Mat

An investor is well aware of the fact that maximization of total return on investment is best possible if the help of Linear Programming technique is taken. For that he has employed you and provided with the following information about his various activities of money making.

Activity A_1 – Invest in Bank Fixed Deposit schemes, Activity A_2 – Invest in Government Bonds, Activity

A_3 – Invest in Midcap Mutual Funds and Activity A_4 – Invest in Equity linked Mutual Funds.

Total amount to be invested – ₹ 5 lakhs

To avoid excessive investment, not more than 50% of the total should be invested in Government Bonds and Midcap Mutual Funds.

Investment in Bank Fixed Deposits is very conservative way of money making, while Investment in Equity linked Mutual Funds is very speculative. To avoid excessive speculation, at least ₹ 1 must be invested in Fixed Deposits for every ₹ 3 invested in Equity linked Mutual Funds.

Data on the ROI of the different activities are as follows –

Activity	A_1	A_2	A_3	A_4
Return on Investment	4%	5.5%	7%	10%

To solve the problem the investor has provided you with software. But you have to arrange for formulating the mathematical model of the problem and subsequent data feeding to use the software.

Explain clearly your first step.

Reference Formulation of LPP	What's New Maximization of total return

Answer

Mathematical formulation of the given LPP would be the first step and that is given as follows.

Maximize $Z = 0.04x_1 + 0.055x_2 + 0.07x_3 + 0.10x_4$ (x_i is the amount invested in Activity A_i and $i = 1, 2, 3, 4$)

Subject to the constraints

$$x_1 + x_2 + x_3 + x_4 \leq 500000, -0.5x_1 + 0.5x_2 + 0.5x_3 - 0.5x_4 \leq 0, 3x_1 - x_4 \geq 0 \text{ \& } x_1, x_2, x_3, x_4 \geq 0$$

Q12

CMA Mat

For a manufacturing company the following data are relevant to its products A and B

Description	Product A	Product B
1) Selling price per unit in Rupees	200	240
2) Cost per unit in Rupees		
2.1) Direct materials	45	50
2.2) Direct wages in		
Department 1	16	20
Department 2	22.50	15.50
Department 3	10/-	30/-
2.3) Variable Overhead	6.50/-	11.50/-
3) Hours required per unit in		
Department 1	8	10
Department 2	10	6
Department 3	4	12

Also it is given that the number of employees in the Departments 1, 2 and 3 are respectively 20, 15 and 18 as well as the maximum available hours per employee per week for each of the three departments is 40.

Formulate the linear programming problem so that the contribution of the company is maximized.

Reference Formulation of LPP	What's New

Answer

Maximize $Z = 100x + 115y$ (x and y are the number of Products A and B are to be manufactured)

Subject to the constraints

$$8x + 10y \leq 800, 10x + 6y \leq 600, 4x + 12y \leq 720 \text{ \& } x, y \geq 0$$

Q13

CMA Mat

A company buying scrap metal has two types of scrap available to them. The first type of scrap has 20% of Metal A, 10% impurity and 20% of Metal B by weight. The second type of scrap has 30% of Metal A, 10% impurity and 15% of Metal B by weight. The company requires at least 120 kgs. of Metal A, at most 40 kgs. of impurity and at least 90 kgs. of Metal B. The price for the two scraps are ₹ 200 and ₹ 300 per kg. respectively.

Determine the optimum quantities of the two scraps to be purchased by the company so that the requirements of the two metals and the restriction on impurity are satisfied at minimum cost. Use graphical method.

Reference Formulation of LPP & Solution	What's New Graphically

Answer

The solution is infeasible.

Q14

CMA Mat

An engineering company dealing with Combustion Equipment and Furnaces is planning its advertising strategy. They have two monthly industrial magazines under consideration. The first magazine has a reach of 2000 potential customers per advertisement and the second magazine has a reach of 3000 potential customers per advertisement. Respective cost per advertisement of the two magazines are ₹ 6000 and ₹ 9000 and the firm has a monthly budget of ₹ 1 lakh. There is an important requirement that the total reach for the business group having annual turnover below ₹ 20 Crores must not exceed 3000 potential customers. The reach of the two magazines having such business group customers are respectively 300 and 150 potential customers per advertisement. How many times the company should advertise in the two magazines to maximize the total reach?

Solve graphically.

Reference Formulation of LPP & Solution	What's New Graphically

Answer

The problem has multiple optimum solution with maximum value of Objective Function = $100000/3$

Q15

CMA Mat

Obtain the Dual from the following Primal.

$$\text{Minimize } Z = x_1 - 3x_2 - 2x_3$$

Subject to the Constraints $3x_1 - x_2 + 2x_3 \leq 7$

$$2x_1 - 4x_2 \geq 12$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

$x_1 \geq 0, x_2 \geq 0$ and x_3 is unrestricted in sign.



Reference

Formation of Dual

What's **New**

Unrestricted in Sign

Answer

$$\text{Maximize } Z^* = -7y_1 + 12y_2 + 10y_3$$

Subject to the constraints

$$-3y_1 + 2y_2 + 4y_3 \leq 1$$

$$y_1 - 4y_2 + 3y_3 \leq -3$$

$$2y_1 - 8y_3 = 2$$

$y_1 \geq 0, y_2 \geq 0$ and y_3 is unrestricted in sign

Q16

CMA Mat

Obtain the Dual from the following Primal.

$$\text{Minimize } Z = x_1 - 3x_2 - 2x_3$$

Subject to the Constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$2x_1 - 4x_2 \geq 12$$

$$-4x_1 + 3x_2 + 8x_3 = 10$$

$x_1 \geq 0, x_2 \geq 0$ and x_3 is unrestricted in sign.

Reference

Formation of Dual

What's New

Unrestricted in Sign

AnswerMaximize $Z^* = -7y_1 + 12y_2 + 10y_3$

Subject to the constraints

$$-3y_1 + 2y_2 + 4y_3 \leq 1$$

$$y_1 - 4y_2 + 3y_3 \leq -3$$

$$2y_1 - 8y_3 = 2$$

 $y_1 \geq 0, y_2 \geq 0$ and y_3 is unrestricted in sign
Q17**CMA Mat**

A company makes three products X, Y and Z using the raw materials A, B and C, Requirement of raw materials for each of the products X, Y and Z are given below.

Each of Product X requires 1 unit of raw material A, 2 units of B and 2 units of C. Each of Product Y requires 2 units of raw material A, 1 unit of B and 5 units of C. Each of Product Z requires 1 unit of raw material A, 4 units of B and 1 unit of C.

Find the optimum product mix when it is given that each unit of X, Y and Z gives profit of ₹40, ₹25 and ₹50 respectively. **Write** the Dual of the given problem. Using the optimum solution of the Primal problem, **write** the solution of the Dual and interpret it.

Reference

Formation of Dual

What's New

Interpretation of Solution

Answer

Optimum Product Mix is $X = 20$ Units, $Y = 0$ and $Z = 5$ Units and Maximum Profit = ₹ 1050 Dual is given as

$$\text{Minimize } Z^* = 36P + 60Q + 45R$$

Subject to the constraints

$$P + 2Q + 2R \geq 40$$

$$2P + Q + 5R \geq 25$$

$$P + 4Q + R \geq 50$$

$$P, Q \text{ \& } R \geq 0$$

From the optimum solution of the Primal problem, the solution of the Dual is given as $P = 0$, $Q = 10$ & $R = 10$ and $Z^* = 1050$

Final table of the Primal problem indicates that the marginal value of the raw material $A = 0$, $B = ₹10$ per unit and $C = ₹10$ per unit respectively. Thus instead of making the products X, Y and Z and selling, if the raw materials A, B and C are sold at the above rates then also the same contribution of ₹1050 can be achieved.

SJC Institute

Q1

CMA Mat

A company dealing with a special type of liquid has three plants P_1 , P_2 & P_3 located throughout the country. Production capacities of these plants are respectively 50, 75 and 25 Gallons. Each day the firm must furnish to four of its Retail shops R_1 , R_2 , R_3 , and R_4 with at least 20, 20, 50 and 60 Gallons of the product. The transportation cost per Gallon (in '000 ₹) between various sets of Plants and Retail Shops are given below –

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	20	20	50	60	

Find the Basic Feasible Solution for the problem of transportation using NWCM.

Reference NWCM	What's New

Answer

Starting from North West corner cell, min. between 50 & 20 (i.e. 20) is allocated to the cell P_1R_1 . It is shown below

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	20 3	5	7	6	30 ~~50~~
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	~~20~~	20	50	60	

From the Table above it is clear that the demand of the Retail Shop (R_1) represented along 1st column is completely satisfied and we shift sideways to cell P_1R_2 for next allocation. Maintaining the constraint between Supply and Demand here also 20 Gallons allocation is possible. As

a result demand of the 2nd column is also exhausted completely. The table below shows this allocation along with the previous one.

from Plants	to Retail Shops				Supply
	R ₁	R ₂	R ₃	R ₄	
P ₁	(20) 3	(20) 5	7	6	30 10 50
P ₂	2	5	8	2	75
P ₃	3	6	9	2	25
Demand	20	20	50	60	

Again we move horizontally to the cell P₁R₃ and allocate 10 Gallons which causes complete exhaustion of Supply from P₁. In other words Supply corresponding to the 1st row is totally exhausted. It is shown in the table below.

from Plants	to Retail Shops				Supply
	R ₁	R ₂	R ₃	R ₄	
P ₁	(20) 3	(20) 5	(10) 5	6	30 10 50
P ₂	2	5	8	2	75
P ₃	3	6	9	2	25
Demand	20	20	50 40	60	

Next allocation of 40 Gallons is done in cell P₂R₃ and the resultant table is shown below. It is clear that the Demand of R₃ is fully meet up due to this allocation. In other words Column R₃ is totally exhausted now.

from Plants	to Retail Shops				Supply
	R ₁	R ₂	R ₃	R ₄	
P ₁	(20) 3	(20) 5	(10) 7	6	30 10 50
P ₂	2	5	(40) 8	2	35 75
P ₃	3	6	9	2	25
Demand	20	20	50 40	60	

Thereafter allocation of 35 Gallons is done in cell P₂R₄. This has completely exhausted the Supply capacity of P₂ or the 2nd row, but the 4th column or R₄ still has a demand of 25 Gallons which has to be fulfilled by the Plant P₃. The results are shown in the table below.

from Plants	to Retail Shops				Supply
	R ₁	R ₂	R ₃	R ₄	
P ₁	3 (20)	5 (20)	7 (10)	6	30 10 50
P ₂	2	5	8 (40)	2 (35)	35 75
P ₃	3	6	9	2	25
Demand	20	20	50 40	60 25	

The final allocation of 25 Gallons is done in the cell P₃R₄ and the complete set of allocations is shown in the Table below.

from Plants	to Retail Shops				Supply
	R ₁	R ₂	R ₃	R ₄	
P ₁	3 (20)	5 (20)	7 (10)	6	30 10 50
P ₂	2	5	8 (40)	2 (35)	35 75
P ₃	3	6	9	2 (25)	25
Demand	20	20	50 40	60 25	

The Basic Feasible Solution of the given Transportation problem by NWCM is given as follows –

from Plant	to Retail Shop	Shipping cost per Gallon (₹ '000)	Quantity allocated (Gallons)	Cost (₹ '000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P ₁	R ₁	3	20	60
P ₁	R ₂	5	20	100
P ₁	R ₃	7	10	70
P ₂	R ₃	8	40	320
P ₂	R ₄	2	35	70
P ₃	R ₄	2	25	50
TOTAL				670

Total Cost of Transportation = ₹ 6,70,000

Q2

CMA Mat

A company dealing with a special type of liquid has three plants P_1 , P_2 & P_3 located throughout the country. Production capacities of these plants are respectively 50, 75 and 25 Gallons. Each day the firm must furnish to four of its Retail shops R_1 , R_2 , R_3 , and R_4 with at least 20, 20, 50 and 60 Gallons of the product. The transportation cost per Gallon (in '000 ₹) between various sets of Plants and Retail Shops are given below –

from Plants	to Retail Shops				Supply
	R_1	R_2	R_3	R_4	
P_1	3	5	7	6	50
P_2	2	5	8	2	75
P_3	3	6	9	2	25
Demand	20	20	50	60	

Find the Basic Feasible Solution by VAM.

Reference	What's New
VAM	Initial Solution

Answer

Penalty values for each row and column are calculated and entered in the table as shown. Of all the penalties, the one corresponding to the 1st Row is highest (background marked with a different colour). Corresponding to 1st Row least cost (3) appears in the cell P_1R_1 . Maximum possible allocation in this cell is 20 Gallons. Also that exhausts the demand of R_1 . Hence Column for R_1 is shaded meaning no further participation of it is possible in the subsequent computations. Results are given in the following table.

from Plants	to Retail Shops				Supply	Row pen. 1
	R_1	R_2	R_3	R_4		
P_1	20 3	5	7	6	30 50	2
P_2	2	5	8	2	75	0
P_3	3	6	9	2	25	1
Demand	20	20	50	60		
Col pen. 1	1	0	1	0		

Row pen. = Row penalty

Col. pen. = Column penalty

Next set of penalties are calculated omitting 1st Column and entered in the table. Of all, that corresponding to 3rd Row is maximum. Again least cost figure in 3rd Row is 2 which corresponds to cell P_3R_4 . Maximum possible allocation here is 25 Gallons which exhausts the supply capacity

of P_3 . It is shaded to indicate its omission from further participation in the new computations. Results are given in the table below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2	
	R_1	R_2	R_3	R_4				
P_1	3 (20)	5	7	6	30 50	2	1	Row pen. = Row penalty
P_2	2	5	8	2	75	0	3	
P_3	3	6	9	(25)	25	1	4	Col. pen. = Column penalty
Demand	20	20	50	60 35				
Col pen. 1	1	0	1	0				
Col pen. 2	-	0	1	0				

Next set of penalties are calculated omitting 1st Column and 3rd Row entered into the table. Of all, that corresponding to 4th Column is maximum. Again least cost figure in 4th Column is 2 which corresponds to cell $P_2 R_4$. Maximum possible allocation here is 35 Gallons which exhausts the demand of R_4 . It is shaded to indicate its omission from further participation in the new computations. Results are given in the table below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2	Row pen. 3
	R_1	R_2	R_3	R_4				
P_1	3 (20)	5	7	6	30 50	2	1	1
P_2	2	5	8	(35)	40 75	0	3	3
P_3	3	6	9	(25)	25	1	4	-
Demand	20	20	50	60 55				
Col pen. 1	1	0	1	0				
Col pen. 2	-	0	1	0				
Col pen. 3	-	0	1	4				

Next set of penalties are calculated omitting 1st Column, 3rd Row and 4th Column & entered into the table. Of all, that corresponding to 2nd Row is maximum. Again least cost figure in the 2nd Row is 5 which correspond to the cell $P_2 R_2$. Maximum possible allocation here is 20 Gallons which exhausts the demand of R_2 . It is shaded to indicate its omission from further participation in the new computations. Results are given in the table below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2	Row pen. 3	Row pen. 4
	R ₁	R ₂	R ₃	R ₄					
P ₁	20 3	5	7	6	30 50	2	1	1	2
P ₂	2	20 5	8	35 2	40 75 20	0	3	3	3
P ₃	3	6	9	25 2	25	1	4	-	-
Demand	20	20	50	60 55					
Col pen. 1	1	0	1	0					
Col pen. 2	-	0	1	0					
Col pen. 3	-	0	1	4					
Col pen. 4	-	0	1	-					

From above it is clear that the two remaining cells P₁R₃ and P₂R₃ should have allocations of the remaining quantities of 30 and 20 Gallons in the 1st and 2nd Rows respectively. These allocations are done and the final table with all possible allocations is shown below.

from Plants	to Retail Shops				Supply	Row pen. 1	Row pen. 2	Row pen. 3	Row pen. 4
	R ₁	R ₂	R ₃	R ₄					
P ₁	20 3	30 5	7	6	50 50	2	1	1	2
P ₂	2	20 5	20 8	35 2	40 75 20	0	3	3	3
P ₃	3	6	9	25 2	25	1	4	-	-
Demand	20	20	50	60 55					
Col pen. 1	1	0	1	0					
Col pen. 2	-	0	1	0					
Col pen. 3	-	0	1	4					
Col pen. 4	-	0	1	-					

The Basic Feasible Solution of the Transportation problem by VAM is given as follows –

from Plant	to Retail Shop	Shipping cost per Gallon (₹000)	Quantity allocated (Gallons)	Cost (₹000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P ₁	R ₁	3	20	60
P ₁	R ₃	7	30	210

P_2	R_2	5	20	100
P_2	R_4	2	35	70
P_2	R_3	8	20	160
P_3	R_4	2	25	50
TOTAL				650

Total Cost of Transportation = ₹ 650,000/-

So improvement is there [by an amount of ₹ (670000 – 650000) = ₹ 20000/-] in the solution obtained by VAM over that by NWCM and also [by an amount of ₹ (655000 – 650000) = ₹ 5000/-] over that by LCM.

Q3

CMA Mat

Test the Basic Feasible Solution obtained by VAM in the previous Illustration for Optimality. In case the solution is non-optimal, **find** the optimum solution to get the minimum Total Cost of Transportation.

Reference	What's New
VAM	Optimum Solution

Answer

The Basic Feasible Solution by VAM in the previous Illustration is considered and the Row Nos. (u_i) and Column Nos. (v_j) are calculated as follows –

1. The Transportation problem is having a (3×4) Matrix. So $m = 3$ and $n = 4$ here. Thus, there are three Row Nos. (u_1 , u_2 and u_3) and four Column Nos. (v_1 , v_2 , v_3 and v_4) i.e. in total 7 Nos. are to be calculated.
2. To find the Nos. we have to use the relation $C_{ij} = u_i + v_j$ for the ALLOCATED cells, but there are only 6 Allocated cells in the solution. In other words, 6 equations of the above type can be formed to find 7 unknown numbers. This is not possible by using general mathematical concepts. So the value of one of the numbers should be assumed.
3. We assume $u_2 = 0$. This is done because 2nd Row is having maximum number of allocations.
4. The calculations are as follows:

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
$P_2 R_2$	$C_{22} = 5$	$C_{22} = u_2 + v_2$ Or, $5 = 0 + v_2$	$v_2 = 5$
$P_2 R_3$	$C_{23} = 8$	$C_{23} = u_2 + v_3$ Or, $8 = 0 + v_3$	$v_3 = 8$
$P_2 R_4$	$C_{24} = 2$	$C_{24} = u_2 + v_4$ Or, $2 = 0 + v_4$	$v_4 = 2$
$P_3 R_4$	$C_{34} = 2$	$C_{34} = u_3 + v_4$ Or, $2 = u_3 + 2$	$u_3 = 0$

$P_1 R_3$	$C_{13} = 7$	$C_{13} = u_1 + v_3$ Or, $7 = u_1 + 8$	$u_1 = -1$
$P_1 R_1$	$C_{11} = 3$	$C_{11} = u_1 + v_1$ Or, $3 = -1 + v_1$	$v_1 = 4$

The results are entered into the table below to show the results at a glance.

from Plants	to Retail Shops				Supply	Row Nos.
	R_1	R_2	R_3	R_4		
P_1	(20) 3	5	(30) 7	6	50	$u_1 = -1$
P_2	2	(20) 5	(20) 8	(35) 2	75	Let $u_2 = 0$
P_3	3	6	9	(25) 2	25	$u_3 = 0$
Demand	20	20	50	60		
Column No	$v_1 = 4$	$v_2 = 5$	$v_3 = 8$	$v_4 = 2$		

For the UNALLOCATED Cells, Opportunity Cost values are calculated using the relation $\Delta_{ij} = C_{ij} - (u_i + v_j)$. The calculations and the results are shown in the table below.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No. (v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) - [(3) + (4)]
$P_1 R_2$	$C_{12} = 5$	$u_1 = -1$	$v_2 = 5$	1
$P_1 R_4$	$C_{14} = 6$	$u_1 = -1$	$v_4 = 2$	5
$P_2 R_1$	$C_{21} = 2$	$u_2 = 0$	$v_1 = 4$	-2
$P_3 R_1$	$C_{31} = 3$	$u_3 = 0$	$v_1 = 4$	-1
$P_3 R_2$	$C_{32} = 6$	$u_3 = 0$	$v_2 = 5$	1
$P_3 R_3$	$C_{33} = 9$	$u_3 = 0$	$v_3 = 8$	1

So, there exists two cells with negative Opportunity Cost. Hence the solution is non-optimal.

Highest negative Opportunity Cost appears at the cell $P_2 R_1$. Starting from here a closed loop (as shown in the table below) is drawn and alternately (+) and (-) signs are assigned to the corners of the loop which starts with a (+).

from Plants	to Retail Shops				Supply	Row Nos.
	R_1	R_2	R_3	R_4		
P_1	(-) (20) 3	5	(+) (30) 7	6	50	$u_1 = -1$
P_2	(-) (20) 2	5	(-) (20) 8	(35) 2	75	Let $u_2 = 0$
P_3	3	6	9	(25) 2	25	$u_3 = 0$

Demand	20	20	50	60		
Column No	$v_1 = 4$	$v_2 = 5$	$v_3 = 8$	$v_4 = 2$		

Allocated quantities of the (-) signed cells are both 20. This amount is subtracted from the quantities of the (-) signed cells and added to the quantities of the (+) signed cells. As a result both the (-) signed cells become unallocated. This turns into a five cell allocation solution which means a **Degenerate solution**. To take care of this an infinitely small allocation of amount (ϵ) is made to the cell having higher cost between the currently vanished cells i.e. the cell P_2R_3 . The new set of allocations is given in the table below along with the new set of Row and Column Nos.

from Plants	to Retail Shops				Supply	Row Nos.
	R_1	R_2	R_3	R_4		
P_1	20 3	5	30 7	6	50	$u_1 = -1$
P_2	20 2	20 5	6 8	35 2	75	Let $u_2 = 0$
P_3	3	6	9	25 2	25	$u_3 = 0$
Demand	20	20	50	60		
Column No	$v_1 = 4$	$v_2 = 5$	$v_3 = 8$	$v_4 = 2$		

For the UNALLOCATED Cells, again the Opportunity Cost values are calculated using the relation given before i.e. $\Delta_{ij} = C_{ij} - (u_i + v_j)$. Results are shown below.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No. (v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) - [(3) + (4)]
P_1R_2	$C_{12} = 5$	$u_1 = -1$	$v_2 = 5$	1
P_1R_4	$C_{14} = 6$	$u_1 = -1$	$v_4 = 2$	5
P_1R_1	$C_{11} = 3$	$u_1 = -1$	$v_1 = 2$	2
P_3R_1	$C_{31} = 3$	$u_3 = 0$	$v_1 = 2$	1
P_3R_2	$C_{32} = 6$	$u_3 = 0$	$v_2 = 5$	1
P_3R_3	$C_{33} = 9$	$u_3 = 0$	$v_3 = 8$	1

As no negative Opportunity Cost values are there, the solution is Optimal. The Optimum solution is given below.

from Plant	to Retail Shop	Shipping cost per Gallon (₹ '000)	Quantity allocated (Gallons)	Cost (₹ '000)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
P_1	R_3	7	50	350
P_2	R_1	2	20	40
P_2	R_2	5	20	100

P_2	R_4	2	35	70
P_2	R_3	8	$\epsilon = 0$	0
P_3	R_4	2	25	50
TOTAL				610

So minimum Total Cost of Transportation = ₹ 610,000/-

Q4

CMA Mat

A multi plant company has three manufacturing plants M_1 , M_2 and M_3 . The company is dealing with a unique product and enjoy monopoly as far as competition is concerned. They have two fixed customers A and B who procures all the items produced by the company.

Cost of Production (₹ per piece of the product) of the Plants M_1 , M_2 and M_3 are respectively 1500, 1600 & 1700. Selling prices to the customers A and B are ₹ 4400 and ₹ 4700 per piece respectively.

Production Capacities of the three Plants, Demands of the two Customers and the Costs of Transportation per unit from various Plants to the different Customer's premises are given as follows:

	Plant M_1	Plant M_2	Plant M_3	Demand
Customer A	₹1000	₹2000	₹1500	3500 units
Customer B	₹1500	₹3000	₹2500	3600 units
Production Capacity	2000 units	3000 units	4000 units	

Formulate the problem as LP Model. Also find the optimum solution using Transportation algorithm.

Reference	What's New
Formulation of LP and Transportation solution	

Answer

This is a problem of Profit maximization. Profit figures per piece are computed using the relation given below. Profit = Selling Price – (Production Cost + Transportation Cost)

Profit figures corresponding to various combinations of Customer and Plant are provided in the matrix below.

	Customer A	Customer B	Production Capacity
Plant M_1	$4400 - (1500 + 1000) = ₹1900$	$4700 - (1500 + 1500) = ₹1700$	2000 units
Plant M_2	$4400 - (1600 + 2000) = ₹800$	$4700 - (1600 + 3000) = ₹100$	3000 units

Plant M ₃	4400 – (1700+1500) = ₹1200	4700 – (1700+2500) = ₹500	4000 units
Demand	3500 units	3600 units	7100 units \ 9000 units

From above it is clear that the given problem is an unbalanced one. Here total Production Capacity (9000 units) is more than the total Demand (7100 units) by an amount of 9000 – 7100 = 1900 units. To make it balanced, a Dummy Customer C having Demand of 1900 units is introduced. Profit figures for it corresponding to different Plants are each zero. The balanced Profit Matrix is given hereunder.

Profit Matrix after making the problem Balanced

	Customer A	Customer B	Customer C	Production Capacity
Plant M ₁	₹ 1900	₹ 1700	0	2000 units
Plant M ₂	₹ 800	₹ 100	0	3000 units
Plant M ₃	₹ 1200	₹ 500	0	4000 units
Demand	3500 units	3600 units	1900 units	9000 units

Let us consider the Decision Variables x_{ij} ($i = 1, 2 \text{ \& } 3$ and $j = 1, 2 \text{ \& } 3$) as the quantities to be shipped from the Plant M_i to the Customer j . So the LPP can be written as –

$$\text{Maximize } Z = 1900 x_{11} + 1700 x_{12} + 800 x_{21} + 100 x_{22} + 1200 x_{31} + 500 x_{32}$$

Subject to the constraints:

$$x_{11} + x_{12} + x_{13} = 2000, x_{21} + x_{22} + x_{23} = 3000, x_{31} + x_{32} + x_{33} = 4000 \text{ (Row constraints or Production Capacity constraints)}$$

$$x_{11} + x_{21} + x_{31} = 3500, x_{12} + x_{22} + x_{32} = 3600, x_{13} + x_{23} + x_{33} = 1900 \text{ (Column Constraints or Demand Constraints)} \quad x_{ij} \geq 0 \text{ for all } i, j = 1, 2, 3$$

To facilitate solution of the problem by using the algorithm of Transportation, above matrix is converted into a Relative Loss matrix by subtracting each of its elements from the highest element i.e 1900.

Relative Loss Matrix

	Customer A	Customer B	Customer C	Production Capacity
Plant M ₁	0	200	1900	2000 units
Plant M ₂	1100	1800	1900	3000 units
Plant M ₃	700	1400	1900	4000 units
Demand	3500 units	3600 units	1900 units	9000 units

To make the calculations simpler, each figure of the above table is divided by 100. The resultant Matrix is given as follows -

	Customer A	Customer B	Customer C	Production Capacity in ₹ '00 units
Plant M_1	0	2	19	20
Plant M_2	11	18	19	30
Plant M_3	7	14	19	40
Demand in 00 units	35	36	19	90

Basic Feasible solution of the problem by VAM is shown in the Table below along with the Row and Column Nos.

From Plants	To	Warehouse			Production Capacity	Row Penalty			Row Nos. (u_i)
		A	B	C		1	2	3	
M_1		0	2 (20)	19	20	2	-	-	$u_1 = 2$
M_2		11	18 (11)	19 (19)	30	7	7	1	$u_2 = 18$
M_3		7 35	14 (5)	19	40	7	7*	5*	$u_3 = 14$
Demand		35	36	19	90				
Column Penalty	1	7	12*	0					
	2	4	4	0					
	3	-	4	0					
Column Nos. (v_j)		$V_1 = -7$	$V_2 = 0$ (let)	$V_3 = 1$					

For the Transportation matrix, No. of Rows = $m = 3$ and No. of Columns = $n = 3$. So $(m + n - 1) = 3 + 3 - 1 = 5$.

In the above solution No. of Allocated Cells = $5 = m + n - 1$

So the solution is Non – degenerate.

Calculation of Row Numbers (u_i) and Column Numbers (v_j) are shown in the table below.

Allocated Cell	Cost (C_{ij})	$C_{ij} = u_i + v_j$	Calculated No.
$M_1 - B$	$C_{12} = 2$	$C_{12} = u_1 + v_2$ Or, $2 = u_1 + 0$	$u_1 = 2$
$M_2 - B$	$C_{22} = 18$	$C_{22} = u_2 + v_2$ Or, $18 = u_2 + 0$	$u_2 = 18$
$M_2 - C$	$C_{23} = 19$	$C_{23} = u_2 + v_3$ Or, $19 = 18 + v_3$	$v_3 = 1$
$M_3 - B$	$C_{32} = 14$	$C_{32} = u_3 + v_2$ Or, $14 = u_3 + 0$	$u_3 = 14$
$M_3 - A$	$C_{31} = 7$	$C_{31} = u_3 + v_1$ Or, $7 = 14 + v_1$	$v_1 = -7$

To test optimality of the solution, Opportunity Costs for each of the Unallocated Cell is computed and shown in the Table follows.

Unallocated Cell	Cost (C_{ij})	Row No. (u_i)	Column No. (v_j)	Opportunity Cost (Δ_{ij})
(1)	(2)	(3)	(4)	(5) = (2) - [(3) + (4)]
$M_1 - A$	$C_{11} = 0$	$u_1 = 2$	$v_1 = -7$	5
$M_1 - C$	$C_{33} = 19$	$u_1 = 2$	$v_3 = 1$	16
$M_2 - A$	$C_{21} = 11$	$u_2 = 18$	$v_1 = -7$	0
$M_2 - C$	$C_{23} = 19$	$u_2 = 18$	$v_3 = 1$	0
$M_3 - C$	$C_{33} = 19$	$u_3 = 14$	$v_3 = 1$	4

As no negative Opportunity Cost values are there, the solution is Optimal.

The Optimum Transportation schedule along with the maximum total profit is given hereunder.

From Plant	To Customer	Profit per unit (₹)	Quantity allocated (₹00 Units)	Profit (₹)
(1)	(2)	(3)	(4)	(5) = (3) × (4)
M_1	B	1700	20	34000
M_2	B	100	11	1100
M_2	C	0	19	0
M_3	A	1200	35	42000
M_3	B	500	5	2500
TOTAL				79600

Maximum Total Profit = ₹ 79,60,000 (After considering proper unit of the Quantity allocated values)

[N.B – Profit figures are taken from the PROFIT MATRIX prepared earlier. Plant M_2 can produce 1900 units less because these many units produced by it are allocated to the Customer C which is actually a DUMMY Customer]

Q5

CMA Mat

ABC Enterprises are having three Plants, for manufacturing Dry Cells, located at three different states of the country. Production cost differs from Plant to Plant. There are four dedicated Sales Offices of the Company located at the four metro cities in the four regions of the country and a Head Office, in the capital of the country, which also takes care of sales. The Sales prices are different from one region to the other and also in the capital city. Unit Production Cost and Maximum Production Capacity of each Plant is provided in the first table below. Also provided are the data on Shipping Cost from each Plant to the different Sales Offices as well as Demand and Price at which they can sell.

Table 1 Showing Production Cost and Capacity of the Plants

Plant Number	Maximum Production Capacity	Production Cost (₹)
1	150 Units	20
2	200 Units	22
3	125 Units	18

Table 2 Showing Shipping Cost per unit from different Plants to various Sales Offices

from Plants	to Sales Offices at				
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City
1	1	1	5	9	4
2	9	7	8	3	6
3	4	5	3	2	7

Table 3 Showing Demand (Units) and Selling Prices (₹) for different Sales Offices

Item	for Sales Offices at				
	Northern Region	Southern Region	Western Region	Eastern Region	Capital City
Demand	80	100	75	45	125
Selling Price	30	32	31	34	29

You have to **find** the Production and Distribution schedule which maximises the Profit of the company.

Reference	What's New

Answer

Production and Distribution Schedule

From Plant	To Sales Office	Number of units allocated
1	Northern Region	50
1	Southern Region	100
2	Eastern Region	25
2	Capital City	125
3	Northern Region	30
3	Western Region	75
3	Eastern Region	20

Q6

CMA Mat

A company wishes to determine an Investment Strategy for each of the next four years. Five investment types have been selected, investment capital has been allocated for each of the coming four years and maximum investment levels have been established for each investment type. An assumption is that an amounts invested in a year will remain invested until the end of the planning horizon of four years. The following table summarises the data for this problem. The values in the body of the table represent net return on investment of one rupee up to the end of the planning horizon. For example a rupee invested in investment type B at the beginning of the year 1 will grow to ₹ 1.90 by the end of the 4th year yielding a net return of ₹0.90.

Table showing Net Return data

Investment made at the beginning of the year	Investment Type					Amount available for Investment (₹000)
	A	B	C	D	E	
1	0.80	0.90	0.60	0.75	1.00	500
2	0.55	0.65	0.40	0.60	0.50	600
3	0.30	0.25	0.30	0.50	0.20	750
4	0.15	0.12	0.25	0.35	0.10	800
Maximum possible investment (₹ '000)	750	600	500	800	1000	

You need to **determine** the amount to be invested at the beginning of each year in different types of investment so as to maximize the net rupee return for the period of four years. Also find the maximum value of net return on investment.

Reference	What's New
Maximum value of net return on investment	

Answer

Schedule of Amounts to be invested in different types of Investment in various years

Year of Investment	Investment type	Amount to be invested (₹000)
1	E	500
2	B	600
3	D	750

4	A	250
4	C	500
4	D	50

Maximum net return on investment = ₹ 14,45,000

Q7

CMA Mat

Consider the problem of scheduling the weekly production of certain items for the next four weeks. The cost of production of the item is ₹10 for the first 2 weeks and ₹15 for the last 2 weeks. The weekly demands are 300, 700, 900 and 800 units, which must be met. The plant can produce maximum 700 units per week. In addition the company can employ overtime during the 2nd and 3rd week. This increases the weekly production by additional 200 units, but the production cost increases by ₹5. Excess production can be stored at a unit cost of ₹3 per week.

How should the production be scheduled so as to minimize the total cost? **How much** production is to be carried out by overtime?

Reference	What's New
Minimisation	Prohibited Transportation

Answer

As per the given condition, Available options of production in different weeks can be written down as – Normal time production for week 1, Normal time production for week 2, Overtime production for week 2, Normal time production for week 3, Overtime production for week 3 and Normal time production for week 4.

Also Cost of producing an item in Normal time for the 1st two weeks = ₹ 10. But if the same item is produced in a week and used in a subsequent week then there is Carrying Cost @ ₹ 3 per week. Hence an item produced in the 1st week and used in the 2nd week will have a cost of ₹ 13, in the 3rd week it will be ₹16 and so on. Similarly Cost of producing an item in overtime = ₹15. Thus cost of using the same item in the next week = ₹18, in the week next to that = ₹21 and so on.

Thus the given information can be tabulated as follows –

Produced during week	Cost (₹ per unit) when used in the week					Quantity available (Nos.)
	1	2	3	4	Dummy	
1	10	13	16	19	0	700
2 (Normal)	M	10	13	16	0	700
2 (Overtime)	M	15	18	21	0	200

3 (Normal)	M	M	15	18	0	700
3 (Overtime)	M	M	20	23	0	200
4	M	M	M	15	0	700
Demand	300	700	900	800	500	3200

[As the production of 2nd week cannot be used in the 1st week, the allocation in the cell (2,1) is not possible. In other words it is a case of prohibited transportation. To take care of this restriction the cost figures in all such cells have been taken as M]

Now the problem can be solved using usual technique of solving Transportation problems.

The optimal schedule is given as follows -

Produced in week	Used in week	Number of units
1	1	300
1	3	200
1	4	100
1	Dummy	100
2 (Normal time)	2	700
2 (Overtime)	Dummy	200
3 (Normal time)	3	700
3 (Overtime)	Dummy	200
4	4	700

Minimum Total Cost = ₹ 36,100.

From the table above it is clear that overtime productions have been allocated only to the Dummy week. Hence no production in overtime is necessary. In fact there will be spare capacity of 100 units in the 1st week because this quantity of the production of 1st week is allocated to Dummy week.

Chapter 12

Assignment

Q1

CMA Mat

A computer centre has three expert programmers and needs to develop three application programs. The head of the centre estimates the time required (in minutes) by the respective experts to develop the application programs are as follows –

		Programmers		
		A	B	C
Programmes	1.	120	100	80
	2.	80	90	110
	3.	110	140	120

Formulate an LP Model.

Reference

Formulation of LP model for Assignment Problem

What's New

Answer

Key decision is to determine which Programmer should be assigned with which Program. Let us designate the assignment of the j th Programmer to the i th Program by the decision variable x_{ij} (where the subscripts $i = A, B, C$ and $j = 1, 2, 3$)

Objective is to minimize the total time required i.e.. to

$$\text{Minimize } Z = 120x_{11} + 100x_{12} + 80x_{13} + 80x_{21} + 90x_{22} + 110x_{23} + 110x_{31} + 140x_{32} + 120x_{33}$$

Subject to the following constraints

- Each Programmer must be assigned with development of one and only one Program. Thus,
 $x_{11} + x_{21} + x_{31} = 1$ for Program A, $x_{12} + x_{22} + x_{32} = 1$ for Program B, $x_{13} + x_{23} + x_{33} = 1$ for Program C
- Each Program must be assigned to one and only one Programmer. Hence
 $x_{11} + x_{12} + x_{13} = 1$ for Programmer 1, $x_{21} + x_{22} + x_{23} = 1$ for Programmer 2, $x_{31} + x_{32} + x_{33} = 1$ for Programmer 3
- $x_{ij} = 0$ or 1 , for all i and j .

Q2

CMA Mat

A city corporation has decided to carry out road repairs on four main arteries of the city. The government has agreed to make a special grant of ₹ 50 lakhs towards the cost with a condition that the repairs must be done at the lowest cost and quickest time. If conditions warrant, then a supplementary token grant will also be considered favourably. The corporation has floated tender and five contractors participated in bidding. In order to expedite work, one road will be awarded to one contractor. The following matrix of Cost of Repairs is prepared by the corporation on the basis of the bids submitted by the participants.

Contractors	Cost of Repairs in ₹ Lakhs			
	Road 1	Road 2	Road 3	Road 4
A	9	14	19	15
B	7	17	20	19
C	9	18	21	18
D	10	12	18	19
E	10	15	21	16

- Find** the best way of assigning the repair work to the contractors and the total cost.
- If it is necessary to seek supplementary grant then **what** should be the amount sought?
- Which** of the five contractors will be unsuccessful in his bid?

Reference	What's New
Unbalanced Problem of Assignment	

Answer

- This is an unbalanced problem of Assignment where No. of rows (5) $\neq$ No. of columns (4) in the given cost matrix. To make the problem balanced we introduce a Dummy Column with all zero elements as shown below.

Table – 1 showing conversion of Unbalanced Cost Matrix to Balanced one

Contractors	Cost of Repairs in ₹ Lakhs				
	Road 1	Road 2	Road 3	Road 4	Dummy Road
A	9	14	19	15	0
B	7	17	20	19	0
C	9	18	21	18	0
D	10	12	18	19	0
E	10	15	21	16	0

As each row of the above matrix contains a zero, it will remain unchanged after Row operation (1st step) of Hungarian Method. Thus, we straight way go for the next step of Column operation and arrive at the Table below.

Table – 2 showing reduced matrix after Column operation

Contractors	Cost of Repairs ₹ in Lakhs				
	Road 1	Road 2	Road 3	Road 4	Dummy Road
A	2	2	1	0	0
B	0	5	2	4	0
C	2	6	3	3	0
D	3	0	0	4	0
E	3	3	3	1	0

So minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 $\neq$ Order (5) of the matrix

Thus, the solution is non-optimal. An improvement of the solution is done by subtracting the minimum of the uncovered elements i.e., 1 from all the uncovered elements and adding the same to the elements at the junction.

Table – 3 showing improved Matrix (Non-optimal)

Contractors	Cost of Repairs ₹ in Lakhs				
	Road 1	Road 2	Road 3	Road 4	Dummy Road
A	2	2	1	0	1
B	0	5	2	4	1
C	1	5	2	2	0
D	3	0	0	4	1
E	2	2	2	0	0

Here also minimum no. of horizontal and vertical straight lines to cover all the zeros = 4 $\neq$ Order (5) of the matrix.

Thus, the solution is non-optimal. An improvement of the solution is done by subtracting the minimum of the uncovered elements i.e., 1 from all the uncovered elements and adding the same to the elements at the junction.

Table – 4 showing improved Matrix (Optimal)

Contractors	Cost of Repairs ₹ in Lakhs				
	Road 1	Road 2	Road 3	Road 4	Dummy Road
A	1	1	0	0	1
B	0	5	2	5	2
C	0	4	1	2	0
D	3	0	0	5	2
E	1	1	1	0	0

Here minimum no. of horizontal and vertical straight lines to cover all the zeros = 5 = Order (5) of the matrix.

So the solution is optimal.

Table – 5 showing Optimal Assignments

Contractors	Cost of Repairs ₹ in Lakhs				
	Road 1	Road 2	Road 3	Road 4	Dummy Road
A	1	1	0	~~0~~	1
B	0	5	2	5	2
C	~~0~~	4	1	2	0
D	3	0	~~0~~	5	2
E	1	1	1	0	~~0~~

Row B is having only one zero at the cell (B - 1). So it is assigned and zero in the same column at the cell (C - 1) is crossed out. Now Row C is having one zero at the cell (C – Dummy). So it is assigned and zero in the same column in the cell (E – Dummy) is crossed out. Again Row E is having only one zero at the cell (E – 4). It is assigned and the zero in the cell (A - 4) of the same column is crossed out. Next we check columns to find single zero in column 2 at the cell (D – 2). It is assigned and zero at the cell (D – 3) of the same row is crossed out. Only the cell (A – 3) is left with a zero and it is assigned.

With the assignments shown above Minimum Total Cost of repairing the roads is calculated as follows -

Calculation of Minimum Total Cost

Road to be repaired	Assigned to Contractor	Cost of Repair (₹ Lakhs)
1	B	7
2	D	12
3	A	19
4	E	16
Total	-	54

So the best way of repairing the roads and associated cost is as shown in the above table

- (ii) As the minimum total cost (₹ 54 Lakhs) exceeds sanctioned amount (₹ 50 Lakhs), a supplementary grant of ₹ 4 Lakhs to be sought.
- (iii) From the Table 5 above we find that the Dummy Road is assigned to the Contractor C. So C will be unsuccessful in his bid because there is no existence of the Dummy road.

Q3

CMA Mat

Five Salesmen are to be assigned to five Districts. Estimates of Sales Revenue (in ₹000 ₹) for each Salesman are given in the table below.

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	32	38	40	28	40
2	40	24	28	21	36
3	41	27	33	30	37
4	22	38	41	36	36
5	29	33	40	35	39

Find the assignment pattern that maximises Revenue.

Reference Maximisation Assignment	What's New

Answer

To solve the sum we have to show all the steps done up to Table - 6 of Illustration 3. Thereafter the alternative solutions have to be shown in the Tables-7, 8 & 9 as follows.

Table-7 showing Alternative Optimal Assignments (2)

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	12	0	1	8	~~8~~
2	~~8~~	10	9	11	0
3	0	8	5	3	~~8~~
4	22	~~8~~	0	~~8~~	4
5	14	4	~~8~~	0	~~8~~

Table-8 showing Alternative Optimal Assignments (3)

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	12	0	1	8	~~8~~
2	~~8~~	10	9	11	0
3	0	8	5	3	~~8~~
4	22	~~8~~	~~8~~	0	4
5	14	4	0	~~8~~	~~8~~

Table-9 showing Alternative Optimal Assignments (4)

Districts	Salesman A	Salesman B	Salesman C	Salesman D	Salesman E
1	12	0	1	8	~~8~~
2	0	10	9	11	~~8~~
3	~~8~~	8	5	3	0
4	22	~~8~~	~~8~~	0	4
5	14	4	0	~~8~~	~~8~~

Calculation of maximum Total Sales Revenue (in ₹000 ₹) for all the Alternative Solutions are done as follows -

Calculation of Maximum Total Sales Revenue (₹000 ₹) for various Alternatives

Alternative 1 (Table-6)*			Alternative 2 (Table – 7)			Alternative 3 (Table – 8)			Alternative 4 (Table – 9)		
Person	District	Sales	Person	District	Sales	Person	District	Sales	Person	District	Sales
A	2	40	A	3	41	A	3	41	A	2	40
B	1	38	B	1	38	B	1	38	B	1	38
C	4	41	C	4	41	C	5	40	C	5	40
D	5	35	D	5	35	D	4	36	D	4	36
E	3	37	E	2	36	E	2	36	E	3	37
Total	-	191	Total	-	191	Total	-	191	Total	-	191

[* For Table 6 refer to the solution of Illustration 3]

From above it is seen that the maximum total Sales Revenue in each case is ₹ 1,91,000. Thus, each solution gives same optimum result. In other words, the problem has multiple optimum solutions.

Q4

CMA Mat

A company has taken on rent three floors (1st, 2nd and 3rd) of a multi storied building for their city office. It has been decided to locate Managers of Marketing, Purchase, HR, Finance and Corporate Law in the office. The management has earmarked in different floors five rooms having numbers 103, 201, 205, 302 and 304 for the above mentioned Managers. But no particular room has been allotted for any particular Manager and rather they have given option to indicate their preference of rooms so that decision can be taken by the management using some scientific method and subsequently arrangement of sitting of the subordinates of various Managers can be made. Managerial preferences are provided in the table below with 1st preference appearing in the top for each and every Manager.

Preference of Rooms of different Managers				
Marketing Manager	Purchase Manager	HR Manager	Finance Manager	Company Secretary
302	302	103	302	201
103	304	201	205	302
304	205	304	304	304
	201	205	103	
		302		

It is evident that most of the Managers have not given preference for all the available rooms because they feel that all the rooms do not have the facility they are looking for. Assuming that the preferences can be quantified by numbers.

Find out which manager should be assigned with which room to minimise the preferential measure.

Reference

What's New

Prohibited Assignment

Answer

Let the preferential ranking of the rooms be quantified as 1, 2, 3, 4 and 5 as well as M for no preference. In fact the managers who have given no preference for certain rooms do not want those rooms to be allotted to them i.e., assignment of certain Rooms to some Managers are restricted. For this a very high value (M) is assigned to those specific cells of the matrix. Following table shows the numerical figures for various combination of Manager and Room.

Table -1 showing Preferential Scores of different Rooms as awarded by various Managers

Managers	Preferential scores of different Rooms by different Managers				
	Room -103	Room - 201	Room -205	Room - 302	Room -304
Marketing Manager	2	M	M	1	3
Purchase Manager	M	4	3	1	2
HR Manager	1	2	4	5	3
Finance Manager	4	M	2	1	3
Company Secretary	M	1	M	2	3

Table -2 showing reduced matrix after Row subtraction operation

Managers	Preferential scores of different Rooms by different Managers				
	Room -103	Room - 201	Room -205	Room - 302	Room -304
Marketing Manager	1	M	M	0	2
Purchase Manager	M	3	2	0	1
HR Manager	0	1	3	4	2
Finance Manager	3	M	1	0	2
Company Secretary	M	0	M	1	2

Table – 3 showing reduced matrix after Column subtraction operation

Managers	Preferential scores of different Rooms by different Managers				
	Room -103	Room - 201	Room -205	Room - 302	Room -304
Marketing Manager	1	M	M	0	1
Purchase Manager	M	3	1	0	0
HR Manager	0	1	2	4	1
Finance Manager	3	M	0	0	1
Company Secretary	M	0	M	1	1

Here minimum number of horizontal and vertical straight lines to cover all the zeros = 5 = Order of the matrix.

So the solution is optimal and assignments are made using standard procedure of Hungarian Method as below.

Table - 4 showing matrix with Optimal assignments

Managers	Preferential scores of different Rooms by different Managers				
	Room -103	Room - 201	Room -205	Room - 302	Room -304
Marketing Manager	1	M	M	0	1
Purchase Manager	M	3	1	~~0~~	0
HR Manager	0	1	2	4	1
Finance Manager	3	M	0	~~0~~	1
Company Secretary	M	0	M	1	1

Thus, the required assignment of Rooms should be –

Room 103 to HR Manager, Room 201 to Company Secretary, Room 205 to Finance Manager, Room 302 to Marketing Manager and Room 304 to Purchase Manager.

Q5

CMA Mat

A travelling salesman has to visit five cities. He wishes to start from a particular city, visit each city once and then return to his starting point. The travelling cost (in ₹00 ₹) between any two cities is given in the table below

From City	To City				
	A	B	C	D	E
A	M	5	8	4	5
B	5	M	7	4	5
C	8	7	M	8	6
D	4	4	8	M	8
E	5	5	6	8	M

Find the cost minimising sequence of visit.

Reference Travelling Salesman	What's New

Answer

Table – 1 showing reduced matrix after Row subtraction operation

From City	To City				
	A	B	C	D	E
A	M	1	4	0	1
B	1	M	3	0	1
C	2	1	M	2	0
D	0	0	4	M	4
E	0	0	1	3	M

Table – 2 showing reduced matrix after Column subtraction operation

From City	To City				
	A	B	C	D	E
A	M	1	3	0	1
B	1	M	2	0	1
C	2	1	M	2	0
D	0	0	3	M	4
E	0	0	0	3	M

Here minimum number of horizontal and vertical straight lines to cover all the zeros = 4 $\neq$ Order (5) of the matrix. So the solution is non-optimal. Improvement of the above matrix is done by subtracting the minimum value of the uncovered elements i.e. 1 from all the uncovered elements and adding the same to the elements at the junction of the horizontal and vertical lines. The resultant matrix is shown below.

Table – 3 showing improved matrix (Optimal)

From City	To City				
	A	B	C	D	E
A	M	0	2	0	1
B	0	M	1	0	1
C	1	0	M	2	0
D	0	0	3	M	5
E	0	0	0	4	M

Here the minimum number of straight lines required to cover all the zeros = 5 = Order of the matrix. So the solution is optimal. Now assignments are done by following the standard rules of Hungarian Method as below.

Table – 4 showing matrix with Optimal Assignments (Alternative – 1)

From City	To City				
	A	B	C	D	E
A	M	0	2	~~8~~	1
B	~~8~~	M	1	0	1
C	1	~~8~~	M	2	0
D	0	~~8~~	3	M	5
E	~~8~~	~~8~~	0	4	M

As per the solution above, the Salesman will travel from A to B, then B to D, then D to A. But this is not meeting the requirement of travelling through all the cities and finally returning to the starting point i.e.. A. Hence the solution is unacceptable.

Table – 5 showing matrix with Optimal Assignments (Alternative – 2)

From City	To City				
	A	B	C	D	E
A	M	~~8~~	2	0	1
B	0	M	1	~~8~~	1
C	1	~~8~~	M	2	0
D	~~8~~	0	3	M	5
E	~~8~~	~~8~~	0	4	M

Again the solution above shows the travelling route as A to D, then D to B, then B to A. This is also not acceptable because of violation of the basic requirement of the problem.

Under the circumstances it is decided to try for the assignment at the cells which are having next highest entry after zero. It can be mentioned that as far as practicable Assignments should be done at the cells having 0 entry. Here next highest entry in the table after 0 is 1 and it appears at the four cells – (A, E), (B, C), (B, E) and (C, A). By arbitrarily choosing any one of these, assignment is done afresh. Let the cell (A, E) be chosen for the purpose

Table – 5 showing matrix with the required solution

From City	To City				
	A	B	C	D	E
A	M	~~0~~	2	~~0~~	1
B	~~0~~	M	1	0	1
C	1	0	M	2	~~0~~
D	0	~~0~~	3	M	5
E	~~0~~	~~0~~	0	4	M

Using the standard procedure of Hungarian Method assignments are made starting from 1st Row cell (A, E) and finally required solution is reached which shows the travel route of the salesman as – A to E, E to C, C to B, B to D and D to A. Minimum Cost of travel is $5 + 4 + 7 + 4 + 6 = 26$

Q6

CMA Mat

Sugarcane is a globally important commercial crop that can be used to produce both direct and indirect products such as Sugar, Ethanol, Fodder etc. Sugarcane is a tall perennial grass reaching 3 to 4 metres in height. Its cultivation is not easy and it needs special tools to be harvested effectively.

To harvest sugarcane, cutting at the ground level is needed because the sweetness level is higher in this part of the cane. The tips of the stem and the leaves are immediately removed. Traditionally hand harvesting is effective for this. Currently mechanical harvesters play an important role in sugarcane harvesting due to shortage of labour and increased labour costs. A harvester is very powerful since it can replace 200 labourers. In the first era of using the harvester instead of human labour, the price of the harvester is still high. The number of harvesters in use is limited but there are many farmers who want to use it. Therefore the scheduling of harvesters is needed to achieve the aim of the farmers or the owner of the harvester.

Currently many brands, types and sizes of harvesters are available in the market. There is limitation for using a particular harvester such as some fields cannot be served by some harvester types due to the steepness of the field or insufficient roads to the field. This makes it more difficult to assign the field to the suitable harvester. Moreover, the cost control is also of interest to the owner of the harvester. The level of effectiveness of the machine has been considered when assigning the harvester to the field. It depends on a few factors, such as the model and number of operating years as well as experience of the user.

One harvester can be assigned to more than one field if it has enough working time available even after serving one field. Different harvesters have to work for different hours in a field because of the variation in their harvesting capacities. Each harvester has limitation on its

available working time. Therefore assigning harvesters to different fields falls under the category of generalized assignment problem or GAP.

As mentioned above, besides assigning harvesters to fields there are various other factors to be considered, too. So the problem here cannot be taken as a simple GAP. One of the important factors is assignment of drivers to harvesters. The skill level of the driver will affect the effectiveness of the harvester, which also has a different capacity. The capacity of the harvester in this case, refers to the driving and harvest speeds which have great influence on the resource usage. Then the fields will be assigned to the harvesters which will affect the profit because each field has different sweetness and density levels. Actually this determines the amount of sugarcane that can be harvested per day. Also it has been mentioned before that one harvester can be assigned to more than one field depending on its working time availability. Thus, instead of considering it as a GAP, the problem should be taken as a specialised version of GAP.

Thus, the problem is composed of three actors – the driver, the harvester and the sugarcane field. Here a driver is to be assigned to a harvester set which is to be paired to a suitable field for harvesting so that the daily profit is maximized. The profit depends on the income and cost generated in the system, while the income is generated from the amount of harvested sugarcane sold multiplied by the sweetness level of the harvested sugarcane (the price of sugarcane sold is based on per ton unit). The profit is the result of total income subtracted by the cost of the harvester's fuel consumption and the corresponding driver's daily wages. There is a pool of drivers with various levels of experience and wages. The harvester sets have a range of machine types and operating years, which determines the fuel consumption rate. The efficiency of harvesting is also affected by the driver of the particular harvester set. After the harvester set and its driver have been assigned, they will start to harvest the assigned sugarcane field. Each sugarcane field has a different size, sweetness level and travelling distance. All these activities generate a special case of GAP. So the solution of the problem will not be an exact one and rather it will be a heuristic one.

A differential evolution (DE) algorithm is proposed to be used for arriving at a solution to the problem. The steps involved in the proposed DE are – (1) Generate initial solution, (2) Implement the mutation process, (3) Execute the recombination process & (4) Complete the selection process. Generally DE has three types of vectors, namely Target Vectors, Mutant Vectors and Trial Vectors. Also the DE Algorithm works on the real numbers generated randomly by the DE operators.

Assuming that there are 6 Fields of sugarcane, 4 Harvesters and 5 Drivers as well as 5 sets of target vectors, the initial set of randomly generated numbers for finding an Initial Solution are given as follows –

Target Vector	Sugarcane Field						Harvester				Driver of the Harvester				
	1	2	3	4	5	6	1	2	3	4	1	2	3	4	5
1	0.08	0.61	0.24	0.22	0.21	0.45	0.72	0.09	0.43	0.47	0.65	0.36	0.63	0.00	0.32
2	0.71	0.45	0.40	0.63	0.78	0.07	0.61	0.16	0.54	0.12	0.56	0.70	0.71	0.70	0.38
3	0.55	0.20	0.63	0.34	0.39	0.14	0.10	0.93	0.61	0.09	0.06	0.35	0.88	0.92	0.06
4	0.80	0.75	0.49	0.76	0.74	0.55	0.18	0.82	0.90	0.57	0.75	0.44	0.86	0.19	0.94
5	0.22	0.20	0.12	0.42	0.74	0.90	0.72	0.09	0.43	0.47	0.65	0.36	0.63	0.00	0.32

A_K	80	50	40	70	32	41									
B_K	1.2	1.5	0.9	0.8	1.0	0.8									
E_J							120	180	110	130					
U_J							1.1	1.3	1.0	1.2					
T_J							9	10	6	8					
R_i											1.2	0.7	0.8	1.1	1.2
P_i											0.9	1.2	1.1	0.9	0.8

The symbols used in the above table have the following meaning –

A_K = Area of the sugarcane field K in Acres & B_K = Sweetness factor of the crop of field K

E_J = Fuel consumption rate of harvester J in ₹/hour, U_J = Age factor of harvester J & T_J = Harvesting rate of harvester in Acres /hour

R_i = Experience factor of driver i (which affects fuel consumption rate) & P_i = Experience level of the driver i (which affects harvesting rate)

Time required in hour to drive the harvester J from the Parking Area to the sugarcane field K are given in the table below –

Harvester	Sugarcane Field					
	1	2	3	4	5	6
1	0.80	0.08	0.99	0.35	0.89	0.15
2	0.32	0.79	0.11	0.04	0.18	0.55
3	0.84	0.65	0.66	0.70	0.08	0.05
4	0.25	0.95	0.08	0.63	0.89	0.21

Additional information is as follows –

1. Fuel cost is ₹115 per hour,
2. Selling price of the sugarcane is ₹600 per ton,
3. 12 tons of standard sugarcane is obtained from 1 Acre of land,
4. All drivers are allowed to work for 9 hours a day,

Use the following methodology to **find** the Initial solution of the problem

1. Sort all the subsets of the vectors (i.e. Field, Harvester & Driver) in ascending order.
2. Assign a driver to a harvester and subsequently assign the harvester to the sugarcane field/s using the order obtained in step (1) above.

While doing this, the following rule should be used –

- (a) Assign the entity (Field, Harvester, Driver) that is in position at the front of the order first
- (b) If the current position violates the capacity constraint of the harvester (Maximum 9 hours of usage per day) the entity that is in the next position is allowed to be replaced.

Do you think there is sufficient resource to harvest all the six Sugarcane Fields?

Reference Assignment	What's New

Answer

As per the given instructions of the problem, we have to first of all sort all the subsets of the vectors in ascending order. The results after sorting the data corresponding to the Target Vector 1 is given in the table below.

Target Vector	Sugarcane Field						Harvester				Driver of the Harvester				
	1	5	4	3	6	2	2	3	4	1	4	5	2	3	1
1	0.08	0.21	0.22	0.24	0.45	0.61	0.09	0.43	0.47	0.75	0.00	0.32	0.36	0.63	0.65

The sequence of Assignment in this case is – (1) Pairing of Driver & Harvester and (2) Pairing of Field with the selected set of Driver & Harvester

As per the table above the first pair of Driver & Harvester should be 4 (1st one of the Driver's order) and 2 (1st one of the Harvester's order)

Real harvesting rate of Harvester 2 = Harvesting rate of Harvester 2 X Experience level of Driver 4 = $T_2 \times P_4 = 10 \times 0.9 = 9$ Acres / Hour

Now this pair of Driver (4) & Harvester (2) should be assigned to Sugarcane field 1 (1st one of the Field's order in the above table)

From the given data in the question we have,

Time required to drive Harvester 2 from the parking area to the Sugarcane Field 1 = 0.32 hour

As each Harvester can work at the most 9 hours a day, the effective working hours for Harvester 2 = $9 - (0.32 \times 2) = 8.36$

So the combination of Driver 4 & Harvester 2 is capable of harvesting at the most $(8.36 \times 9) = 75.24$ Acres / day But the Area of Sugarcane Field 1 = 80 Acres which is more than 75.24 Acres

Thus, the combination of Driver 4 & Harvester 2 cannot be assigned to Field 1.

So the combination should be tried for the next Field in the order – Field 5 which has an Area of 32 Acres

Also Harvester 2 requires 0.18 hour to be driven from parking area to Field 5. So effective working hours for this Harvester = $9 - (0.18 \times 2) = 8.64$

Thus, the combination of Driver 1 & Harvester 2 can harvest at the most $(8.64 \times 9) = 77.76$ Acres which is more than 32 Acres by $77.76 - 32 = 45.76$ Acres

So Driver 4 & Harvester 2 combination can be utilised to harvest another field.

Next field in the order is Field 4 with an Area of 70 Acres which is much more than the balance capacity of 45.76 Acres of the particular Driver – Harvester combination. But the next field in

the order i.e. Field 3 has an Area of 40 Acres. Hence it can be assigned to the Driver – Harvester combination of 4 – 1.

Thus, the combination of Driver 4 & Harvester 2 should be assigned to the Fields 5 and 3.

Following same technique other combinations of Driver and Harvester are assigned to the remaining Fields. Results of the assignment are shown below.

Driver	Experience level of Driver	Harvester	Harvesting Capacity (Acres / hour)	Actual Harvesting Capacity (Acres/ hour)	Assigned Sugarcane Field	Area of the field (Acres)	Time required to harvest & movement (hours)	Total time during which Harvester is busy
(1)	(2)	(3)	(4)	(5) = (2) × (4)	(6)	(7)	(8) = [(7) ÷ (5)] + Movement time	(9)
4	0.9	2	10	9	5	32	$3.55 + 0.36 = 3.91$	$3.91 + 4.66 = 8.57$ Hrs.
					3	40	$4.44 + 0.22 = 4.66$	
5	0.8	3	6	4.8	6	41	$8.54 + 0.10 = 8.64$	8.64 Hrs.
2	1.2	4	8	9.6	1	80	$8.33 + 0.50 = 8.83$	8.83 Hrs.
3	1.1	1	9	9.9	4	70	$7.07 + 0.70 = 7.77$	7.77 Hrs.

[Note – Movement time refers to the to and fro time required to drive a Harvester from parking area to the Field]

From the column no. (9) of the above table it is seen that all the assigned Harvesters are remaining busy for a period less than 9 hours. Hence the time constraint is not violated.

But Field No. 2 could not be assigned due to non-availability of Harvester. Hence there is insufficient resource to harvest all the six fields as far as the initial solution is concerned.

Q7

CMA Mat

A Methods Engineer wants to assign four new methods to three work centres. The assignment of the new methods will increase production and they are given in the matrix below. If only one method can be assigned to a work centre, determine the optimum assignment.

Methods	Increase in production (Units) at the Work Centres		
	A	B	C
1	10	7	8
2	8	9	7

3	7	12	6
4	10	10	8

Is this a problem of multiple optimum solution? If yes, then **find** the alternative solution/s also.

Reference	What's New
Multiple optimum solution	

Answer

Assignments are – 1 to A, 2 to Dummy, 3 to B and 4 to C

Optimum assignment = 30 units

Alternative solution – 1 to C, 2 to Dummy, 3 to B and 4 to A

Q8

CMA Mat

Consider a problem of assigning four junior assistants to four tasks. The time (hours) required to complete the tasks are given in the table below.

CLERKS	TASKS			
	A	B	C	D
1	4	7	5	6
2	-	8	7	4
3	3	-	5	3
4	6	6	4	2

Assistant 2 cannot be assigned to task A and Assistant 3 cannot be assigned to task B. **Find** the optimal assignment schedule

Reference	What's New
Prohibited Assignment	

Answer

Assignments are – 1 to B, 2 to D, 3 to A and 4 to C

Minimum total time required = 18 hours

Alternative solution – 1 to C, 2 to D, 3 to A and 4 to B

Q9

CMA Mat

An air-line operates seven days a week has time-table shown below. Crews must have a minimum layover (rest) time of 5 hrs, between flights. Obtain the pair of flights that minimizes layover time away from home. For any given pair the crews will be based at the city that result in the smaller layover.

Delhi-Jaipur			Jaipur-Delhi		
Flight No.	Depart	Arrive	Flight No.	Depart	Arrive
1	7.00 AM	8.00 AM	101	8.00 AM	9.15 AM
2	8.00 AM	9.00 AM	102	8.30 AM	9.45 AM
3	1.30 PM	2.30 PM	103	12.00 NOON	1.15 PM
4	6.30 AM	7.30 PM	104	5.30 PM	6.45 PM

For each pair, **mention** the town where the crews should be based.

Reference

Airline Idle Time Optimisation

What's New

Answer

[Hints – First of all construct a layover matrix showing times between flights when the crew is based in Delhi. For the sake of simplicity assume 15 minutes = 1 unit.

Flight	101	102	103	104
1	96	98	112	38
2	92	94	108	34
3	70	72	86	108
4	50	52	66	88

Since, the crew must have a minimum layover of 5 hrs between flights

The layover time between flights 1 and 101 will be 24 hrs (96 units) from 8.00 AM to 8.00 AM next day i.e. flight 1 arrives Jaipur at 8.00 am and leaves Jaipur 8.00 am next day because the minimum layover is 5 hrs between flights and other flights are there in between. So flight will be there next day only.

Flight 1 to 102 will be (98 units) 8.00 am arrives at Jaipur and leaves Jaipur at 8.30 am next day = 24 hrs + 30 minutes

Flight 1 to 103 will be (112 units) 8.00 am arrives at Jaipur and leaves Jaipur at 12.00 noon next day = 24 hrs + 4 hrs = 112 units

Flight 1 to 104 will be (38 units) 8.00 am arrives at Jaipur and leaves Jaipur at 5.30 pm on the same day = 9 hrs + 30 min = 38 mins

Similarly timings for other flights are also computed.

Next the layover matrix is formed for the situation when the crew is based at Jaipur and shown below.

Flight	101	102	103	104
1	87	85	71	49
2	91	89	75	53
3	113	111	97	75
4	37	35	21	95

Now a table for minimum layover times between the flights is constructed using the data of the above two tables. It is shown below. [Data marked with (*) denote the crew is based at Jaipur.

Flight	101	102	103	104
1	87*	85*	71*	38
2	91*	89*	75	34
3	70	72	86	75
4	37*	35*	21*	88

Using Hungarian Method the problem is solved now and the optimal assignment is given below. Flight 1 – 103, Flight 2 – 104, Flight 3 – 101 and Flight 4 – 102]

Chapter 13

Simulation

Q1

CMA Mat

Generate a string of 12 pseudo random numbers by Mixed type Residue Method when it is given that the values of the constants are $a = 4$, $b = 7$, $m = 9$ and the Seed is 1.

Reference	What's New
Generate Random nos.	

Answer

The expression for Random Number Generator by Mixed type Residue Method is given as follows –

$$r_{i+1} = (a.r_i + b) \text{ (modulo } m) \text{ where } a = 4, b = 7, m = 9 \text{ and Seed } = r_0 = 1$$

Calculations for Random Number generation by Mixed type Residue Method

r_i	$r_{i+1} = (a.r_i + b) \text{ (modulo } m)$	Result with Residue
r_1	$r_1 = (a.r_0 + b)/m = (4.1+7)/9 = 11/9$	1 with residue 2
r_2	$r_2 = (a.r_1 + b)/m = (4.2+7)/9 = 15/9$	1 with residue 6
r_3	$r_3 = (a.r_2 + b)/m = (4.6+7)/9 = 31/9$	3 with residue 4
r_4	$r_4 = (a.r_3 + b)/m = (4.4+7)/9 = 23/9$	2 with residue 5
r_5	$r_5 = (a.r_4 + b)/m = (4.5+7)/9 = 27/9$	3 with residue 0
r_6	$r_6 = (a.r_5 + b)/m = (4.0+7)/9 = 7/9$	0 with residue 7
r_7	$r_7 = (a.r_6 + b)/m = (4.7+7)/9 = 35/9$	3 with residue 8
r_8	$r_8 = (a.r_7 + b)/m = (4.8+7)/9 = 39/9$	4 with residue 3
r_9	$r_9 = (a.r_8 + b)/m = (4.3+7)/9 = 19/9$	2 with residue 1
r_{10}	$r_{10} = (a.r_9 + b)/m = (4.1+7)/9 = 11/9$	1 with residue 2
r_{11}	$r_{11} = (a.r_{10} + b)/m = (4.2+7)/9 = 15/9$	1 with residue 6

So the required string of Pseudo Random Numbers – 1, 2, 6, 4, 5, 0, 7, 8, 3, 1, 2, 6,

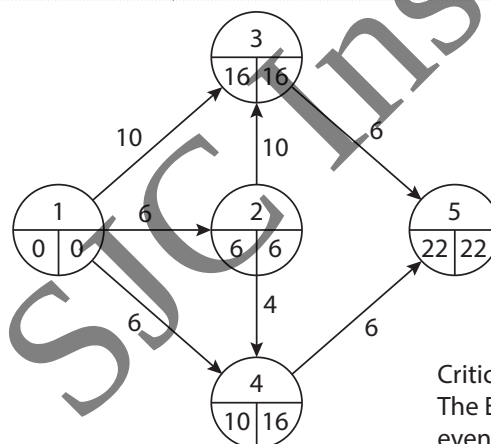
Network Analysis - PERT, CPM

Q1

CMA Mat

Consider the following problem of project scheduling to obtain a schedule which will minimize the peak manpower requirement and smooth out period to period variation of manpower requirement.

Activity	Duration (Days)	Manpower requirement
1-2	6	8
1-3	10	4
1-4	6	9
2-3	10	7
2-4	4	6
3-5	6	17
4-5	6	6



Critical Path 1 – 2 – 3 – 5
The EST & LFT for each event are presented in boxes

Critical Path : 1 – 2 – 3 – 5

Reference

Resource levelling

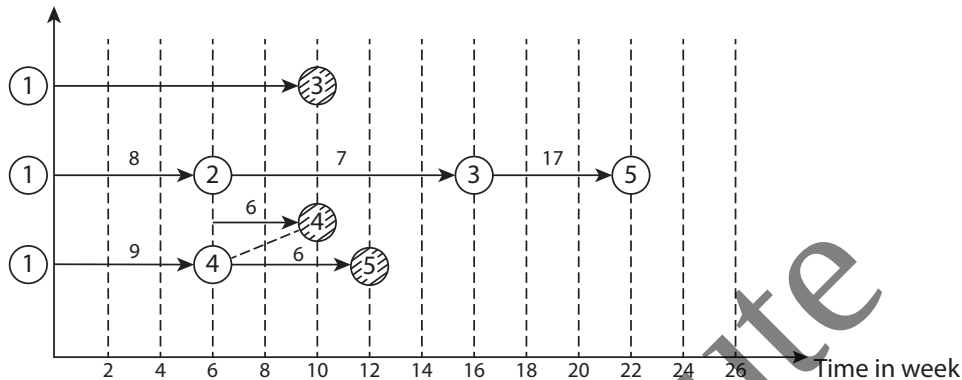
What's New

Answer

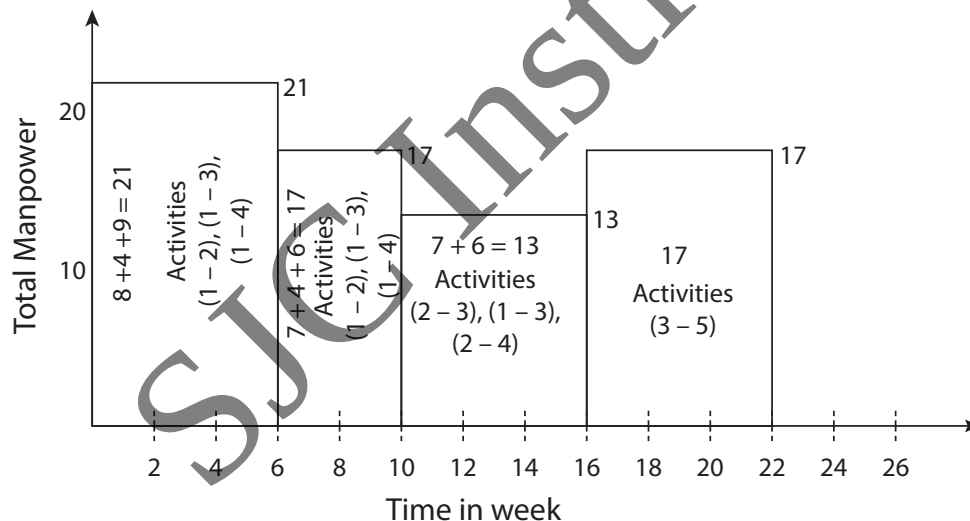
The Network Diagram is shown in the figure above. Critical path = 1-2-3-5

The Earliest & Latest Times for each event are presented in quadrants of the Node Circles.

Now the activities are represented on a time scale as shown below. The corresponding Manpower requirement are mentioned above the arrows representing Activities in the diagram.

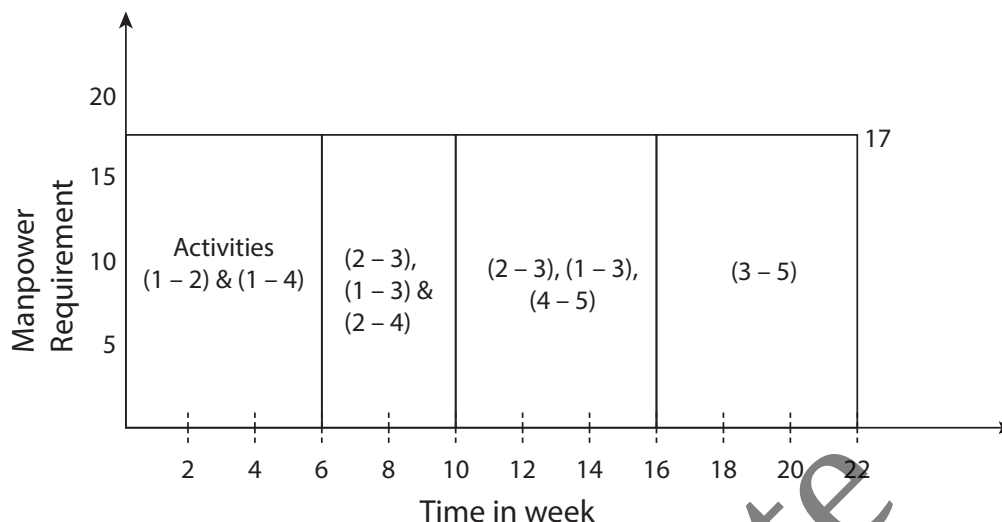


The corresponding Manpower Requirement Histogram (shifting activity 4 – 5 to the right) is as follows.



The peak manpower requirement is 21 and it occurs between 0-6 weeks. The activities which are scheduled during this period are (1 – 2), (1 – 3) and (1 – 4). Activity (1 – 2) is a Critical Activity. So it should not be disturbed. Between activities (1 – 3) and (1 – 4), the activity (1 – 3) has a Total Float of 6 weeks. So it has to be delayed to the maximum possible extent (i.e it can be started at the end of the 6th week).

The modified Histogram is shown below.



Q2

CMA Mat

Information about a project are given as follows

Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor Activity	None	None	A	None	B, C	D	D	E, F	E, F	G, I	H
Least time	4	5	8	2	4	6	8	5	3	5	6
Greatest time	8	10	12	7	10	15	16	9	7	11	13
Most likely time	5	7	11	3	7	9	12	6	5	8	9

Draw the Network diagram and **identify** the Critical Path. **Find** the Earliest and Latest times of each Event. Also **calculate** the Total Float for each activity. Consider the unit of various times given above is weeks.

Reference	What's New
Network diagram, Critical Path, Earliest and Latest times, Total Float for each activity	

Answer

Critical Path: A – C – E – H – K,

Values of Total Floats (in weeks) for the Activities A through K are respectively 0, 8.83, 0, 10, 0, 10, 15, 0, 2.5, 2.5, 0

Q3

CMA Mat

M/S Ramswarup Limited is a renowned trader of Steel Structural with head office in Kolkata, has planned to diversify and decided to enter into manufacturing sector. They want to put up a Foundry with an aim to produce high quality abrasion resistant castings which has a very good export potential. A detailed study has been carried out by the team of M/S Ramswarup with the help of consultant Dr. P.K.Mathur who is a management consultant with vast hands on experience in manufacturing of castings. After the study, Dr. Mathur suggested not to go for a huge installed capacity at the beginning. Rather he is of the opinion to go in phases. Accordingly he listed the following necessary activities with associated precedence relationships as well as Expected Durations in weeks for the first phase operation.

Activity	A	B	C	D	E	F	G	H	I	J
Prede- cessor	None	None	A	A	A	C	D	B, D, E	H	I, G, F
Dura- tion	6	3	5	4	3	3	5	5	2	3
S.D	1	0.5	1	1	0.5	0.5	1	1	0.5	1

You are entrusted with the responsibility of assisting Dr. Mathur in different areas as he deemed necessary. To properly Plan, Schedule and Control the first phase of the project, Dr. Mathur wants you to draw the Network diagram and identify the Critical Path. Also he wants to know the project duration.

After getting the Network diagram from you, Dr. Mathur submitted the same to the top management of the company for getting a green signal to start the work. Till the permission is obtained, Dr. Mathur envisaged some delay in the Activities B, E and G by 3, 2 and 2 weeks respectively. He wants to know from you the effect of these delays on the project completion. It has to be reported to him urgently so that he can subsequently bring the same to the notice of the management.

The durations mentioned in the above table are expected values which are Normally distributed with the Standard Deviations (S.D) mentioned above. Ignoring the delays mentioned above for the activities B, E and G and possible effect of uncertainty in the non - critical activities, determine 95% Confidence Interval for the Expected Project Completion.

The Cost of the project is estimated to be ₹ 10,00,000/-. If it is completed within 24 weeks the expected returns should be ₹10,000,000/-. But if the deadline fails the export market will be penetrated by competitors from South Africa and Australia. That will result in a net revenue of only ₹ 2,00,000/-.

You need to **determine** the expected returns ignoring the delays mentioned before and report it to Dr. Mathur.

Reference
Network diagram, Critical Path, Earliest and Latest times, Total Float for each activity

What's **New**

Answer

Critical Path is A – D - Dummy – H – I – J and project duration is 20 weeks (Can be obtained by drawing the Network diagram)

On calculation of **Total Float**, the values of the same for Activities B, E and G are respectively 7, 1 and 2 weeks. So delay of 3 weeks in B and 2 weeks in G are not going to hamper the project completion. But delay of 2 weeks in E is going to increase the project duration by 1 week because it is having a Total Float of 1 week.

Mean project duration = 20 weeks (from Network diagram)

Variance of project duration = Sum of the variances of the Critical Activities i.e A, D, H, I & J = $1^2 + 1^2 + 1^2 + (0.5)^2 + 1^2$

S.D of project duration = $\sqrt{(1^2 + 1^2 + 1^2 + (0.5)^2 + 1^2)} = 2.06$ weeks

So 95% Confidence Interval of the expected project completion = Mean duration ± 1.96 (S.D)
= $20 \pm 1.96 \times 2.06 = 15.96$ to 24.04 weeks

P (Project duration exceeds 24 weeks) = $P(X > 24) = P[Z > (24 - 20) / 2.06] = P[Z > 1.94] = 1 - P(Z \leq 1.94)$

= $1 - 0.9738 = 0.0262$

Expected return from the project = $10,000,000 \times P(\text{Project will be completed within 24 weeks}) + 2,00,000 \times P(\text{Project will not be completed within 24 weeks}) - \text{Initial investment} = 10,000,000 \times (1 - 0.0262) + 2,00,000 \times 0.0262 - 10,00,000 = ₹ 87,43,240/-$

Q4

CMA Mat

Activity P is followed by Activity Q which in turn, is followed by Activity R. The Direct Cost of these Activities in relation to the choice of feasible durations is given in the Table below. For all the three activities taken together **what** is the minimum possible Direct Cost for a total duration of 21 days.

	Activity P			Activity Q			Activity R		
Duration (Days)	7	6	5	8	7	6	9	8	7
Direct Cost (₹ '000)	12	14	15	20	23	27	40	42	45

Reference	What's New
Minimum Possible Direct Cost	

Answer

₹ 77,000

Q5**CMA Mat**

A project has four activities P, Q, R and S as shown below.

Activity	Normal Duration (Days)	Predecessor Activity	Cost Slope (₹ / DAY)
P	3	-	500
Q	7	P	100
R	4	P	400
S	5	R	200

Normal Cost of the project is ₹ 10000/- and the Overhead Cost is ₹ 200 per day. If the project duration has to be crashed down to 9 days, then **what** will be the Total Cost of the project?

	Reference	What's New
	Crashing	

Answer

₹ 12,500

Chapter 15

Learning Curve

Q1

CMA Mat

A manager wants to determine an appropriate learning percentage for a new type of work his company will undertake. He has obtained unit times for the initial six repetitions which are given below.

Unit	1	2	3	4	5	6
Completion time (Minutes.)	46	39	35	33	32	30

Determine –

- The learning percentage.
- Estimate the average time if a total of 30 units are planned.
- After producing how many units, will completion time less than 15 Minutes per unit be achieved?



Reference

Learning rate, Average time, No. of units to achieve target labour time.

What's New

Back Calculation

Answer

- As per the concept of doubling of Learning Curve theory we have,
Completion time of 2nd Unit = Completion time of 1st Unit $\times$ Learning percentage or, 39
 $= 46 \times LP$ or, $LP = 0.8478$
Completion time of 4th Unit = Completion time of 2nd Unit $\times$ Learning percentage or, 33
 $= 39 \times LP$ or, $LP = 0.8461$
 So the Learning Percentage (LP) can be taken as 0.85 (Approx.) i.e. 85%
- $y_x = ax^b$ where $a = \text{Completion Time of the 1st Unit} = 46 \text{ Mins}$, $X = \text{Cumulative no. of units produced} = 30$
 $b = \text{Learning Index} = [\log(\text{learning percentage})] / \log 2 = \log(0.85) / \log 2 = -0.2344$ & $y_x = y_{30} = ?$
 So average completion time of 30 units $= y_{30} = 46.(30^{-0.2344}) = 20.73 \text{ Mins.}$

(iii) Using the above formula we have, $15 = 46(X^{-0.2344})$

$$\text{or, } \log 15 = \log 46 - 0.2344 \log X$$

$$\text{or, } \log X = 2.0762$$

$$\text{or, } X = 10^{2.0762} = 119.179$$

Average completion time will be less than 15 minutes if 120 units are produced.

Q2

CMA Mat

XYZ Co. does not have any costing department. You **need** to help them out in finding the total variable cost for a contract of a single order of 600 units for one of their products.

The average unit variable cost data for an initial batch of 200 units are as follows –

(i) Direct Material – 15 m² @ ₹ 80 per m²

(ii) Direct Labour –

8 Hours in Department 1 @ ₹ 75 per hour and 100 Hours in Department 2 @ ₹ 100 per hour

(iii) Variable Overhead – 25% of Direct Labour

Labour times in Departments 1 and 2 are expected to follow 80% and 70% Learning Curves respectively.

Reference	What's New
Total Variable Cost of Single Order	

Answer

From Learning Curve Theory, we have $y = aX^b$

where y = Cumulative average direct labour hours per unit

a = Direct labour hours for the first unit

X = Cumulative number of units

b = Learning Index

As this is a case of Batch Production, we define 'a' as the time for production of the first batch of 200 units

For Department 1, we have $a = 200 \times 8 = 1600$ hours and Department 2, $a = 200 \times 100 = 20,000$ hours

Department 1 has Learning efficiency of 80%. So $b = \log 0.8 / \log 2 = -0.322$

Department 2 has Learning efficiency of 70%. So $b = \log 0.7 / \log 2 = -0.515$

So formula for Department 1 is $y = 1600(X^{-0.322})$ and for Department 2, $y = 20000(X^{-0.515})$

Calculations

Direct Material Cost = $(15 \text{ m}^2 \times ₹80 \text{ per m}^2) \text{ per unit} \times 600 \text{ units} = (₹1200) \times 600 = ₹ 72,000$

Direct Labour Cost –

As this is a case of Batch Production and 1st batch is of 200 units, the order of 600 units will be equal to 3 batches.

For Department 1, Cumulative average time to produce 600 units = $y = 1600(3^{-0.322}) = 1123.277$ hours per batch

Total time required for 3 batches = $1123.277 \times 3 = 3369.83$ hours

So the Labour Cost is $3369.83 \times ₹75 \text{ per hour} = ₹2,52,737.25$

For Department 2, Cumulative average time to produce 600 units = $y = 20000(3^{-0.515}) = 11358.279$ hours per batch

Total time required for 3 batches = $11,358.279 \times 3 = 34,074.84$ hours

So the Labour Cost is $34,074.84 \times ₹ 100 \text{ per hour} = ₹ 34,07,484$

Total Direct Labour Cost = $₹ (2,52,737.25 + 34,07,484) = ₹ 36,60,221.25$

Variable Overhead – 25% of Direct Labour Cost = $0.25 \times 36,60,221.25 = ₹ 9,15,055.31$

Total Variable Cost for 600 units is – $₹ (72000 + 36,60,221.25 + 9,15,055.31) = ₹ 46,47,277$

Q3

CMA Mat

An automobile manufacturer is conducting a product recall after it was discovered that a possible defect in the braking mechanism could cause loss of braking in certain cars. The recall covers a span of three model years. The company sent out letters to the car owners promising to repair the defect at no cost at any dealership.

The company's policy is to pay the dealer a fixed amount for each repair. The repair is somewhat complicated and the company expects learning to be a factor. In order to set a reasonable rate for repairs, the company conducted a number of repairs in house. Thereafter it was decided that the appropriate pay is ₹7000 per repair based on a flat hourly rate of ₹1700 and a learning percentage of 90.

Shortly after the dealers began making the repairs, the company received complaint from several dealers that they were encountering resistance from the workers who felt the flat rate was way too low and were threatening to refuse the work on those jobs. One of the dealers collected data on task times and sent it to the company given as follows

1. Three mechanics each completed two repairs.
2. Average time for the first unit was 9.6 hours and for the second unit was 7.2 hours.

The dealer has suggested a rate of ₹8500 per repair. You have been asked to **investigate** the situation and **prepare** a report.

1. **Prepare** a list of questions that you will need to have answered in order to analyze the situation.
2. **Comment** on the information provided in the case.
3. **What** preliminary thoughts do you have on solutions to the points you have raised?

Reference	What's New
 List of Questions, Comments, Solution using Learning Curve	

Answer

1. The list of questions for analyzing the situation could be as follows
 - Is the learning percentage considered by the manufacturer correct?
 - What time per repair has been allocated by the manufacturer?
 - After how many repairs this time is achievable?
 - What is the total number of repair jobs?
 - Is the demand of the workers for higher rate justified?
 - Should the rate suggested by the dealer be acceptable?
2. Information provided in the case is somewhat incomplete. Though it has been mentioned that the cars manufactured within a span of 3 model years are recalled but no numerical figure about the total number of cars recalled is clearly mentioned. In absence of this figure, it is not possible for anyone to understand exactly after how many repairs the rate suggested by the manufacturer becomes feasible. Moreover due to the very nature of learning phenomenon, the time required to complete the initial jobs of repair are bound to be more resulting in demand for higher rate by the workers.
3. Solutions to the points raised in (1) above are as follows
 - From the dealer's data of repair of 1st and 2nd cars we find the same 9.6 and 7.2 hours respectively. That means the Learning percentage is $7.2/9.6 = 0.75$ or $75\% < 90\%$ Hence the actual learning percentage is less than that considered by the manufacturer.
 - The manufacturer's rate per repair is ₹7000 @ ₹1700 per hour. Thus, time allocated per repair is $7000 / 1700 = 4.12$ hours
 - Considering the dealer's figure of 9.6 hours for the 1st job to be correct the calculations for finding the number of repairs required to achieve 4.12 hours per repair is done as below.

$$Y = aX^b$$

where Y = Average time to complete X number of jobs = 4.12 hours,

a = Time to complete the 1st job = 9.6 hours and

b = Learning Index = $\log(\text{learning percentage}) / \log 2 = \log(0.75) / \log 2 = -0.415$

$$4.12 = 9.6 \cdot X^{-0.415} \text{ Or, } X^{-0.415} = 4.12 / 9.6$$

$$\text{Or, } X^{-0.415} = 0.429 \text{ Or, } -0.415 \log X = \log(0.429) \text{ Or, } \log X = 0.886 \text{ Or, } X = 10^{0.886} = 7.7$$

Thus, the time considered by the manufacturer is achievable in 8th repair

- Total number of repair jobs is not clearly spelt here, but it can be well assumed that the same would definitely be a few thousands.
- Demand of the workers is not justified because after doing only 8 repairs the rate suggested by the manufacturer is achievable and thereafter the actual labour hours will be further reduced causing the workers to earn good amount because whatever hours they take the amount of ₹7000 they will definitely get.
- As explained above the fixed rate of ₹7000 is quite a good rate and hence the same does not need a higher revision.

Q4

CMA Mat

A tool manufacturing company wants to sell 49 units to a customer. **Calculate** the minimum learning rate if the 49th unit cannot take more than 5.6 hours. Given:- 1st unit takes 16 hours

Reference	What's New
Minimum learning rate	Back Calculation

Answer

Minimum learning rate = 83%

Q5

CMA Mat

Assume that the first unit of a very special product takes 15 labour hours to produce and the Learning rate is 74%. **How many** units should at least be produced to reduce the time per unit to 6 hours or less?

Reference	What's New
Minimum production	Back Calculation

Answer

At least 9 units should be produced.

Q6

CMA Mat

A lot of 20 units is to be produced. Labour cost is ₹ 150 per hour. Set up cost is ₹ 3500 and Material cost is ₹ 1500 per unit. The learning percentage is expected to be 90. Overhead is charged at the rate of 50% of total Material, Labour and Set Up cost. **Determine** the average unit cost for the lot when the first unit took 5 hours to complete.

Reference

Average unit cost using Learning Curve

What's **New**

Answer

Average unit cost = ₹ 3226

Q7

CMA Mat

The organisation to which you belong is a regular supplier of Indian Railways. Submission of bids against various tenders is a regular affair of the firm. As a part of the organisation's Costing Department your job is to **help** the Department who are dealing with these bids by calculating the costs.

Recently there is a limited tender for supply of 1600 pieces of a product which is somewhat new to your organisation. Indian Railways want supply of batches of 100 at a time. It has been estimated that the average cost of the first batch of supply is ₹ 100 per piece. 90% Learning Curve applies to the cumulative labour cost on this contract.

- On the basis of the information above **prepare** an estimate of labour cost for the full contract.
- As per the tender, a supplier will be awarded with another order of 800 pieces after the successful completion of supply of the first 1600. **Find out** the incremental labour cost for producing an additional 800 unit?

Reference

Labour cost, incremental labour cost

What's **New**

Answer

- (a) Total cost of 1600 pieces = ₹ 104976,
(b) Incremental cost for the additional 800 pieces = ₹ 43078

[Hints : Additional 800 pieces after 1600 means a total of 2400 pieces which means 24 batches of 100 each. Also 2400 pieces is not the double of 1600. So the formula to be applied here is $Y = aX^b$ with $a = ₹ 100$, $X = 24$ and $b = \log (0.90) / \log 2$]

Q8

CMA Mat

A firm has a training program for an operation. The progress of the trainees is carefully monitored. An established standard requires a trainee to be able to complete the sixth repetition of the operation in 6 hours or less. Those who are unable to do this are assigned to other jobs. Currently, three trainees have each completed two repetitions: Trainee Ajay had times of 9 hours for the first and 8 hours for the second repetition; Trainee Badal had times of 10 hours and 8 hours for the first and the second repetitions and trainee Chanchal had times of 12 and 9 hours. **Which** trainee or trainees will achieve the standard? **Explain** with reasons.

Reference

Meeting the standard

What's New

Back Calculation

Answer

Both Badal and Chanchal will achieve the standard because for them the time to complete the 6th repetition are respectively 5.6 hours and 5.7 hours i.e. less than 6 hours, but for Ajay, the time is 6.64 hours.

Q9

CMA Mat

The vendor of an automobile company is approached by the principals to build a special component for them. Based on the previous experience of the similar components, the following data have been made ready by the Costing Department of the vendor.

Breakeven quantity = 24 units, Time for producing the 24th unit = 161 hours, Learning rate = 80%

Direct labour cost = ₹ 120 per hour

- (a) Based on the above data **what** quote should be given to the principals for the direct labour cost for an order of 30 units?
(b) The quotation given as per above is accepted by the principals and a green signal is given to the vendor to start production. Although the first few units took longer than the

estimated labour hours but thereafter it has started to stabilize. However the customer has made some changes in design after the production has started. They want the change to be implemented after the 15th unit. As per the estimation, the design change will cause 30 hours of new work for the 24th unit when the Learning Curve remains same. In fact the new work replaces the same amount of work of 30 hours for the 24th unit. **What** should be the revised quotation, if any, for the order of 30 units?

 **Reference****Quotation using Learning Curve****What's New****New Learning effect from 16th unit****Answer**

- (a) Quote for the direct labour cost for an order of 30 units = ₹ 539421
(b) Revised quote for the direct labour cost to make 30 units = ₹ 583000

[Hints : Due to design change there will be new learning effect from 16th unit onwards]

NOTES

SJC Institute

Part V

Case

Studies

SJC Institute



Cost Control and Cost Reduction

– Contemporary Techniques

Case Study 1

How a Reputed Consulting Firm (RCF) assisted a Client in Cost Reduction

The client is world leader in the food and beverage industry. As his legacy, the departing executive committed the company to adding hundreds of basis points to the bottom line. The CFO of the client approached RCF in 2011 to help the company establish a strategic cost reduction program. Understanding the culture of the company, the CFO wanted to start small and expand as milestones were achieved.



Although there are a number of approaches to cost reduction, many companies opt for zero-based budgeting. Zero-based budgeting removes a budget's baseline, which means that every cost identified in the budget must be approved. The client, under discussion, preferred to take a priority-based budgeting approach where marginal services or costs need to be justified.

RCF began to prioritize cost savings by looking at half of the company's cost base. RCF team recommended that the company first seek savings in non-headcount-related areas, such as travel and entertainment, as well as consultant fees. The goal was to remove 10% of the costs in these areas in the first year and then 5% to 10% annually thereafter for the following two years.

Once the company was well on its way to removing costs from these areas, RCF team helped the company renegotiate procurement costs to gain greater efficiencies. After that, RCF team set to work determining cost savings in targeted emerging market locations to expand "efficient growth."

Priority-based budgeting formed one part of the cost reduction equation for all of RCF team's efforts. Culture formed another. For every cost savings RCF team members identified, they had to then put it through the client's cultural lens to confirm that the decision was a good fit for both the company and its employees.

The company had spent years cultivating a youthful and energetic culture for the organisation. As such, it needed to balance shareholder and analyst demands for cost reductions with a culture that formed the company's identity and its brand — with employees and in the market.

Working closely with the client in targeted areas of the organisation, RCF helped the company carve out more than US\$250 million in costs from its bottom line over three years by reducing or eliminating activity in nonheadcount areas. RCF expects the company would save another US\$50 million by renegotiating contracts and generating efficiencies in the company's procurement processes.

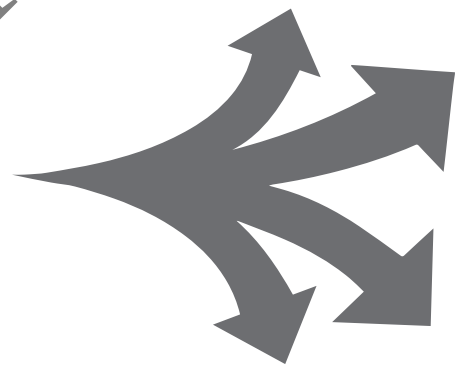
In addition to the savings identified, the client's cost reduction program is also expected to help the departing executive to reach his legacy goal. Because culture is such an important factor for the client, the company took a conservative approach to its priority-based decision-making. However, the CFO was willing — and prepared — to take more aggressive action, should the need arise.

Case Study 2

A Four-pronged Approach to input steel prices and commodity purchases

A research paper by 'Atkearney' suggests that there are four ways that the companies can improve their steel and other commodity purchases.

1. **Capture the value of Scrap:** Steel scrap, a by-product of the manufacturing process, is typically not well managed. Depending on the process, upto 30% of input steel is unused and considered waste. This is true for numerous manufacturing industries from automotive, white goods and electronics to heavy industries. While manufacturers focus on minimizing waste, they often fail to capture the value of material scrap in the part price. Companies that account for scrap material value in the component price can reduce their material costs by 5 to 8 percent.
2. **Increase Sourcing Power:** Steel is often purchased from an intermediary, such as a steel service center, resulting in an extended supply chain that includes numerous service centers and different pricing levels. It is possible to increase negotiating leverage and reduce costs by gaining more visibility and control over material supply chain – specifically by optimizing the material purchases directly from the large steel mills.
3. **Optimize Material Usage:** The third component of the strategy is to focus on reducing costs through technical improvements including reducing complexity, shrinking, part design costs, and segmenting supplies. To reduce complexity, the focus turns to portfolio rationalization to reduce specifications such as gauges and grades and then implementing processes to prevent re-proliferation. Reducing part design costs begin with collaborative reviews with internal and supplier engineering teams to evaluate all parameters that affect material costs and utilisation. In the context of segmentation, sourcing parts that use the same grade or gauges to the same supplier allows for nesting parts more effectively. Optimization of material usage can result in cost reduction from 5 to 8 percent.



4. **Include Scrap in Material Supply Chain:** Material scrap generated – both internally and externally – shall form part of material supply chain. The goal is to create a closed loop network whereby the company uses the scrap dealer for processing and transportation and sells scrap directly to a scrap consuming supplier.

Case Study 3

Ichalkaranji Power-Loomers minimise their Fixed Cost Burden

Ichalkaranji is a moderate industrial town located in the western parts of Maharashtra. The place is well known for skilled weavers and a prosperous power-loom industry. Hence, the town has acquired the fame as the Manchester of Maharashtra. The power-loom is considered as a diligent economic model between well-organized mill sector and outdated handlooms. An important characteristic of the power loom industry in Ichalkaranji is that most of the looms are owned by the local micro entrepreneurs with an average holding of about four looms.

One power-loom unit, on an average, produces about 80 metres of grey cloth per day from either cotton or polyester yarn. The cloth is used as the primary material by the textile and garment industries. As the textile industry is prone to seasonal fluctuations, i.e., surging in sales during festive and marriage seasons and declining sales in slack seasons; the power-looms also used to suffer from lack of buyers during the slack season. In order to retain the skilled labour during the slack season, the loom owners used to pay wages to the workers even when there is no work. As such, the industry is burdened with considerable retaining costs (i.e., idle labour costs) which tend to remain fixed for the entire slack-period. These of the idle costs used to cripple the earnings of the loom owners severely.

Grey cloth is one of the primary packing materials for the agriculture-seeds because of the fact that most of the seeds are packed in cloth bags. One of the renowned seed companies, which was having international presence, was looking for ways and means of reducing its packing costs. The company came to know about the slack season cost-burden of the power loom industry of Ichalkaranji. The company has also observed that peak demand for the seed packing coincides with the slack-season of the power looms.

The seed company came up with a pricing proposal for buying the cloth from the power loom owners during the slack season on variable cost-plus basis whereby the fixed cost burden of the looms would be reduced substantially. The loom owners came forward willingly and supplied the cloth to the seed company at fairly cheaper prices. The end result was a win-win cost reduction both for the loom owners as also the seed company. The loom owners were able to reduce their slack season idle costs by about eighty per cent and the seed company was able to reduce its packing material costs by about twenty percent.

Case Study 4

MEC multiplies its profit through Outsourcing

MEC (name changed) is a medium scale electronic manufacturer located in Central India. Its annual turnover used to be about five crores of rupees. The components used in the company's products could be conveniently divided into A, B and C; A items accounting for ten percent in quantity and seventy percent in value, B items twenty percent in quantity and twenty percent in value and C items seventy percent in quantity and ten percent in value. Over a period of time, the company established a niche for quality in its field and started experiencing an upswing.



For the year under consideration the company was flooded with profitable orders worth twenty crores of rupees, i.e., four times of its existing turnover. After a diligent review, the management felt that its existing capacity can, at best, be stretched by fifty percent whereby orders to the extent of ₹7.5 crore can be executed.

The management had several sessions of brainstorming. The executives realised that the company is capable of producing the entire quantity of A & B items needed for the orders; but the challenge was about C items. Assembly labour was identified as the major limiting factor for the production of C items. The deliberations were extended to the vendors and a viable solution brought out. The vendors were willing to undertake the work of assembling the C items, if they were given to them in SKD (Semi Knocked Down) form. The proposition was readily accepted.

At the end MEC was able to execute the orders worth ₹ 20 crores successfully by outsourcing the assembly operations relating to C items. The productivity as also the profitability of MEC and its vendors were multiplied by means of prudent deployment and optimum utilisation of their labour resources.



Value Analysis & Value Engineering

- Business Process Reengineering

Case Study 1

Aadarsh Instruments, located in Ambala, is a medical instrument manufacturing company considered to apply Value Engineering in to the Focus Adjustment Knob in one of their model SL 250 for Slit Lamp in microscope. This microscope has found application in the field of eye inspection. The value engineering analysis may help company in running its export business of medical microscope. This firm is producing different types of microscopes which they export to various countries around the globe. All of the products manufactured here are conforming to the international standards. It is an ISO certified company

The following are the steps to be used for carrying out the Value Engineering exercise by Aadarsh Instruments in their model SL 250 for Slit Lamp in Microscope for the Focus Adjustment Knob:

- (i) Selection of the Product Plan.
- (ii) Gathering Product Information
- (iii) Functional Analysis
- (iv) Creativity Phase and preparing the work-sheet
- (v) Evaluation Sheet
- (vi) Cost Analysis
- (vii) Result and Conclusion
- (viii) Implementation.

The total savings after the implementation of value engineering are as given below:

- Cost before analysis — ₹ 29.99
- Total Cost of Nylon Knob — ₹ 18.40
- Saving per product — ₹ 11.59
- Percentage saving per product — 38.64%
- Annual Demand of the product — 8,000
- Total Annual Saving — ₹ 92,720
- Value Improvement — ₹ 62.98% $\left(\frac{92,720}{18.40 \times 8,000} \times 100 \right)$

With a critical evaluation of this study, Aadarsh Instruments has been able to increase the value of the product by substituting another material in place of the one currently in use. The % value improvement is to the tune of 62.98% and the total annual saving has been ₹92,720. The various advantages have been observed in terms of:

- Cost Reduction
- Increase in overall production
- Reduction in man-power
- Reduction in scrap.

Thus, the cost has been brought down by a substantial margin and thereby the value of the product has been increased.

Case Study 2

Credit Card Approval

An applicant submits an application. The application is reviewed first to make sure that the form has been completed properly. If not, it is returned for completion. The complete form goes through a verification of information. This is done by ordering a report from a credit company and calling references. Once the information is verified, an evaluation is done. Then, a decision (yes or no) is made. If the decision is negative, an appropriate rejection letter is composed. If the decision is positive, an account is opened, and a card is issued and mailed to the customer. The process, which may take a few weeks due to workload and waiting time for the verifications, is usually done by several individuals. Business processes are characterized by three elements:

- The inputs, (data such customer inquiries or materials),
- The processing of the data or materials (which usually go through several stages and may necessary stops that turns out to be time and money consuming), and
- The outcome (the delivery of the expected result).

The problematic part of the process is processing. Business process reengineering mainly intervenes in the processing part, which is reengineered in order to become less time and money consuming.



Case Study 3

Ford Motors

One of the best-known examples of organisations that used BPR in an effort to become more efficient is Ford Motors, a car manufacturer. Ford Motor Company is the world's second largest manufacturer of cars and trucks with products sold in more than 200 markets.

With inherent large-scale growth issues, more demanding customers, and mounting cost pressures, Ford needed to transform from a linear, top-down bureaucratic business model to an Internet ready, nimble organisation that engages and integrates customers, suppliers, and employees. Working with Cisco, Ford integrated and leveraged their supplier base by designing Covisint, an end-to-end infrastructure that enables an online, centralized marketplace connecting the automotive industry supply chain. Ford also enhanced the customer buying experience through redesigned and more user friendly Web sites.

As a result, Ford is enjoying an increase in customer satisfaction, sees huge revenue opportunities for developing and retaining loyal product advocates, and has taken both complexity and cost out of the supply chain.



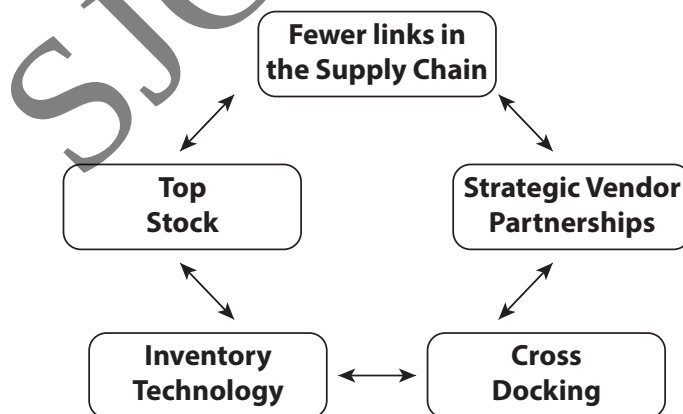


Supply Chain Management

Case Study 1

Walmart's Inventory Innovations

Fewer links in the Supply Chain: Even in its early years, Walmart's supply chain management contributed to its success. Walmart's supply chain innovation began with the company removing a few of the chain's links, right from the very beginning. Founder Sam Walton, who owned several Ben Franklin franchise stores before opening the first Walmart in Rogers, Arkansas in 1962, selectively purchased bulk merchandise and transported it directly to his stores. Later, in the 1980s, Walmart began working directly with manufacturers to cut costs and more efficiently manage the supply chain. In the process, Walmart has pioneered the concept of Vendor Managed Inventory (VMI) with an added competitive advantage.



Strategic Vendor Partnerships: Walmart has long practiced strategic sourcing to find products at the best price from suppliers who are in a position to ensure they can meet demand. The company then establishes strategic partnerships with most of their vendors, offering them the potential for long-term and high-volume purchases in exchange for the lowest possible prices.

Furthermore, Walmart streamlined supply chain management by constructing communication and relationship networks with suppliers to improve material flow with lower inventories. The

network of global suppliers, warehouses, and retail stores has been described as behaving almost like a single firm.

Cross-docking: Cross-docking is a logistics practice that is the centerpiece of Walmart's strategy to replenish inventory efficiently. It means the direct transfer of products from inbound or outbound truck trailers without the need for extra storage, by unloading items from an incoming semi-trailer truck or railroad car and loading these materials directly into outbound trucks, trailers, or rail cars (and vice versa), with no storage in between.

Suppliers have been delivering products to Walmart's distribution centers where the product is cross-docked and then delivered to Walmart stores. Cross-docking keeps inventory and transportation costs down, reduces transportation time, and eliminates inefficiencies.

Walmart's truck fleet of drivers continuously deliver goods to distribution centers (located an average 130 miles from the store), where they are stored, repackaged and distributed without sitting in storage. Goods will cross from one loading dock to another, usually in 24 hours or less, and company trucks that would otherwise return empty "backhaul" unsold merchandise.

This strategy has reduced Walmart's costs significantly, allowing the company to pass those savings on to their customers with highly competitive pricing.

Advanced Inventory Technology: In its relentless pursuit of low consumer prices, Walmart embraced and invested in technology to become an innovator in the way stores track inventory and restock their shelves. Thus, allowing them to cut costs. In 2015, the company spent a reported \$10.5 billion on information technology and has also invested significantly in improving their e-Commerce capability.

Technology plays a key role in Walmart's supply chain, serving as the foundation of their supply chain strategy. Walmart has the largest information technology infrastructure of any private company in the world, and it is this state-of-the-art technology and network design that allows Walmart to accurately forecast demand, track and predict inventory levels, create highly efficient transportation routes, manage customer relationships, and service response logistics.

For example, Walmart implemented the first company-wide use of Universal Product Code (barcodes) in 1983, through which store level information was immediately collected and analyzed. Later, Walmart leveraged this now-everyday technology into a further innovation: 'Savings Catcher', which allows consumers to scan product barcodes on their smartphones to compare best prices.

The company then devised 'Retail Link', a mammoth database. Through a global satellite system, Retail Link is connected to analysts who forecast supplier demands to the supplier network, which displays real-time sales data from cash registers and to Walmart's distribution centers.

Suppliers and manufacturers within the supply chain synchronize their demand projections under a collaborative planning, forecasting and replenishment scheme, and every link in the chain is connected through technology that includes a central database, store-level point-of-sale systems, and a satellite network.

What made Walmart so innovative? Was that it had been sharing all this information with their partners? Back in the day, a lot of companies weren't doing that, but rather using third-party services where they had to pay for the information.

Walmart's approach allows for frequent, informal cooperation among stores, distribution centers and suppliers, and less centralized control. Furthermore, the company's supply chain, by tracking customer purchases and demand, allows consumers to effectively pull merchandise to stores through demand, rather than having the company push goods onto shelves.

In recent years, Walmart has used radio frequency identification tags (RFID), which use numerical codes that can be scanned from a distance to track pallets of merchandise moving along the supply chain. As inventory must be handled by both Walmart and its suppliers, Walmart has encouraged its suppliers to use RFID technology as well.

Even more recently, the company has begun using smart tags, read by a handheld scanner, that allow employees to quickly learn which items need to be replaced so that shelves are consistently stocked and inventory is closely watched.

According to researchers at the University of Arkansas, there has been a 16% reduction in out-of-stocks since Walmart introduced RFID technology into its supply chain. The researchers also pointed out that the products using an electronic product code were replenished three times as fast as items that only used barcode technology.

In addition, Walmart also networked its suppliers through computers. It entered into collaboration with P&G for maintaining the inventory in its stores and built an automated re-ordering system, which linked all computers between the P&G factory through a satellite communication system. P&G then delivered the item either to a Walmart distribution center or directly to the concerned stores.

Top Stock: And it's not just high-tech innovation that Walmart innovates on: in the recent past, Walmart announced the trial of a new system to manage its stock, called Top Stock, in which the top shelves are utilized for more storage, freeing up back rooms. This move is designed to get products on the shelves sooner, creating more space for fulfilling online delivery orders and allowing more visibility of stock levels for both staff and customers. The move also means that customers don't have to wait to find a staff member to track down an item they don't see on a shelf.

Competitive Advantages: Walmart's supply chain management strategy has provided the company with several sustainable competitive advantages, including lower product costs, reduced inventory carrying costs, improved in-store variety and selection, and highly competitive pricing for the consumer. This strategy has helped Walmart become a dominant force in a competitive global market. As technology evolves, Walmart continues to focus on innovative processes and systems to improve its supply chain and achieve greater efficiency.

Case Study 2

Deere & Company

Deere & Company (brand name John Deere) is famed for the manufacture and supply of machinery used in agriculture, construction, and forestry, as well as diesel engines and lawn care equipment. Despite the ongoing challenges associated with the pandemic, John Deere delivered a year of solid performance in 2020. The company's Net sales and revenues for the year were \$35.54 billion, and net income was \$2.75 billion. Deere also delivered solid returns to investors.



Supply Chain Cost Reduction Challenges: Deere and Company has a diverse product range, which includes a mix of heavy machinery for the consumer market, and industrial equipment, which is made to order. Retail activity is extremely seasonal, with the majority of sales occurring between March and July.

The company was replenishing dealers' inventory weekly, using direct shipment and cross-docking operations from source warehouses located near Deere & Company's manufacturing facilities. This operation was proving too costly and too slow. So the company launched an initiative to achieve a 10% supply chain cost reduction within four years.

The Path to Cost Reduction: The company undertook a supply chain network-redesign program, resulting in the commissioning of intermediate "merge centers" and optimization of cross-dock terminal locations. Deere & Company also began consolidating shipments and using break-bulk terminals during the seasonal peak. The company also increased its use of third-party logistics providers and effectively created a network that could be optimized tactically at any given point in time.

Supply Chain Cost Management Results: Deere & Company's supply chain cost-management achievements included an inventory decrease of \$1 billion, a significant reduction in customer delivery lead times (from ten days to five or less) and annual transportation cost savings of around 5%.

Case Study 3

Intel

One of the world's largest manufacturers of computer chips, Intel needs little introduction. However, the company needed to reduce supply chain expenditure significantly after bringing its low-cost "Atom" chip to market. Supply chain costs of around \$5.50 per chip were affordable for units selling for \$100; but the price of the new chip was a fraction of that, at about \$20.

The Supply Chain Cost Reduction Challenge: Somehow, Intel had to reduce the supply chain costs for the Atom chip, but had only one area of leverage—inventory. The chip had to work, so Intel could make no service trade-offs. With each Atom product being a single component, there was also no way to reduce duty payments. Intel had already whittled packaging down to a minimum, and with a high value-to-weight ratio, the chips' distribution costs could not be pared down any further.

The only option was to try to reduce levels of inventory, which, up to that point, had been kept very high to support a nine-week order cycle. The only way Intel could find to make supply chain cost reductions was to bring this cycle time down and therefore to reduce inventory.

The Path to Cost Reduction: Intel decided to try what was considered an unlikely supply chain strategy for the semiconductor industry: make to order. The company began with a pilot operation using a manufacturer in Malaysia. Through a process of iteration, they gradually sought out and eliminated supply chain inefficiencies to reduce order cycle time incrementally. Further improvement initiatives included:

- Cutting the chip assembly test window from a five-day schedule, to a bi-weekly, 2-day-long process
- Introducing a formal Standard Operating Planning Process (S&OP).
- Moving to a vendor-managed inventory model wherever it was possible to do so

Supply Chain Cost Management Results: Through its incremental approach to cycle time improvement, Intel eventually drove the order cycle time for the Atom chip down from nine weeks to just two. As a result, the company achieved a supply chain cost reduction of more than \$4 per unit for the \$20 Atom chip — a far more palatable rate than the original figure of \$5.50.



Quality Cost Management

Part A

- Managing Quality in Competitive Environment

Case Study 1

Although it may appear easier to find success with TQM at a boutique-sized endeavour, the philosophy's principles hold true in virtually every sector. Educational institutions, for example, have utilized quality management in much the same way – though to tackle decidedly different problems.

The global financial crisis hit higher education harder than many might have expected, and nowhere have the odds stacked higher than in India. The nation pays home to one of the world's fastest-growing markets for business education. Yet over recent years, the relevance of business education in India has come into question. A report by one recruiter recently asserted just one in four Indian MBAs were adequately prepared for the business world.

At the Ramaiah Institute of Management Studies (RIMS) in Bangalore, recruiters and accreditation bodies specifically called into question the quality of students' educations. Although the relatively small school has always struggled to compete with India's renowned Xavier Labour Research Institute, the faculty finally began to notice clear hindrances in the success of graduates. The RIMS board decided it was time for a serious reassessment of quality management.

The school nominated Chief Academic Advisor Dr Krishnamurthy to head a volunteer team that would audit, analyse and implement process changes that would improve quality throughout (all in a particularly academic fashion). The team was tasked with looking at three key dimensions: assurance of learning, research and productivity, and quality of placements. Each member underwent extensive training to learn about action plans, quality auditing skills and continuous improvement tools – such as the 'plan-do-study-act' cycle.

Once faculty members were trained, the team's first task was to identify the school's key stakeholders, processes and their importance at the institute. Unsurprisingly, the most vital processes were identified as student intake, research, knowledge dissemination, outcomes evaluation and recruiter acceptance. From there, Krishnamurthy's team used a fishbone diagram to help identify potential root causes of the issues plaguing these vital processes. To illustrate just how bad things were at the school, the team selected control groups and administered domain-based knowledge tests.

The deficits were disappointing. A RIMS students' knowledge base was rated at just 36 percent, while students at Harvard rated 95 percent. Likewise, students' critical thinking abilities rated nine percent, versus 93 percent at MIT. Worse yet, the mean salaries of graduating students averaged \$36,000, versus \$150,000 for students from Kellogg. Krishnamurthy's team had their work cut out.

To tackle these issues, Krishnamurthy created an employability team, developed strategic architecture and designed pilot studies to improve the school's curriculum and make it more competitive. In order to do so, he needed absolutely every employee and student on board – but there was some resistance at the onset. Yet the educator asserted it didn't actually take long to convince the school's stakeholders as the changes were extremely beneficial.

"Once students started seeing the results, buy-in became complete and unconditional," he says. Acceptance was also achieved by maintaining clearer levels of communication with stakeholders. The school actually started to provide shareholders with detailed plans and projections. Then, it proceeded with a variety of new methods, such as incorporating case studies into the curriculum, which increased general test scores by almost 10 percent. Administrators also introduced a mandate saying students must be certified in English by the British Council – increasing scores from 42 percent to 51 percent.

By improving those test scores, the perceived quality of RIMS skyrocketed. The number of top 100 businesses recruiting from the school shot up by 22 percent, while the average salary offers graduates were receiving increased by \$20,000. Placement revenue rose by an impressive \$50,000, and RIMS has since skyrocketed up domestic and international education tables.

(The case study is taken from the website www.europeanceo.com/business-and-management/total-qualitymanagement-three-case-studies-from-around-the-world on 21.03.2019)

No matter what the business is, total quality management can and will work. Yet this philosophical take on quality control will only impact firms that are in it for the long haul. Every employee must be in tune with the company's ideologies and desires to improve, and customer satisfaction must reign supreme.



Total Quality Management

Praise

Case Study 1

The Customer Knows Best (AtlantiCare)

TQM isn't an easy management strategy to introduce into a business; in fact, many attempts tend to fall flat. More often than not, it's because firms maintain natural barriers to full involvement. Middle managers, for example, tend to complain their authority is being challenged when boots on the ground are encouraged to speak up in the early stages of TQM. Yet in a culture of constant quality enhancement, the views of any given workforce are invaluable.

One firm that's proven the merit of TQM is New Jersey-based healthcare provider AtlantiCare. Managing 5,000 employees at 25 locations, AtlantiCare is a serious business that's boasted a respectable turnaround for nearly two decades. Yet, in order to increase that margin further still, managers wanted to implement improvements across the board. Because patient satisfaction is the single-most important aspect of the healthcare industry, engaging in a renewed campaign of TQM proved a natural fit. The firm chose to adopt a 'plan-do-check-act' cycle or (PDCA Cycle), revealing gaps in staff communication – which subsequently meant longer patient waiting times and more complaints. To tackle this, managers explored a sideways method of internal communications. Instead of information trickling down from top-to-bottom, all of the company's employees were given freedom to provide vital feedback at each and every level.

AtlantiCare decided to ensure all new employees understood this quality culture from the onset. At orientation, staff now receive a crash course in the company's performance excellence framework – a management system that organises the firm's processes into five key areas: quality, customer service, people and workplace, growth and financial performance. As employees rise through the ranks, this emphasis on improvement follows, so that managers can operate within the company's tight-loose-tight process management style.

After creating benchmark goals for employees to achieve at all levels – including better engagement at the point of delivery, increasing clinical communication and identifying and prioritising service opportunities – AtlantiCare was able to thrive. The number of repeat customers at the firm tripled, and its market share hit a six-year high. Profits unsurprisingly followed. The firm's revenues shot up from \$280m to \$650m after implementing the quality improvement strategies, and the number of patients being serviced dwarfed state numbers.



Six Sigma

Case Study 1

Six Sigma Reduces Costs & Improve Environmental Impact (Ford Motors)

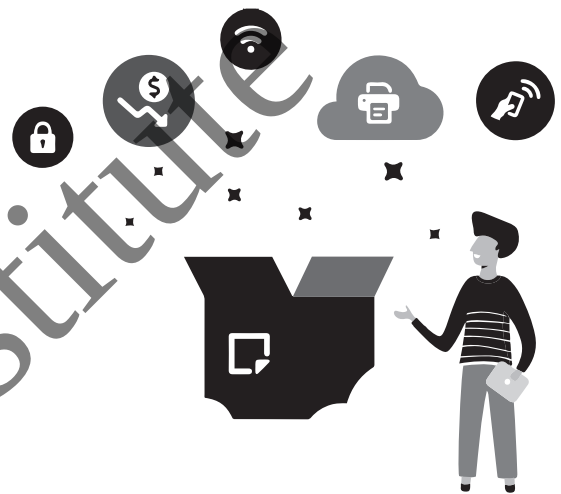
The Red Flag: Ford's balanced scorecard system provides reporting tools that offer monthly values versus target figures, year-to-date/year-end values against target, and a prioritization system using red/green/yellow evaluations to pinpoint where improvement is needed. Using this evaluation system, the auto-maker classifies data as:

- **Green:** measures are on or over target.
- **Yellow:** metrics are under target, but better than last year.
- **Red:** results are under target.

In the fall of 2009, data for body paint consumption for the Focus and Kuga were classified as red, thus capturing the attention of plant officials. A quick review of historical data showed basecoat paint consumption stood at 3.74 kg/unit in 2007, while current consumption was 4.18 kg/unit. Sensing an opportunity, Ford officials selected this improvement as a Six Sigma Black Belt project, offering an ideal fit with the One Ford strategy that focuses on "working together effectively as one team."

Six Sigma Black Belt Project: The project began in October 2009 with team member selection. Of the plant's 7,000 employees, more than 50 are Six Sigma Black Belts and another 400 are trained as Green Belts, thus providing a pool of qualified team members to assist with the project. Team leader and Six Sigma Black Belt Martin Fischer based his selections on a candidate's responsibilities, subject-matter expertise and process ownership, and on relative need throughout project development, planning, implementation, and follow up. Other factors included communication skills and the candidate's ability to interact in a team-based structure.

Applying the define, measure, analyze, improve, and control (DMAIC) approach, the team began by defining project stakeholders using a SIPOC (Suppliers, Inputs, Process, Outputs, Customers) analysis. This analysis led to three groups— internal, external, and a mixed group that contained both internal and external customers. The mixed group included not only customers who purchase the cars, but also internal customers such as the process owners, in this case the paint shop and the quality control group.



Define: The goals of the project were threefold:

1. Reduce costs: Reduce paint consumption to lower production costs.
2. Improve customer satisfaction: Improve process capability to better meet customer needs.
3. Lower environmental impact: Reduce solvent consumption to achieve a better VOC1 balance.

The team predicted the degree of impact for each goal by measuring anticipated benefits against organizational goals and measures.

They determined:

- The degree of impact for cost reduction was high, as \$1.5 million could be saved annually.
- Customer satisfaction impact was medium with a target of 127,000 ppm (defective parts per million) reduction.
- Environmental impacts were also medium with a projected 50,000 kg annually in VOC1 savings.

Measure: Several tools were used early in the measurement phase. For example, value-stream mapping served as a visual tool to help the team understand the flow of material and the paint application process. Statistical measures helped them filter, evaluate, and obtain strong data for the project. Cause-and effect diagrams were useful for identifying the root causes of consumption and performance issues, and brainstorming sessions were used to rate all potential causes. The next step was creating a data collection plan to help narrow the list of potential root causes by focusing on the following factors or critical Xs:

1. Daily basecoat consumption : Is there any dependency based on day or shift?
2. Paint film thickness check : Is there an increase, and if so, why?
3. Consumption per robot (automated painter) : Are there differences, and if so, why?
4. Consumption per manual painter : Monitor consumption to check the process capability.
5. First-time through rate versus consumption : A low rate means more repairs, which translates to higher basecoat use.
6. Application equipment : Check for damages or technical problems.

Analyze: The Six Sigma team conducted a '5 Why analysis,' as well as test trials on the six potential root causes. The results showed that factors one, two, four, and five were not significant. Factor three, consumption per robot, showed an increase for the liftgate robot. Through testing of factor six—application equipment—the team discovered a damaged solvent recovery valve that warranted further investigation. Additional testing uncovered that a defective solvent recovery valve was causing a direct paint flow from the color changer to the recycling tank, thus increasing consumption. Normally, the solvent recovery valve opens only for the cleaning program to bring the cleaning solvent back to a recycling tank.

Improve: The team used a variety of tools to develop solutions/improvement actions to address the two likely root causes. Value-stream mapping and benchmarking activities proved useful in the search for a manual solution to monitor the valve. On the other hand, while zeroing

in on the robot issue, the team reviewed the value-stream map and discovered they could change the automatic process to a manual one for painting the liftgates. Also, through research and discussions with suppliers, they realized the plant could apply paint more efficiently by upgrading to an electrostatic paint application process.

Based on the outcome of the analyze phase, four potential improvement actions were identified for the defective solvent recovery valve factor:

1. Replace plastic valves with stainless steel valves.
2. Create an automatic recovery valve check system.
3. Check the valves weekly.
4. Eliminate the solvent recovery process.

The team used four primary methods to select the final improvement actions: test trials to evaluate stainless steel valves against plastic valves, technical research to develop an automatic recovery valve check system, brainstorming and value-stream mapping to determine the effectiveness of a weekly valve check, and the elimination of the solvent recovery process.

The test results revealed that a quick, inexpensive change from plastic to stainless steel valves would result in a 45 percent performance improvement. Testing also demonstrated that an automatic recovery valve check system would be cost effective and could offer an effective error-proofing device.

For the liftgate robot factor, three potential solutions were identified:

1. Develop a new cleaning program.
2. Change the robot process to a manual paint application.
3. Upgrade to an electrostatic paint application.

Testing focused on improving the existing cleaning program and then comparing the consumption data from the robot process to a manual process. The team also created a cost-benefit analysis for an upgrade to an electrostatic paint application. Tests showed there was no significant difference between the old and the new cleaning program but, by simply changing to manual only painting processes for interior painting, it was estimated that Ford could save 0.28 kg/unit. Finally, the team also determined that upgrading to an electrostatic paint application system would not be cost effective.

Once the solutions were finalized, the team created a three-step implementation plan that included the following steps:

- **Think:** Plan all necessary implementation activities.
- **Act:** Implement the solutions.
- **Control:** Check if solutions were correctly implemented.

Yet another critical element in the project was overcoming stakeholder resistance to the solutions. This was accomplished through effective relationship building as well as providing data, training, and opportunities to discuss the project solutions.

Once the solutions were implemented, the team achieved every project goal and even exceeded the expected cost reduction by a half million dollars annually. More specifically, in meeting these goals, the basecoat paint consumption dropped from 4.18 kg/unit to a mean consumption of 3.3 kg/unit.

Control: The new monitoring system and standard operating procedures are vital to helping the Saarlouis plant sustain the results gained in this project. This system provides a real-time view of paint consumption in detail for each of the four paint booths. All of the plant's standard operating procedures are part of the plant's ISO 9001 compliant quality management system and are therefore included in routine audits. This helps assure that paint consumption will remain within specifications.

Performance metrics signaled increase in basecoat paint consumption at Ford's vehicle operations center in Saarlouis, Germany



A cross-functional Six Sigma team was chartered to solve the problem using a DMAIC approach



Using a variety of quality tools, the team identified root causes before developing and testing potential solutions



By reducing paint expenditures, the team achieved a \$2 million annual savings



Ford entered this project in ASQ's 2011 International Team Excellence Award competition where it earned finalist honours



Pricing Decision & Strategies

Case Study 1

Wal-Mart pulls off its growth through Price Perfection

Many retailers are trying to replicate the success of Wal-Mart Stores Inc., the world's largest retailer that was founded over 60 years ago, on "Everyday Low" prices. The central goal of Wal-Mart is to keep retail prices low -- and the company has been very successful at this. Experts estimate that Wal-Mart saves shoppers at least 15 percent on a typical cart of groceries.

Wal-Mart has grown, over the years, into more than just the world's largest retailer. It is an economic force and a cultural phenomenon. It all started with a simple philosophy from founder Sam Walton: "Offer shoppers lower prices than they get anywhere else". That basic strategy has shaped Wal-Mart's culture and driven the company's growth.

Sam Walton opened his first 'five-and-dime' in 1950 and his vision was to keep prices as low as possible. Even if his margins weren't as fat as competitors, he figured he could make up for that in volume. Wal-Mart has been able to keep its prices low through cutting-edge technology, a push to make suppliers sell merchandise at cheaper and cheaper prices, and a frugal corporate culture.

Wal-Mart pushed the retail industry to establish the universal bar code, which forced manufacturers to adopt common labelling. Over a decade back, Wal-Mart became the first major retailer to demand manufacturers use Radio Frequency Identification Technology (RFID).

The stories of how Wal-Mart pushes manufacturers into selling the same product at lower and lower prices are legendary. One example is Lakewood Engineering & Manufacturing Co. in Chicago, a fan manufacturer. In the early 1990s, a 20-inch box fan costs \$20. Wal-Mart pushed the manufacturer to lower the price, and Lakewood responded by automating the production process. Lakewood also badgered its own suppliers to knock down the prices of parts. Then, in 2000, Lakewood opened a factory in China, where workers could be as cheap as 25 cents an



hour. By 2003, the price on the fan in a Wal-Mart store had dropped to about \$10.

In a 2003 Los Angeles Times article, part of a Pulitzer Prize-winning series about Wal-Mart, tells of a Wal-Mart buyer named Celia Clancy, who was in charge of clothing and demanded that each supplier either lower the price or increase the quality every year on every item. This philosophy is known as "plus one." In "The Wal-Mart Effect," author Charles Fishman discusses how the price of a four-pack of GE light bulbs decreased from \$2.19 to 88 cents during a five-year period.

Wal-Mart's impact extends beyond just small suppliers. It also affects how even major, established companies like Coca-Cola and PepsiCo do business. At Wal-Mart's request, Coke and its largest bottler Coca-Cola Enterprises announced that they are changing the way they deliver Powerade in the United States, altering a basic distribution method for drinks that has been in place for more than a century. Coke also now allows Wal-Mart in on the research-and-development process. In 2005, Coke planned to launch one new diet cola called Coke Zero. At Wal-Mart's request, it changed the name to Diet Coke with Splenda and launched a separate product called Coke Zero.

This kind of retailer involvement was unheard of at Coke decades ago. Pepsi also came up with a line of diet drinks, called Slice One, to initially be sold exclusively in Wal-Mart.

Walton continued to drive an old pickup truck and share budget-hotel rooms with colleagues on business trips, even after Wal-Mart made him very rich. He demanded that his employees also keep expenses to a bare minimum -- a mentality that is still at the heart of Wal-Mart culture. The company has continued to grow rapidly even after his death in 1992 and now operates many retail divisions.

Now that Wal-Mart is so huge, it has unprecedented power to shape labour markets globally and change the way entire industries operate. Ninety percent of the U.S. population is stated to be living within 15 miles of a Wal-Mart, according to "The Wal-Mart Effect."

The key learning could be that Wal-Mart considers 'Everyday Low Prices' are the prices that would be afforded by its customers which in turn would enable customer retention and customer attraction whereby the revenues are propped up. In the process Wal-Mart pulls off its voluminous growth, through perfecting 'Everyday Low Prices' revealing that Price Perfection enables Perpetual Growth.

Case Study 2

McDonald's Cost Leadership Strategy

The restaurant industry is known for yielding low margins that can make it difficult to compete with a cost leadership marketing strategy. However, McDonald's - the near-eighty-year-old restaurant chain - has been extremely successful with Cost Leadership Strategy by offering basic fast-food meals at low prices.

McDonald's has made itself to be the family friendly low-cost restaurant in the fast-food business all over the world. McDonald's



have stuck to their core market throughout the years even through the changing times. The term happy meal is said and begged for by children worldwide and has become a household name. McDonald's does things differently than its competitors by marketing to the exclusive family market.

The entity realizes that when a customer goes into McDonald's he or she expects two things. They expect the food will come out fast, and it will be inexpensive. That is what McDonald's aim, i.e., (a) cheap and (b) fast, and everything they do within the organization works towards these specific goals.

These two competitive advantages, cheap and fast, comply directly with the vision of the company which reads: "McDonald's vision is to be the world's best quick service restaurant experience. Being the best means providing outstanding quality, service, cleanliness, and value, so that we make every customer in every restaurant smile."

McDonald's strive to be cost leaders and offer their food at prices that cannot be matched by their competitors. They ensure that their chain-stores are efficient enough to keep everyday operations costs as low as possible.

They have the most modern and technologically advanced equipment in their restaurants to make the processes easier. The computer operated machinery allows the franchisee outlets to keep costs low by needing only a few employees to do the work of several. The automation also enables the employees to do the job quicker. Many of the McDonald's have dual drive-through to decrease wait time and to increase volume of customers served.

In order to ensure that the products remain value oriented, McDonald's make sure costs do not get out of hand, with practices such as wage controls and ingredient standardization. While the fast-food industry experiences a high-turnover rate, McDonald's have learned to work with the system. They make the trade-off of an advanced training program, with that of a simplified, pictographic, "assembly line" procedure, which ensures quality and consistency by narrowing employees' task scope, and therefore their required training. Such of these internal tradeoff practices have allowed McDonald's to beat out cost competition effectively.

All of the activities of McDonald's work towards cutting costs. Instead of buying high grade meat and ingredients, McDonald's settle for a standard grade meat that fits exactly into their needs. Also, they keep employee wages low, and training costs minimal. They are able to keep wages low through a division of labour that allows it to hire and train inexperienced employees rather than trained cooks. It also relies on a few numbers of managers who typically earn higher wages. These two characteristics, viz. low-cost materials and low wages, go a long way in keeping McDonald's food cheaper than many of its competitors.

Another important competitive advantage they aim at McDonald's is the speedy delivery of the food. In order to maintain this advantage over other fast-food chains, the processes of cooking food are made simple for all employees at the restaurants. They target a low failure rate to ensure the quick production and delivery of food.

The speedy service fits well throughout McDonald's organization and is very convenient to the customer. They built on this idea of convenience by building a McDonald's store everywhere.

History and prestige give McDonald's an advantage against other chains. Prices have remained low due to their enormous wealth's ability to take a hit on price margins and the international

expansion making up for the declining U.S. sales. The numerous stores also satisfy consumers on convenience. Being in the industry for so long has allowed McDonald's to lock in certain suppliers at beneficial prices.

McDonald's are, thus, able to maintain their edge on cost leadership through a meticulously systematised cost management activities comprising:

- a. Process Automation & Simplification
- b. Standardisation of Ingredients
- c. Achieving lower Employee Costs
- d. Chain of Convenient Stores
- e. Low price margins driving High Volumes

The synergic impact of McDonald's Cost Leadership is that of maintaining its position as one of the Multinational Industry Leaders.

Case Study 3

Apple earns its pie through Product Differentiation

Apple Inc. is an American multinational corporation headquartered in Cupertino, California. The company designs, develops, and sells consumer electronics, computer software, online services, and personal computers. Its best-known hardware products are the Mac line of computers, the iPod media player, the iPhone smartphone, etc. Apple is the world's second-largest information technology company by revenue after Samsung Electronics, and the world's third-largest mobile phone maker.

The markets for the Apple's products and services are highly competitive and the Company is confronted by aggressive competition in all areas of its business. These markets are characterized by frequent product introductions and rapid technological advances. Principal competitive factors important to the Company include price, product features, relative performance, product quality and reliability, design innovation, software and peripherals, marketing and distribution capability, service and support, and corporate reputation.

The Company's business strategy, therefore, leverages on its unique ability to design and develop its own operating systems, hardware, application software, and services to provide its customers new products and solutions with superior ease-of-use, seamless integration, and innovative design.

Apple became the most valuable consumer-facing brand in the world In June 2011. Apple Inc. reported that the company sold 51 million iPhones in the Q1 of 2014 (an all-time quarterly record), compared to 47.8 million in the year-ago quarter. Apple also sold 26 million iPads during the quarter, also an all-time quarterly record, compared to 22.9 million in the year-ago quarter. The Company sold 4.8 million Macs, compared to 4.1 million in the yearago quarter.

Apple's high level of brand loyalty is considered unusual for any product. Fortune magazine named Apple the most admired company in the United States in 2008, and in the world from 2008 to 2012. On September 30, 2013, Apple surpassed Coca-Cola to become the world's most valuable brand in the Omnicom Group's "Best Global Brands" report. Boston Consulting Group has ranked Apple as the world's most innovative brand for a number of years since 2005.

Apple attempts to increase market demand for its products through differentiation, which entails making its products unique and attractive to consumers. The company's products have always been designed to be ahead of the curve compared to its peers. Despite high competition, Apple has succeeded in creating demand for its products, giving the company power over prices through product differentiation, innovative advertising, ensured brand loyalty, and hype around the launch of new products. By focusing on customers willing to pay more and maintaining a premium price at the cost of unit volume, Apple also set up an artificial entry barrier to competitors.

The renowned Steve Jobs is stated to have built the strategy for Apple on four pillars, viz.

1. Offer a small number of products
2. Focus on the high-end
3. Give priority to profits over market share
4. Create a halo effect that makes people starve for new Apple products

Jobs' vision for Apple was always to create a premier product and charge a premium price. Apple's cheapest products are usually priced in the mid-range, but they ensure a high-quality user experience with their features.

Drawing an example from the mobile market, as in May 2015 Apple charges ₹46,000 for its iPhone 6 whereas Galaxy S5 mini a compatible product of Samsung is priced at ₹21,000.

Apple is able to collect a premium of 119% for its carefully carved out characteristic of unique Product Differentiation.

The reading is that carefully carved out Product Differentiation holds the key for the success story of Apple Inc.; Thus, Apple earns its pie through the strategy of Product Differentiation.

Case Study 4

Yield Management in Indian Railways

Rail vs Air: Captain Gopinath dreamt of bringing air travel within the reach of the common man and conceived Air Deccan which became the first low-cost airline to fly pan-India in 2003. Air Deccan took to the strategy of Yield Management and gradually pioneered the concept of low-cost travel. Air travel was offered at peanut prices as low as one rupee per ticket. The market was taken by storm, and air travel was perceived as an affordable comfort. By 2006 Air Deccan was perceived to have converted air travel into a



mass commodity and Thus, changed the face of aviation sector in India. For the first time, train travellers started shifting from rail journey to flight journey.

It was in 2006 that Indian Railways decided to introduce a Dynamic Pricing Policy for freight as well as passenger, for peak and non-peak seasons, premium and non-premium services, and for busy and non-busy routes. As per this policy the rates for non-peak season, non-premium service and empty flow directions would be less than the general rates and the rates for peak season and premium services could be higher than normal. For the freight the non-peak season would be 1st July to 31st October. For the passenger segment this period would be 15th January to 15th April and 15th July to 15th September.

The tricky issue in dynamic pricing is to figure out how high the tariff can go and to what extent a passenger will pay higher rail tariffs, given that one can get an air-ticket at around the same price. A smart-pricing formula had been developed to compete with airways and increase fares as per the demand curve. The formula has a floor price equivalent or more than the Tatkal Rajdhani/Mail tariff. The price will further increase in tune with the demand

curve – the rate at which the tickets are booked. The prices of air tickets at that time will be monitored and Thus, keep the ticket price for premium trains lower than airfares.

The revenue managers of Indian Railways had perceived the market threat posed by the low-cost air carriers to the rail traffic in a proper perspective and have gone forward in adopting the technique of yield management to optimise its capacity utilisation and counter the switchovers.

Reservation Against Cancellation (RAC): As a corollary to its governmental and social obligations, Indian Railway earmarks considerable portion of its accommodations towards various categories of passengers such as Government Exigencies, Defence, Foreign Tourist, Specially Abled, Women, Senior Citizens, etc. The quota is kept open till the last moment and the unfilled seats and berths are allotted to accommodate the passengers from the general wait list. However, the risk of some of the seats and births remaining vacant would always remain. Similarly, last minute cancellations and no shows add up to the idle berths and seats.

The concept of Reservation Against Cancellation (RAC) was evolved with a view to fill up vacant berths and seats that go idle due to passenger cancellations, quota vacancies, no shows, etc. Each train is given a fixed number of RAC allocations by splitting some of the berths into seats. RAC passengers are offered confirmed sitting accommodation with the assurance of providing berths against probable vacancies arising from any sort of quota lapses or cancellations. The system facilitates assured accommodation to the travellers while, at the same time, yielding additional earnings to the Railways. RAC is a practical example of programmed overbooking towards enabling better capacity utilisation.

Auto Up-gradation: In case of upper-class seats remaining vacant, lower-class passengers are upgraded to higher class travel whereby more lower-class vacancies can be offered to waitlisted passengers. It is reported that on the average there are two upgrades for every wait listed upgraded confirmation. Impliedly, whenever an upgrade does happen, then at least two passengers are happy, viz. the one who is upgraded to the higher class and as also the one who is confirmed in the lower class. The process of auto up-gradation, certainly, maximises the passenger capacity utilisation. In addition, this is one measure that had generated a lot of good will and proved to be a brand builder for the Indian Railways.

Tatkal: Tatkal is an example of demand driven differential pricing wherein the last-minute passengers are offered tickets at a premium. Tatkal bookings start a day before the scheduled journey. The tatkal charges are levied as a percentage of the basic fare, i.e., at the rate of 10% for second class and 30% for all other classes, subject to certain minimum and maximum limits. No refunds are granted on cancellation of confirmed tatkal tickets. The tatkal scheme has gained popularity amongst the rail travellers, eventually leading to Premium Tatkal. Premium Tatkal is an advancement of tatkal wherein dynamic pricing has been introduced by increasing the fare for the subsequent bookings.

Premium special train are being run by Indian Railways with dynamic fare pricing, where dynamic fare stands for the fare component that may be increased with the subsequent bookings. Advance Reservation Period (ARP) of this train will be a maximum of 15 days. Only E-tickets will be permitted for booking. No concession shall be applicable in this train. Vacant berths left at the time of charting will be offered for current booking at current booking counters of train originating stations. Cancellation is not allowed.

Slack Season: Towards addressing the supply driven constraints, Indian Railways have implemented the concept of differential pricing by offering lower train fares during the non-peak travel periods. The slack season fares are kept marginally lower than the normal, the objective being to attract more customers for the rail travel, and thereby minimise capacity losses.

Clone Trains: Clone trains are run on high-demand routes within an hour of a scheduled train's departure to accommodate those on its waiting list. The idea behind such real-time demand-driven trains is to ensure that the wait listed passengers reach their destination around the same time they had originally envisaged. A premium train would automatically get announced on the net/ on the system whenever waitlisted passengers went up beyond a point - one way or both ways. The passengers would have to pay a 'premium' if they want to avail of it.

Add On Revenue: Add On Revenue (AOR) is the Revenue generated by any of the idle resources. Post Budget 2016-17, Indian Railways has initiated several measures to make use of its idle resources and double up its revenues from non-tariff during the next five years. The spread of the activities, spelt out by the railway minister in his budget speech, included:

- i. Monetization of land and buildings through commercial exploitation of vacant land and space rights over station buildings;
- ii. Leasing out huge tracks of land available adjacent to rail network to promote horticulture and tree plantation; Exploring the possibility of using this track for generating solar energy;
- iii. Monetizing data, software and some of the free services provided by IR such as PNR enquiry, currently being commercially exploited by other players;
- iv. Exploiting advertising potential of stations, trains and land adjacent to tracks outside of big stations with a focused target of increasing the advertising revenues by more than four times the current level;
- v. Liberalizing the current parcel policies including opening the sector to container train operators to effect a quantum jump in IR's share of the national CEP (Courier, Express and Parcel) market; and
- vi. Augmentation of revenues from manufacturing activity

Such of these Add On Revenue initiatives do focus on enhancing the existing avenues by putting the idle and dormant resources to demand oriented utilisation in line with the market trend. The propositions are quite innovative and appealing. They would certainly shore up the revenues substantially when implemented.

Surge Pricing: Surge Pricing is the latest adoption of Indian Railways. Rajdhani Express models introduced in 1985, Durontos of 2010 and Shatabdis of the present day - all of them can be construed as demand driven & need warranted premium trains. Effective from 9th September 2016, i.e., exactly ten years after the introduction of dynamic pricing, IR has gone for Surge Pricing on an experimental basis. Ticket prices of Rajdhani, Duronto and Shatabdi trains will keep increasing by ten percent of the basic fare with every ten percent of the tickets sold. The fares will keep raising progressively as the tickets are sold out. Of course, the surging is subject to maximum ceiling.

Being a Social Enterprise, Indian Railways do adopt the concept of 'cap' on fares for each class of travel. A sleeper fare would only increase to one and a half times its base fare and then remain constant; and so also for other classes. Pricing is linked to the rate of sale of tickets, read in buckets, and capped to a maximum for each class of travel.

Overnight Tangle: Wherever the train travel is overnight such that by many a Rajdhani or Duronto, for example Mumbai to Delhi, full travel cost is the key factor that impacts the choice of the mode of journey of a regular passenger. Full cost, here, implies the travel cost and the hoteling cost added together. The traveller starts from home the previous evening, reaches the destination early in the morning, checks into a hotel for the daily chores, completes the day's work, vacates the hotel, takes the evening train and is home the next day morning.

In case of air, the traveller can start from home in the morning, catch the flight in the early hours, attend to the work during the day, board a return flight in the evening and be home by night. The journey having been started from and to home on the same day, the air traveller does not have to incur any hoteling cost. Therefore, an air traveller would opt for train travel only if the to & fro train fare and hoteling cost put together are less than the to & fro air fare.

If a train traveller is extended the facility of having a bath and resting for an hour in an exclusive room in the railway station on marginal rents, Rajdhani and Durontos can pose fierce competition to the airlines. Designated special rest rooms at the Railway Stations can make huge difference to the Class Travel being offered by the IR. The challenging equation, however, is "To & Fro Train Fare + Rest Room Tariff" should be less than "To & Fro Air Fare". This is an idea that can be toyed with by the railways in a true perspective.

Learning Track: The capacity for every train trip remains fixed; once the train is scheduled the seats and berths acquire the nature of perishability; trip based marginal costs are practically nil; the demand for travel can be influenced by the pricing mechanism; passenger market is segmented with different categories of travellers; and the facility of advance booking exists. All the major features of yield management are, thus, visible in the Indian Railway Model.

General wait list for the reserved accommodation is an age-old tradition of the Indian Railways and reflects a chance overbooking whereas RAC is a programmed overbooking, both the measures being in the direction of augmenting the passenger capacity utilisation. Differential pricing strategy is put to use by IR through the medium of demand driven tatkal and

real time-based clone trains. Both of these measures are premium revenue propellers. Auto up-gradation and lower fares during slack season are aimed at improving the capacity utilisation and generating marginal revenues. Non fare revenue is a movement towards next best utilisation of the idle resources. Surge Pricing is a market prone approach.

The endeavours are good, but it is still a long journey for the Indian Railways because of its social and economic limitations. These are the endeavours that can provide the Indian Railways a competitive posture towards perfecting affordable passenger fares and compatible goods tariff; These are the endeavours that shall be pursued for effective and efficient implementation; and these are the endeavours that can make Indian Railways a role model Cost Leader. In the entire process, Yield Management could be the cutting edge for the Indian Railways.

SJC Institute



Target Costing

Case Study 1

Target Costing at Caterpillar

The application of target costing is best illustrated by Dan Swenson, Shahid Ansari, Jan Bell, and IL Woon Kim, in their research article 'Best Practices in Target Costing' (Management Accounting Quarterly, Winter 2003) by drawing a practical example from Caterpillar. The illustration runs as follows.

Caterpillar offers a good illustration to highlight the target costing process as was applied to one of its new products. For this particular vehicle, management set the target cost at 94.6% of a comparable model, creating an initial gap of 5.4%. The cost of the comparable model is based on current manufacturing capabilities. Therefore, to achieve the target, costs must be reduced by 5.4%.

A cost improvement team is then formed comprising representatives from product design, manufacturing engineering, production, marketing, and purchasing to determine how to close the gap. Initially, the group evaluates component part substitutions that would reduce costs but still provide the product features and benefits necessary to satisfy customer requirements. The group also considers opportunities to reduce costs through efficiency improvements. Table 1 shows the outcome from the team evaluations.

Table 1

Modification of Current Product: Known Adjustments

Activity	Current Costs (%)	Projected Costs (%)	Adjusted Savings (%)	Explanation of Known Adjustments
Assembly	5.4	3.9	1.5	Efficiency improvements due to redesigning sheet metal, as documented on current production models.
Cab	7.9	7.1	0.8	Replace current cab with the "Classy Cab." Price Final quote already received.

Engine	8.6	7.9	0.7	Cost estimate from Engineering for switching to different configuration.
Hydraulics	19.1	17.5	1.6	New pump design
Power Train	12.0	12.0	0	
Structures	20.0	20.0	0	
Linkage	18.0	18.0	0	
Others	9.0	9.0	0	
Total	100.0	95.4	4.6	

As may be observed from the table, the cost improvement team identified 4.6% in “known” savings through an initial evaluation of cost savings opportunities. Having reduced the gap by 4.6%, the team must find an additional 0.8% in savings to achieve the 5.4% cost reduction target.

At this stage, the cost improvement team surveys the operational groups, through a questionnaire, to identify potential cost savings opportunities. The responses to the questionnaire do not recommend specific solutions, but they do identify where improvement opportunities are more likely to be successful (see Table 2). Each “yes” response on the questionnaire indicates an opportunity for cost reduction, and the component part category (cab, engine, hydraulics, etc.) that has the largest number of positive responses is viewed as having the greatest potential for saving money.

Table 2

Modification of Current Product: Sample Questionnaire (Yes = 1, No = 0)

	Assembly	Cab	Engine	Hydraulics	Power Train	Structures	Linkage	Others	Total
1. Are there more than five suppliers from whom you can purchase materials?	0	0	0	1	1	1	1	0	
2. Are you costlier than best-in-class supplier (either Caterpillar or non-Caterpillar)?	0	0	0	1	0	0	0	0	
3. Do you plan to survey your supplier cost breakdown?	0	0	0	1	0	0	1	0	
4. Is the current manufacturing process younger than two years?	0	0	0	1	0	0	0	0	
5. Does labour represent more than 40% of your total cost?	0	0	1	1	0	0	1	0	
6. Is your “unit setup cost/total unit cost” ratio greater than 5%?	0	0	1	1	0	0	0	0	

7. Do you see potential for material specification changes?	0	0	0	1	0	0	0	0	
8. Do you see potential for tolerance loosening?	0	0	0	1	0	0	1	0	
9. Does the current family of parts contain non-approved parts?	0	0	0	1	0	1	1	0	
10. Can the current design or manufacturing processes be subjected to emerging innovative technologies?	0	0	0	1	0	0	0	0	
Total	0	0	2	10	1	2	5	0	20
Distribution of 0.8% in Cost Reduction (%)	0	0	10	50	5	10	25	0	100

Table 2 highlights a sample questionnaire, and a tally of the responses indicates the extent to which each part category will be targeted for cost reduction. In this case, hydraulics will be responsible for achieving the highest percentage (50%) of the cost savings that are needed. Therefore, the cost of hydraulics must be reduced by 0.4% (0.50×0.08). Table 3 illustrates the final step in the process. It takes the adjusted costs column from Table 1 and subtracts the additional savings that are required for each component part category. The right-hand column in Table 3 illustrates the target cost for the new vehicle, broken down to the component level.

Table 3

Modification of Current Product: Final Target Cost Assignments

Activity	Adjusted Costs (%)	Distribution of 0.8% in Cost Reduction (%)	Target Cost for New Product (%)
Assembly	3.90	0.00	3.90
Cab	7.10	0.00	7.10
Engine	7.90	0.08	7.82
Hydraulics	17.50	0.40	17.10
Power Train	12.00	0.04	11.96
Structures	20.00	0.08	19.92
Linkage	18.00	0.20	17.80
Others	9.00	0.00	9.00
Total	95.40	0.80	94.60

To recap, Caterpillar began with current costs for a comparable product (100%) and, after deducting known savings based on existing technology (Table 1) and potential savings based on an analysis of the questionnaire (Table 2), established cost targets for each component of the new vehicle whereby the target cost is brought down to 94.60% of the current costs.

The researchers observe that target costing is being adopted in some key industries, namely the transportation and heavy equipment industries. Intensive competition, extensive supply chains, and relatively long product development cycles characterize these industries. The best practice companies were relatively consistent in the way in which they applied target costing. The other companies follow a similar approach to the target costing steps at Caterpillar.

All of the best practice companies employ a cross-functional organizational structure, listen to the “voice of the customer,” emphasize cost reduction during the new product development cycle, and are very effective at removing costs throughout the supply chain. For these companies, target costing has proven to be a very effective means of cost control and profit enhancement.

Case Study 2

Using Target Costing to Increase Value

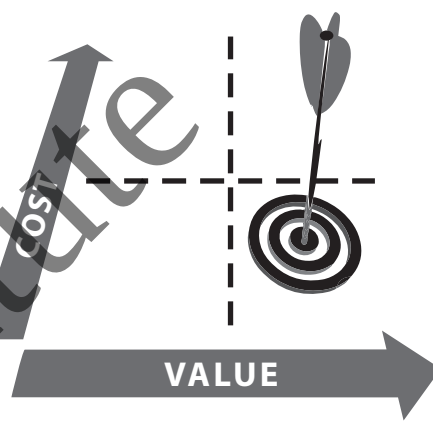
Here under is an interesting field story on ‘Using Target Costing to Increase Value’ posted in February, 2004, by Brian H. Maskell, President BMA Inc.

The Opportunity: In order to capture an Original Equipment Manufacturing (OEM) business, Major-Cable quoted a very low price and bagged an opportunity. The Target Costing project was designed to improve the profitability of these cables to this customer and other OEM customers.

The Issue: The exercise began with one of the key steps in Target Costing where the customer’s needs were nailed down by specifying them exactly. One need from this customer was to have the cable with a minimum of 20 meters length. This has been a problem for Major Cable because delivery is made from their centralized distribution center which cuts the cable to the customers required length and they cannot always guarantee to have the right quantity of 20-meter lengths. The plant provides the distribution center with much longer lengths, but the specific 20-meter length is difficult to maintain.

The Solution: The initial solution to this problem was for the production plant to create a new product number for 20-meter lengths and manufacturing them specially for the OEM customer. The team then asked the question; “Why does the customer need 20-meter minimum lengths?” It turned out that the machines the customer manufactures require several pieces of cable and the total amount of cable for any machine never exceeds 20 meters. They wanted the cables in 20-meter lengths so that they can issue them to the manufacturing location who in turn can then cut the cables into the lengths they require for the specific machine they are currently manufacturing.

Major Cable’s plant has “off-cuts” of cable that are too short to be sent to the distribution center. These off-cuts are stored as finished goods in case there is occasional demand for a short length; but these off-cuts are mostly scrapped at the year-end. The customer’s need for short lengths interested the production manager. He suggested that instead of supplying the product from the distribution center, they can deliver directly to the customer. Instead of delivering 20-meter lengths, they can deliver the cable cut to the precise lengths required by the customer that day. Instead of supplying the product on large round spools, they can place the cut pieces into production kits in cardboard tubes or boxes to suit the customer’s needs.



After this was discussed and agreed with the customer, the result was:

- The customer is delighted to have just-in-time deliveries of cable kits. This has reduced their manufacturing costs.
- The customer is paying a higher price because they are receiving kits instead of spooled cable.
- Many of the cable lengths can be provided from the off-cuts that were being scrapped most of the time previously. The cost of these pieces is effectively zero.
- The cardboard boxes or tubes are much less expensive than the previously used spools and the overall cost of the packed product has turned out to be less.

The sales people were also delighted. They did not know the production plant could supply cut pieces. They were under the impression that the production plant would only make long lengths (economic order quantities) on large spools.

The Power of Target Costing: Without Target Costing Major Cable would not have had the opportunity to bring together the cross-functional teams needed to understand the customer's needs and find manufacturing, logistics, and marketing methods to create more value for the customer. In the process, everybody has "won". The customer has reduced their costs, their inventory, and their production lead time. The production plant has reduced its material costs and scrap. The company's revenues have increased owing to the higher price of the cut piece kits. It is the power of Target Costing in practical action. Put it rightly, it is a focus on increasing value as well as reducing cost."

Case Study 3

Unveiling the Indian Nano

Talking of the Indian scenario, no other car launch in the history of Indian Auto Industry has received as much global press as the "people's car", the 'Tata Nano'. No other car promised to revolutionise motoring as the Nano has. Clever marketing apart, some frugal and out-of-the-box engineering has gone into the making of Nano. Nano modelled Indian Target Costing too.

Much like what Henry Ford had in the beginning of the 20th century with his 'Model T' in 1908 at a price of \$825, exactly 100 years after Ratan Tata unveiled Nano, the Indian ultra-low-cost car. At ₹ 1 lakh, the Nano was the world's cheapest car and makes motoring affordable to millions of Indians. Even its deluxe models, featuring air-conditioning and power windows, are fairly cheaper than the then cheapest car in the country, the Maruti 800.

Every component in the Nano is stated to have been studied from a functionality, cost and performance requirements as there was no other way to reduce costs. From an outsourcing perspective, the company put in place an Early Vendor Integration Programme. The company had a lot of design inputs from vendors that either facilitated manufacturing or brought the cost down. This could be for lamps, seats or for any other component.

The Nano is completely indigenised. At the same time over 85 per cent of the Nano is sourced from outside vendors. Vendor parks have been put in place with the objective of ensuring that the components between vendors and the assembly line move smoothly and just in time. Keeping costs down was a major problem for vendors, and they found innovative ways to achieve it. The initial effort was towards cost prevention, which involved selecting a design concept with the least cost. Later on, it is a perpetual cost-reduction effort.

When Ratan Tata addressed an Automotive Component Manufacturers' Association (ACMA) meeting saying that "Can we all get together to produce an Asian peoples' car?"; the response was lukewarm – as in the case of Henry Ford. Tata too encountered considerable amount of ridicule from certain close quarters. Even the vendors took it to be a hypothetical project. But, Ratan Tata didn't budge; he went ahead and did it. The initial idea was to come up with a low-cost car that Malaysia, Indonesia and India could produce jointly. As it turned out, it was left to Ratan Tata to respond to the FT Reporter at Geneva Motor show to commit an Indian Nano at about ₹ 1,00,000. The news got flashed, and it happened. Nano has restructured the dynamics of the car manufacturers all over the world.

As Ratan Tata put it in his interview to the Economic Times in January 2008; "The real challenge is when you have some strength and you really choose to throw out the gauntlet that you can do X and it ought to be the kind of challenge which somebody says that can't be done because then that really becomes the engine of innovation. We haven't said we will send a man to Mars, we may put landers on Mars, but we have not done those kinds of things. It is those areas which really create the innovation that we need".



Asset Life Cycle Costing

Case Study 1

LC Margin for an Orange Plantation Project

The application of LCC can be universal across the multiple sectors of the economy. Here follows the summarised (basic) computations of LCC relating to an Orange Plantation Project. These computations have been carried out on the basis of inputs provided by a couple of farmers.

Annual Cash Outflow

The annual cash outflow budget for the said plantation would be as follows:

Annual cash outflow Budget of Orange Plantation Project

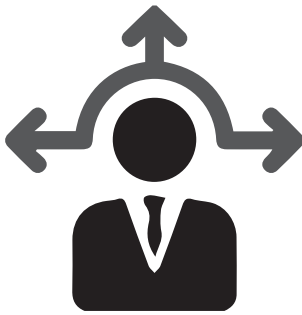
Serial	Item	Year 1 (₹)	Year 2 to Year 5 (₹)	Year 6 to Year 18 (₹)
A	Month wise Details			
1	April			
	Land Acquisition	40,00,000		
	Levelling & Dressing	1,00,000		
	Fencing	96,000		
	Pits	10,000		
	Nutrients in Pit	50,000		
	Attached Labour	7,000	7,000	7,000
	Hired Labour			6,000
	Organic Manure @ 2 trolleys per acre			32,000
	Transport and Labour Charges			8,000
	Sub Total	42,63,000	7,000	53,000
2	May			
	Dug Well (10ft dia × 50ft deep)	2,45,000		

	Attached Labour	7,000	7,000	7,000
	Sub Total	2,52,000	7,000	7,000
3	June			
	Dug Well (10ft dia × 50ft deep)	1,05,000		
	Electric Connection	25,000		
	Motor (5HP)	30,000		
	Cables & Other Equipment	10,000		
	Piping	50,000		
	Drip System	2,40,000		
	Misc. Works	25,000		
	Electricity	4,000	4,000	4,000
	Attached Labour	7,000	7,000	7,000
	Interest on Working Capital			4,617
	Sub Total	4,96,000	11,000	15,617
4	July			
	Plants (125 per acre × 8)	50,000		
	Attached Labour	7,000	7,000	7,000
	Hired Labour	4,000		
	Sub Total	61,000	7,000	7,000
5	August			
	Nutrients	14,000	14,000	10,700
	Misc. Works	7,500	7,500	
	Attached Labour	7,000	7,000	7,000
	Sub Total	28,500	28,500	17,700
6	September			
	Interculture & Weed Control	7,500	7,500	5,075
	Pesticides	7,500	7,500	10,500
	Plant Support Wood			10,000
	Electricity	4,000	4,000	4,000
	Attached Labour	7,000	7,000	7,000
	Hired Labour	5,000	5,000	9,000
	Interest on Working Capital			4,617
	Sub Total	31,000	31,000	50,192
7	October			
	Replacement Plants	10,000		
	Nutrients	14,000	14,000	
	Pesticides			10,500
	Misc. Works	7,500	7,500	

	Attached Labour	7,000	7,000	7,000
	Hired Labour	3,000	3,000	
	Sub Total	41,500	31,500	17,500
8	November			
	Interculture & Weed Control	7,500	7,500	5,075
	Pesticides	7,500	7,500	
	Attached Labour	7,000	7,000	7,000
	Hired Labour	4,000	4,000	3,000
	Sub Total	26,000	26,000	15,075
9	December			
	Nutrients	12,000	12,000	
	Pesticides			9,000
	Misc. Works	3,750	3,750	
	Electricity	4,000	4,000	4,000
	Attached Labour	7,000	7,000	7,000
	Hired Labour	2,000	2,000	3,000
	Interest on Working Capital			4,617
	Sub Total	28,750	28,750	27,617
10	January			
	Misc. Works	3,750	3,750	
	Attached Labour	7,000	7,000	7,000
	Hired Labour	2,000	2,000	3,000
	Sub Total	12,750	12,750	10,000
11	February			
	Misc. Works	3,750	3,750	
	Attached Labour	7,000	7,000	7,000
	Hired Labour	2,000	2,000	3,000
	Sub Total	12,750	12,750	10,000
12	March			
	Misc. Works	3,750	3,750	
	Electricity	4,000	4,000	4,000
	Attached Labour	7,000	7,000	7,000
	Hired Labour	2,000	2,000	3,000
	Interest on Working Capital			4,617
	Sub Total	16,750	16,750	18,617
B	Summary			
	April	42,63,000	7,000	53,000
	May	2,52,000	7,000	7,000

June	4,96,000	11,000	15,617
July	61,000	7,000	7,000
August	28,500	28,500	17,700
September	31,000	31,000	50,192
October	41,500	31,500	17,500
November	26,000	26,000	15,075
December	28,750	28,750	27,617
January	12,750	12,750	10,000
February	12,750	12,750	10,000
March	16,750	16,750	18,617
Total	52,70,000	2,20,000	2,49,318

The total cash outflow during the first year would aggregate to ₹52.70 lakhs, consisting of ₹50.50 lakhs of initial investment and annual nurturing cost of ₹2.20 lakhs. The nurturing outflow would continue at the rate of ₹2.20 lakhs per year till the 5th year. The annual operating outflow would work out to ₹2.49 lakhs from the 6th year to the 18th year. Such of these annual cash budgets would go a long way in facilitating the farmers in planning, coordinating and controlling their cash outflows.



Decision Making Using Probability

Case Study 1

SCM can boost up the Benefits from IQF

Introduction

SA Limited is a moderate MSME located in rural Maharashtra. It is engaged in the business of processing of raw vegetables through the technique of Individual Quick Freezing (IQF). IQF enhances the shelf-life of vegetables to about 18 to 24 months from the date of the packing. The frozen vegetables are to be stored below minus twenty degrees centigrade. Fresh vegetables of various kinds are the raw material. The capacity of the unit is 600 MT of vegetables per month.

IQF process is continuous and, broadly, consists of:

- i. Raw Material Preparation
- ii. Blanching
- iii. Freezing
- iv. Primary Packing
- v. Cold Storage
- vi. Secondary Packing



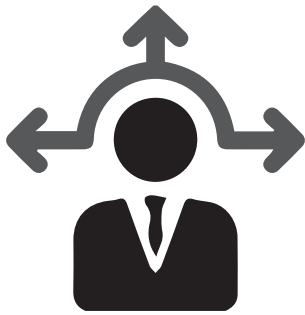
Raw Material Preparation involves cleaning, washing, peeling, cutting, slicing, etc. of the fresh vegetables; which are then taken up for blanching wherever necessary; and processed through the freezing machinery. Primary packing consists of bulk packing for cold storage and secondary packing consists of custom packing made to orders. The frozen vegetables can be sold in domestic market and exported to various developed countries all over the world. Refrigerated containers are used for the carriage of finished goods.

Lucrative aspects of the business include:

1. **Increasing Demand:** Vegetables are essential commodities. Demand is ever increasing with the increase in population and change in consumption pattern i.e., more inclination towards the vegetarian food. As a consequence, the demand and supply gap has also been increasing.

2. **Cheap Labour:** The unit is located in a rural area. Cheap and trained labour is available locally.
3. **Export Potential:** Export market exists to absorb the 100% production capacity of the plant.
4. **Strong Bottom Line:** The industry is quite profitable and the bottom line is very strong.

SJC Institute



Activity Based Cost Management

Case Study 1

How Xu Ji achieved standardisation in working practices and processes (CIMA case study, 2011)

The Chinese electricity company Xu Ji used ABC to capture direct costs and variable overheads, which were lacking in the state-owned enterprise's (SOE) traditional costing systems. The ABC experience has successfully induced standardisation in their working practices and processes. Standardisation was not a common notion in Chinese culture or in place in many Chinese companies. ABC also acts as a catalyst to Xu Ji's IT developments – first accounting and office computerisation, then ERP implementation.

Prior to the ABC introduction in 2001, Xu Ji operated a traditional Chinese state-enterprise accounting system. A large amount of manual bookkeeping work was involved. Accounting was driven predominantly by external financial reporting purposes, and inaccuracy of product costs became inevitable. At this time, Xu Ji underwent a series of flotations following China's introduction of free market competition.

The inaccuracy of the traditional costing information seriously impeded Xu Ji's ability to compete on pricing. The two main tasks for the ABC system were to: trace direct labour costs directly to product and client contracts; and allocate manufacturing overheads on the basis of up-to-date direct labour hours to contracts.

The learning was that the common 'top-down' management style and organisational culture among SOEs worked well when instigating innovative ideas and inducing corporate-wide learning. Top management's commitment to trying out new management ideas and investing in new technology has been the unique feature.

Case Study 2

ABC in Automobile Industry

Many automotive companies use activity-based costing to determine their costs and pricing. For example, Ford decided the company needed this sort of determined, specific look into individual cost steps to see which steps could be eliminated or improved. Chrysler has seen hundreds of millions of dollars saved using activitybased costing to identify and eliminate useless, inefficient or redundant steps in its production, which has also significantly streamlined product development. Since car and truck manufacturing is an established industry, where the manufacturing process has many steps that combine to make the final product, activity-based costing is the ideal analysis.

Case Study 3

Activity-Based Costing of Coca-Cola

Coca-Cola is another company that uses activity-based costing to determine its price points. Coca-Cola offers a large portfolio of products and carries a huge amount of inventory, which can be a significant portion of production cost that is often overlooked. Coca-Cola has used activity-based costing to evaluate the differences between its bigger, world-wide products and its specialty, regionalized products that it may not offer on the global market. This understanding of how production costs are different between established, familiar types and specialized types has enabled them to set price points in each market that ensure them significant profit.



Benchmarking

Case Study 1

Drive thru Practice

A quick service (fast food) restaurant chain dependent upon speedy, accurate service in the drive-thru to maximize efficiency, cut costs and increase profits may study the drive-thru practices of key competitors. Every second gained without sacrificing customer quality allows the firm to increase profits. Over the years, competitors have consistently innovated in their drive-thru operations with configuration, number of windows, menu and speaker boards and ordering approaches in an attempt to improve in this area. They are constantly watching and benchmarking against each other.



Pal's Sudden Service, a small hamburger and hot dog chain and a Baldrige Quality Award winner, is very successful at achieving best-in-class performance for drive-thru and overall restaurant operations. Pal's does not offer sit-down service inside its restaurants. Instead, customers pull up to a window, place their orders face-to-face with an employee (no scratchy loudspeakers), pull around to the other side of the facility, take their bag, and drive on. All this happens at a lightning pace – an average of eighteen seconds at the handout window to place an order, an average of twelve seconds at the drive-up window to receive the order. That's four times faster than the secondfastest quick serve restaurant, which requires more than a minute on average to take an order.

Many companies in the fast-food market use Pal's as a best-in-class benchmark for their own firms. It is no wonder that Pal's opened an educational institute to train other organisations.

NOTES

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